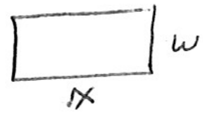
1. A rectangle has perimeter 30 inches. Express the area, A, of the rectangle as a function of the length, x, of one of its sides.

(a) $A = 30x - x^2$
(b) $A = 15x - x^2$
(c) $A = x^2 - 30x$
(d) $A = x^2 - 15x$
(e) $A = 2x + \frac{60}{x}$



$P = 30 = 2x + 2w \Rightarrow 2w = 30 - 2x$
$w = 15 - x$

$A = xw = x(15 - x) = 15x - x^2$

2. Explain how to get the graph of $y = f(x - 2)$ from the graph of $y = f(x)$.

(a) Move the graph up two units
(b) Move the graph down two units
(c) Move the graph right two units.
(d) Move the graph left two units.
(e) Reflect the graph across the x-axis

3. A ball is thrown into the air . Its height in feet after t seconds is given by $y = 40t - 16t^2$. What is its velocity when $t = 2$?

(a) $-$ 24 ft/sec.
(b) $-$ 8 ft/sec.
(c) 8 ft/sec.
(d) 24 ft/sec.
(e) 40 ft/sec

$V = y' = 40 - 32t$
when $t = 2$, $V = 40 - 32(2) = -24$ ft/sec

4. Let $f(x) = (5x + 2)e^{-x}$. Find an equation of the line tangent to the graph of f at the point where $x = 0$.

(a) $y = 3x + 2$
(b) $y = -3x + 2$
(c) $y = 5x$
(d) $y = -5x$
(e) $y = -5x + 2$

$y' = 5 \cdot e^{-x} + (-e^{-x})(5x + 2)$
$= 5e^{-x} - 5xe^{-x} - 2e^{-x}$
$= e^{-x}(3 - 5x)$
when $x = 0$, $y' = e^0(3 - 0) = \boxed{3 = m}$

eq of tan : $y - y_1 = m(x - x_1)$, $\boxed{x = 0}$, $y = (5 \cdot 0 + 2)e^0$
$\boxed{y = 2}$

$y - 2 = 3(x - 0)$
$y = 3x + 2$

5. Suppose $f(1) = 1$, $f'(1) = 2$, $g(1) = 3$ and $g'(1) = 4$.
Let $h(x) = \frac{f(x)}{g(x)}$. Evaluate $h'(1)$.

[✓ (a) 2/9]
(b) $-2/9$
(c) $-1/9$
(d) 1/9
(e) 1/3

$$h' = \frac{f'g - g'f}{g^2}$$

$$h'(1) = \frac{2\cdot 3 - 4\cdot 1}{3^2} = \frac{2}{9}$$

6. Let $f(x) = e^{2x}cos(3x)$. Find the derivative, $f'(0)$.
(Note that $sin\, 0 = 0$ and $cos\, 0 = 1$.)

(a) 0
(b) 1
(c) -1
[✓ (d) 2]
(e) -2

$$f'(x) = 2e^{2x}\cdot\cos(3x) + [-3\sin(3x)]\cdot e^{2x}$$

$$= e^{2x}(2\cos 3x - 3\sin(3x))$$

$$f'(0) = e^0(2\cos 0 - 3\sin 0)$$

$$= e^0(2 - 0) = 2$$

7. Suppose that $x^2y + y^3 = 3x - 1$. Find the derivative $\frac{dy}{dx}$ at the point $(1, 1)$.

[✓ (a) $\frac{1}{4}$]
(b) $-\frac{1}{4}$
(c) $\frac{1}{2}$
(d) $-\frac{1}{2}$
(e) 0

$$2xy + y'x^2 + 3y^2y' = 3$$

$$y'(x^2 + 3y^2) = 3 - 2xy$$

$$y' = \frac{3 - 2xy}{x^2 + 3y^2} = \frac{3 - 2(1)(1)}{1^2 + 3(1)^2} = \frac{1}{4}$$

8. On which interval below is the function $f(x) = 2x^3 + 9x^2 - 24x$ decreasing? $f' < 0$

(a) $(-\infty, -3)$
(b) $(-3, 1)$
[✓ (c) $(-4, 1)$]
(d) $(-3, 2)$
(e) $(2, \infty)$

$$f'(x) = 6x^2 + 18x - 24$$

$$= 6(x^2 + 3x - 4)$$

$$= 6(x+4)(x-1)$$

+ − +
−4 1

$f' < 0$ on the interval $(-4, 1)$

9. Consider the function $f(x) = 4x^3 - 3x^2 - 18x$ on the interval $[0, 2]$. For which value of x does the the function attain its minimum value?

