Systems of Term Equations Over Finite Algebras

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For a fixed finite algebra A, we consider the decision problem $\operatorname{SysTerm}(A)$: does a given system of term equations have a solution in A? This can be formulated as a constraint satisfaction problem (CSP) with relations the graphs of the basic operations of A. From the complexity dichotomy for CSP due to Bulatov and Zhuk, it follows that $\operatorname{SysTerm}(A)$ for a finite algebra A is in P if A has a not necessarily idempotent Taylor polymorphism and is NP-complete otherwise. We show more explicitly that for a finite algebra A in a congruence modular variety, $\operatorname{SysTerm}(A)$ is in P if the core of A is abelian and is NP-complete otherwise. Given A by the graphs of its basic operations, this condition can be decided in quasi-polynomial time.

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