That Damned Problem of Eilenberg and Schützenberger

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In a 1976 paper, Samuel Eilenberg and M.P. Schützenberger posed a provocative problem about finite semigroups, which Mark Sapir solved in 1988: if the pseudovariety generated by a finite semigroup S is finitely axiomatizable (relative to the class of all finite semigroups), is this so because the variety generated by S is finitely axiomatizable? (Sapir's answer: yes). The same question can be posed for any finite algebra in a finite signature, and this expanded question is what universal algebraists call the Eilenberg-Schützenberger problem.

In this talk I will state the problem more carefully and explain just how far we are from a solution. I will describe an "obvious reason" for a positive answer which, so far, works in all special cases for which the problem has been answered. Time permitting, I will describe our recent work on this problem in the context of finite-dimensional nonassociative algebras over finite fields.

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