McKinsey-Tarski Algebras: An Alternative Approach to Pointfree Topology

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An *interior algebra* is a pair $(B, \Box)$ where B is a boolean algebra and $\Box$ is a unary function on B satisfying the well-known Kuratowski axioms: $\Box a \leq a$, $\Box\Box a \leq \Box a$, $\Box(a \wedge b) = \Box a \wedge \Box b$, and $\Box 1 = 1$. Interior algebras were introduced by McKinsey and Tarski in 1944 and have since been studied extensively by numerous authors. We call a complete interior algebra a *McKinsey-Tarski algebra* or *MT-algebra*, and propose the category MT of MT-algebras as an alternative, more expressive, language to study point-free topology. We show that taking the open elements of an MT-algebra yields an essentially surjective functor from MT to the category Frm of frames. We also show that the well-known dual adjunction between Frm and the category Top of topological spaces extends to a dual adjunction between MT and Top, which restricts to a dual equivalence between Top and the category SMT of spatial MT-algebras. This extends the well-known dual equivalence between the categories of spatial frames and sober spaces. We also present the study of separation axioms in the language of MT-algebras, which is more expressive than the corresponding language of frames. In addition, we develop the Hofmann-Mislove theorem for MT-algebras, which allows us to obtain dual adjunctions and dual equivalences for the categories of locally compact spaces and compact Hausdorff spaces, and their corresponding categories of MT-algebras. This yields an alternative proof of Hofmann-Lawson and Isbell dualities in frame theory. We show that unlike the situation in frames, in MT-algebras spatiality is not a consequence of local compactness. In the talk we explain the reason for this discrepancy and show that it disappears once we add the T_D separation axiom, which is easily expressible in the language of MT-algebras.

Keywords: Pointfree topology, Duality theory, Interior algebras