(a) The minimum value occurs at $x = 0$
(b) The minimum value occurs at $x = \frac{1}{2}$
(c) The minimum value occurs at $x = 1$
(d) The minimum value occurs at $x = \frac{3}{2}$
(e) There is no minimum value on this interval

$f'(x) = 12x^2 - 6x - 18$
$= 6(2x^2 - x - 3) = 0$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(2)(-3)}}{2(2)} = \frac{1 \pm 5}{4} = \frac{3}{2}, -1$

critical #'s : $-1, \frac{3}{2}$

$f(-1) = 4(-1)^3 - 3(-1)^2 - 18(-1) = -4 - 3 + 18 = \boxed{11}$ but -1 is not in the interval $[0, 2]$

$f(\frac{3}{2}) = 4(\frac{27}{8}) - 3(\frac{9}{4}) - 18(\frac{3}{2}) = \frac{27}{2} - \frac{27}{4} - 27 = \frac{54 - 27 - 108}{4} = -\frac{81}{4} = \boxed{-20.25}$ → min

$f(0) = \boxed{0}$ and $f(2) = 4(8) - 3(4) - 18(2) = 32 - 12 - 36 = \boxed{-16}$

10. Use L'Hospital's rule to evaluate $\lim_{x \to 0} \frac{\cos(ax) - 1}{x^2}$.

(a) $-\frac{a}{2}$
(b) $\frac{a}{2}$
(c) $-\frac{a^2}{2}$
(d) $\frac{a^2}{2}$
(e) The limit does not exist.

$(\frac{0}{0}) = \lim_{x \to 0} \frac{-a \sin ax}{2x} \quad (\frac{0}{0})$

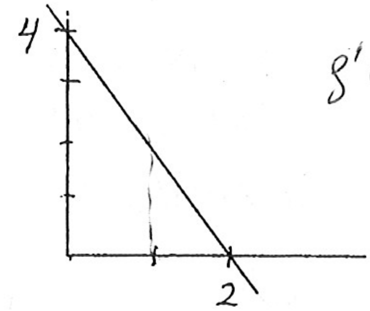
$= \lim_{x \to 0} \frac{-a^2 \cos ax}{2} = -\frac{a^2}{2}$

11. Find the general antiderivative of the function $f(x) = \sin x + 3e^x + 3x^2$

(a) $\cos x + 3e^x + 6x + C$
(b) $-\cos x + 3e^x + 6x + C$
(c) $\cos x + 3xe^{x-1} + x^3 + C$
(d) $-\cos x + 3\frac{e^{x+1}}{x+1} + x^3 + C$
(e) $-\cos x + 3e^x + x^3 + C$

12. The graph of $y = f(x)$ is shown. Estimate the value of the derivative $f'(1)$.

(a) -2
(b) -1
(c) 0
(d) 1
(e) 2



$f'(1) = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{-4}{2} = -2$

PART II

1. Let $f(x) = 2x + 1$. Note that this function is invertible. Find $f^{-1}(3)$.
(a) -1
(b) 1
(c) -3
(d) $\frac{1}{3}$
(e) 7

$y = 2x+1$
$x = 2y+1$
$x-1 = 2y \Rightarrow y = \frac{x-1}{2} = f^{-1}(x) \Rightarrow f^{-1}(3) = \frac{3-1}{2} = 1$

2. Which of the following functions are one to one? Horizontal line test
I. $f(x) = sin\, x$, the domain is $-\infty < x < \infty$ not
II. $g(x) = \frac{1}{x}$, the domain is all x except $x = 0$. yes
III. $h(x) = x^2$, the domain is $-\infty < x < \infty$ not
(a) I only
(b) II only
(c) III only
(d) II and III only
(e) None of these functions are one to one

3. Let $f(x) = \begin{cases} x+1, x \leq 1 \\ 3x-2,\ x > 1 \end{cases}$. Evaluate $\lim_{x \to 1^+} f(x)$. $= 3(1) - 2 = 1$

(a) The limit does not exist.
(b) 0
(c) 1.
(d) 2
(e) 3

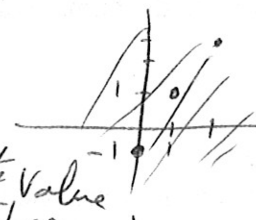
4. Evaluate $\lim_{x \to 2} \frac{x^2+3x-10}{x-2}$ $= \lim \frac{(x+5)(x-2)}{(x-2)} = \lim_{x \to 2} (x+5) = 7$
(a) The limit does not exist.
(b) 1
(c) 3
(d) 5
(e) 7

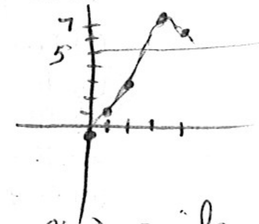
5. Given that $\lim_{x \to 3} f(x) = 2$ and $\lim_{x \to 3} g(x) = -1$. Evaluate $\lim_{x \to 3} \frac{f(x)}{g(x)+x}$ $= \frac{2}{-1+3} = 1$

(a) The limit does not exist
(b) 0
(c) 1
(d) 2
(e) 3

6. Suppose that f is a continuous function and that $f(0) = -1$, $f(1) = 1$, $f(2) = 3$, $f(3) = 7$, and $f(4) = 6$. Which of the following intervals MUST contain a solution of $f(x) = 5$?

(a) [0, 1] $3 < 5 < 7$
(b) [1, 2] $f(2) < f(x) < f(3)$
(c) [2, 3]   so $2 < x < 3$ (the Intermediate Value Theorem)
(d) [3, 4]
(e) There may be no solution of $f(x) = 5$.





$f(x) = 5$ if $2 < x < 3$

7. Suppose that $f(x) = \begin{cases} \frac{x^2-1}{x-1}, & x \neq 1 \\ k, & x = 1 \end{cases}$. Determine the value of k that makes f continuous at $x = 1$.

$\lim_{x \to 1} \frac{x^2-1}{x-1} = f(1) = k$

$= \lim \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1}(x+1) = 2$

so $k = 2$

(a) 3
(b) 2
(c) 1
(d) 0
(e) The function f is never continuous at $x = 1$.

8. Evaluate $\lim_{x \to \infty} \frac{x^2+3x+1}{x^3+5x+2}$ $= \lim \frac{\frac{x^2}{x^3} + \frac{3x}{x^3} + \frac{1}{x^3}}{\frac{x^3}{x^3} + \frac{5x}{x^3} + \frac{2}{x^3}} = \lim \frac{\frac{1}{x} + \frac{3}{x^2} + \frac{1}{x^3}}{1 + \frac{5}{x^2} + \frac{2}{x^3}} = \frac{0}{1} = 0$

(a) 0
(b) 0.1
(c) $\frac{1}{x}$
(d) $\frac{1}{2}$
(e) ∞

9. $\lim\limits_{h \to 0} \frac{(1+h)^2+(1+h)-2}{h}$ represents $f'(a)$ for some function f and some number a. Which of the following is correct?

(a) $f(x) = x^2$, $a = 1$
(b) $f(x) = x^2$, $a = 2$
(c) $f(x) = x^2 + x$, $a = 1$
(d) $f(x) = x^2 + x$, $a = 2$
(e) None of the above

$a = 1$

$f(x) = x^2 + x$

10. Find the linear approximation for the function $f(x) = \sqrt{x}$ at $a = 4$.

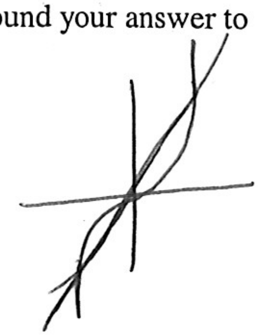
(a) $L(x) = \frac{1}{2}(x-2)+4$
(b) $L(x) = \frac{1}{2}(x-4)+2$
(c) $L(x) = \frac{1}{4}(x-2)+4$
(d) $L(x) = \frac{1}{4}(x-4)+2$
(e) $L(x) = \frac{1}{8}(x-2)+4$

$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$, $f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$f(4) = \sqrt{4} = 2$

11. Use your calculator to find all solutions of $x^3 = 6x + 5$. How many solutions are there? What is the largest value of x that satisfies this equation? (Round your answer to two decimal places.)

(a) There is 1 solution. The solution is $x = -1$.
(b) There is 1 solution. The solution is $x = 2.91$
(c) There are 2 solutions. The largest solution is $x = 3.12$
(d) There are 3 solutions. The largest solution is $x = 2.79$.
(e) There are 3 solutions. The largest solution is $x = 1.81$.



12. We wish to solve $x^3 - 20 = 0$ using Newton's method. Use $x_1 = 3$ as your initial approximation and find x_2, the second approximation. (You are not being asked for the exact solution.) Round your answer to two decimal places.

(a) 2.71
(b) 2.72
(c) 2.73
(d) 2.74
(e) 2.75

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

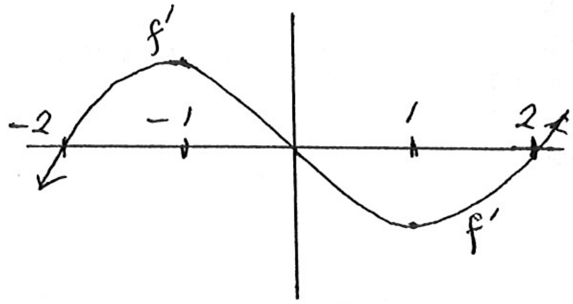
$f(x) = x^3 - 20$, $f(3) = 7$

$f'(x) = 3x^2$, $f'(3) = 27$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

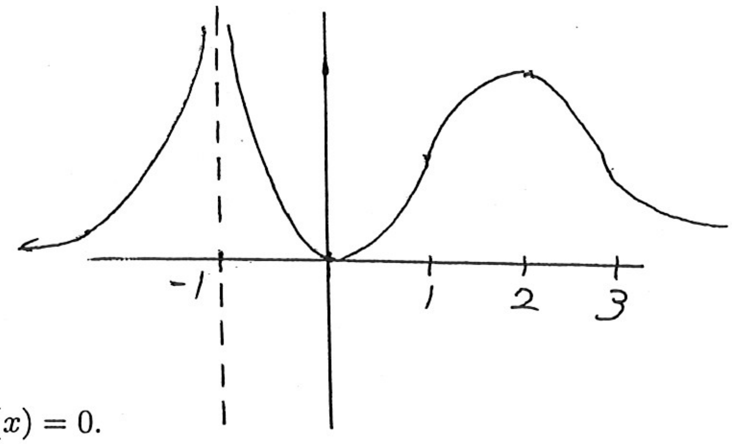
$= 3 - \frac{7}{27} = 2.74$

13. The graph of $y = f'(x)$ is shown. On which of the following intervals is f decreasing?
(You are being asked about f, but you are shown the graph of the **derivative** of f.)
(a) $(-1, 1)$
(b) $(0, 2)$
(c) $(-2, 2)$
(d) $(-\infty, 0)$
(e) $(0, \infty)$



f decreasing if $f' < 0$ on $(-\infty, -2) \cup (0, 2)$

1. The graph of $y = f(x)$ is shown



(a) On which interval(s) is f increasing?

$(-\infty, -1) \cup (0, 2)$

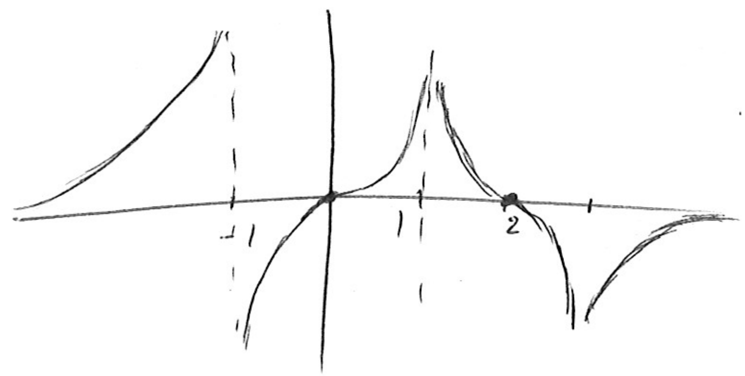
(b) On which interval(s) is f concave down?

$(1, 3)$

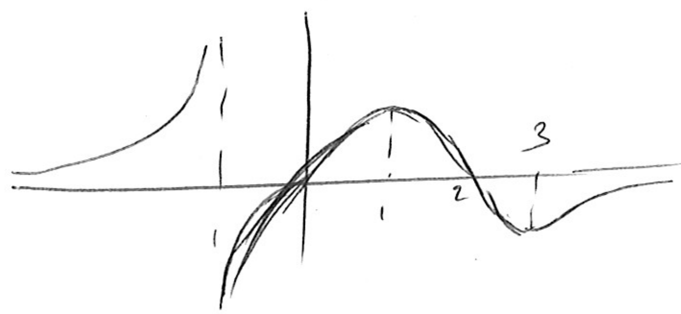
(c) List the x-coordinates of any points where $f'(x) = 0$.

$x = 0, 2$

(d) Sketch a graph of $y = f'(x)$



Or

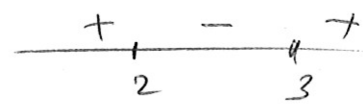


2. A particle moves along the x-axis according to the law of motion $x(t) = 2t^3 - 15t^2 + 36t$ where t is measured in seconds and x is measured in feet.

(a) Find the velocity at time t.

$$V = \boxed{6t^2 - 30t + 36} = 6(t^2 - 5t + 6) = 6(t-2)(t-3)$$

(b) Find the time interval(s) during which the particle is moving to the right. Find the time interval(s) during which the particle is moving to the left.

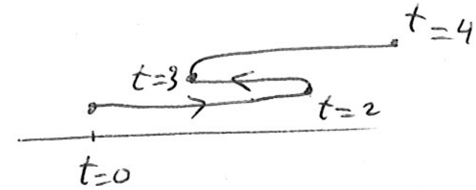


moving right $\Rightarrow$ $v > 0$

if $t < 2$ or $t > 3$

interval $(0, 2) \cup (3, +\infty)$

moving left $\Rightarrow$ $v < 0$ on $(2, 3)$



(c) Find the total distance traveled between $t = 0$ and $t = 4$.

$x(0) = 0$

$x(2) = 2(2^3) - 15(2^2) + 36(2) = 16 - 60 + 72 = 28$

$x(3) = 2(3^3) - 15(3^2) + 36(3) = 54 - 135 + 108 = 162 - 135 = 27$

$x(4) = 2(4^3) - 15(4^2) + 36(4) = 128 - 240 + 144 = 272 - 240 = 32$

total distance $= |x(2) - x(0)| + |x(3) - x(2)| + |x(4) - x(3)|$

$= |28 - 0| + |27 - 28| + |32 - 27| = 28 + 1 + 5 = \boxed{34}$

(d) Find the time interval(s) during which the acceleration is positive.

$a = v' = 12t - 30 > 0 \Rightarrow t > \frac{30}{12} = \frac{5}{2} = 2{,}5$

3. Let k be a positive constant and consider the function $f(x) = x\,ln(kx)$. (Note that this function is defined only for $x > 0$.)

(a) Find the first derivative $f'(x)$ $= 1 \cdot \ln(kx) + x \cdot \frac{k}{kx} = \ln(kx) + 1$

(b) Find the second derivative $f''(x)$ $= \frac{k}{kx} + 0 = \frac{1}{x}$

(c) Determine the interval(s) (if any) where the function f is concave up and the interval(s) (if any) where f is concave down. (Remeber that that $f(x)$ is defined only for $x > 0$)

concave up if $f'' > 0$ on $(0, +\infty)$

$f''(x) = \frac{1}{x} > 0 \Rightarrow \frac{1}{x} > 0$

$\Rightarrow x > 0$

but $x > 0$ (Domain)

~~so $f''(x)$~~

since $x > 0$, f'' cannot be negative so f cannot be concave down. $\boxed{\text{always upward}}$

(d) Use Calculus (not a graphing calculator) to determine the critical numbers of f and the interval(s) where f is increasing. (You must show your work to get credit.)

Critical #'s when $f'(x) = 0 \Rightarrow \ln(kx) + 1 = 0 \Rightarrow \ln(kx) = -1$

$\Rightarrow e^{\ln(kx)} = e^{-1} \Rightarrow kx = \frac{1}{e} \Rightarrow \boxed{x = \frac{1}{ke}}$ critical #

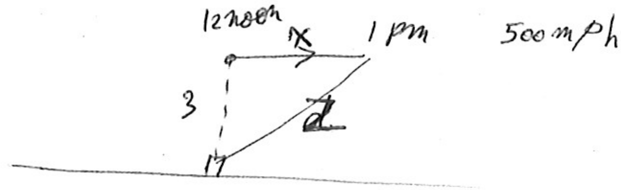
increasing when $f' > 0 \Rightarrow \ln(kx) + 1 > 0 \cdots\cdots x > \frac{1}{ke}$

4. A plane flying horizontally at 500 miles per hour and an altitude of 3 miles, passes directly over the Fretwell building at noon. We wish to find the rate at which the distance from the plane to the Fretwell building is increasing at 1 PM. We will do this in several steps.

(a) draw a diagram related to this situation. Label the important distances with constants or variable names.

distance = z

horizontal distance = x



(b) Find an equation relating the variables in your diagram.

$z^2 = x^2 + 9$

(c) How far has the plane traveled horizontally between noon and 1 PM?

$x = 500 \times 1 = 500$

(d) Use the information from (b) and (c) to determine the rate at which the distance from the plane to the Fretwell building is increasing at 1 PM.

at 1 pm: $z^2 = 500^2 + 9$

$z = 500.009$

$(z^2)' = (x^2+9)'$

$\cancel{2} z z' = \cancel{2} x x' + 0$

$500.009 z' = 500(500)$

$z' = \frac{500^2}{500.009} = 499.99$

5. A function f satisfies the conditions below. Use this information to sketch a graph of f.

> The domain of f is all real numbers except $x = 0$.
> $x = 0$ is a vertical asymptote
> $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0.$ $\longrightarrow$ HA : $y = 0$
> $f(-2) = \frac{1}{2}$, $f(-1) = 2$, $f(1) = 1$.
> $f'(x) > 0$ on the interval $(-\infty, -1)$
> $f'(x) < 0$ on the intervals $(-1, 0)$ and $(0, \infty)$
> $f''(x) > 0$ on the intervals $(-\infty, -2)$ and $(0, \infty)$ $\longrightarrow$ inflection pt
> $f''(x) < 0$ on the interval $(-2, 0)$

(a) List any horizontal asymptotes for f

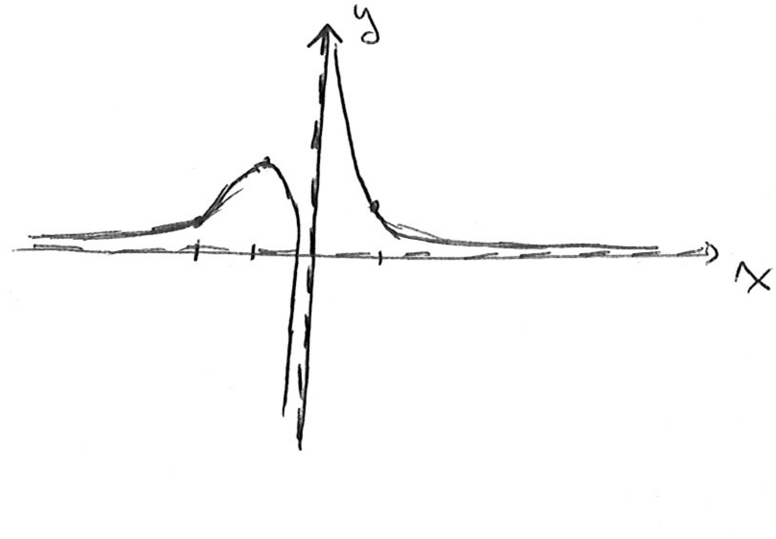
$y = 0$

(b) List the x-coordinates of any inflection points for f

$x = -2$, $\cancel{0}$ not in domain

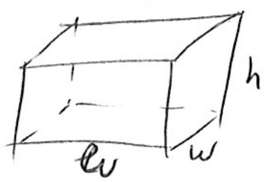
(c) Sketch a graph of the function f.

x	$-\infty$ to -2	-2 to -1	-1 to 0	0 to 1	1 to ∞
f'	+	+	−	−	−
f''	+	−	−	+	+
f	↗ ∪	↗ ∩	↘ ∩	↘ ∪	↘ ∪



6. A box with a square base and an open top must have a volume of 6400 cubic inches. We wish to find the dimensions of the box that minimize the amount of material used. (This is the same as minimizing the surface area of the box.)

(a) draw a diagram of the box labeling the length, width, and height with appropriate variable names. Be sure to take into account all the information that was given above.



$V = w^2 h = 6400 \Rightarrow h = \frac{6400}{w^2}$

(b) Write an equation that relates the volume to the variables you introduced in part (a)

$V = w^2 h = 6400$

(c) Write an equation that expresses the surface area of the box in terms of the variables in part (a).

$A = w^2 + 4wh = w^2 + 4w \cdot \frac{6400}{w^2} = w^2 + \frac{25600}{w}$

(d) Use the answers to (b) and (c) to determine the dimensions of the box that minimize the surface area.

minimize the area : $A' = 2w - \frac{25600}{w^2} = 0$

$2w = \frac{25600}{w^2}$

$w^3 = 12800$

$w = \sqrt[3]{12800}$