MATH 1241 **FINAL EXAM** **FALL 2011**

Part I, No Calculators Allowed

1. Find an equation for the line that is tangent to the graph of $f(x) = x^3 - 5x + 8$ at $x = 2$. $x_1 = 2$

(a) $y = x - 4$
(b) $y = 2x - 4$
(c) $y = 2x + 2$
(d) $y = 7x - 8$
(e) $y = 7x - 14$

$f'(x) = 3x^2 - 5$

$m = f'(2) = 3(2)^2 - 5 = 7$

$y_1 = f(2) = (2)^3 - 5(2) + 8 = 6$

$y - y_1 = m(x - x_1)$

$y - 6 = 7(x - 2)$

$\boxed{y = 7x - 8}$

2. Which of the following is the derivative of $g(x) = \sqrt{5x+3}$?

(a) $\dfrac{5}{2\sqrt{5x+3}}$
(b) $\dfrac{1}{\sqrt{5}}$
(c) $\dfrac{1}{2\sqrt{5x+3}}$
(d) $\dfrac{1}{2\sqrt{5}}$
(e) $\dfrac{2}{5\sqrt{5x+3}}$

$\frac{d}{dx}\left[(5x+3)^{\frac{1}{2}}\right] = \frac{1}{2}(5x+3)^{-\frac{1}{2}} \frac{d}{dx}[5x+3]$

$= \frac{1}{2}(5x+3)^{-\frac{1}{2}} \cdot 5 = \frac{5}{2(5x+3)^{1/2}}$

$= \boxed{\frac{5}{2\sqrt{5x+3}}}$

3. Evaluate the limit: $\lim_{x\to 2} \dfrac{x^2 - 7x + 10}{5x - 10}$.

(a) -0.6
(b) -0.4
(c) 0
(d) 1
(e) Does not exist

$\overset{\left(\frac{0}{0}\right)}{=} \lim_{x\to 2} \frac{2x-7}{5} = \frac{2(2)-7}{5} = -\frac{3}{5}$

$= \boxed{-0.6}$

4. Which of the following is the derivative of $f(x) = \dfrac{x^2 - 3}{5x + 7}$?

(a) $\dfrac{2x}{5}$
(b) $\dfrac{2x}{25}$
(c) $\dfrac{5x^2 + 14x + 15}{(5x+7)^2}$
(d) $\dfrac{-5x^2 - 14x - 15}{(5x+7)^2}$
(e) $\dfrac{-5x^2 - 14x + 15}{(5x+7)^2}$

$\frac{d}{dx}\left[\frac{x^2-3}{5x+7}\right] = \frac{(5x+7)\frac{d}{dx}[x^2-3] - (x^2-3)\frac{d}{dx}[5x+7]}{(5x+7)^2}$

$= \frac{(5x+7)\cdot 2x - (x^2-3)\cdot 5}{(5x+7)^2}$

$= \frac{10x^2 + 14x - 5x^2 + 15}{(5x+7)^2}$

$= \frac{5x^2 + 14x + 15}{(5x+7)^2}$

5. Which of the following is the derivative of $f(x) = \cos(x^2 + 3)$?

(a) $-\sin(x^2 + 3)$
(b) $-2x\sin(x^2 + 3)$
(c) $2x\sin(x^2 + 3)$
(d) $-\sin(2x)$
(e) $\sin(2x)$

$\frac{d}{dx}\left[\cos(x^2+3)\right] = -\sin(x^2+3)\frac{d}{dx}\left[x^2+3\right]$

$= -\sin(x^2+3)\cdot 2x$

6. For what values of x, if any, does the function $f(x) = 3x^4 - 16x^3 + 24x^2 + 10$ have a local minimum?

(a) There is no local minimum
(b) Only at $x = -2$
(c) Only at $x = 0$
(d) Only at $x = 2$
(e) At $x = 0$ and at $x = 2$

Decr ; Decr
2

$f'(x) = 12x^3 - 48x^2 + 48x$

$12x(x^2 - 4x + 4) = (x-2)(x-2) = 0$

$\boxed{x=0}$
$x=2$

$f''(x) = 36x^2 - 96x + 48$ | $f''(2) = 36(2)^2 - 96(2) + 48 = 0$

$f''(0) = 48$
✓
be
min

7. The derivative of the function $f(x)$ is given by $f'(x) = 6x^2 + 3$. Find a formula for the function $f(x)$ given that $f(1) = 25$.

(a) $f(x) = 18x^3 + 3x + 4$
(b) $f(x) = 6x^3 + 19$
(c) $f(x) = 6x^3 + 3x + 16$
(d) $f(x) = 2x^3 + 23$
(e) $f(x) = 2x^3 + 3x + 20$

Antiderivative

$f(x) = 2x^3 + 3x + C$

$25 = f(1) = 2(1)^3 + 3(1) + C$

$C = 20$ $\boxed{f(x) = 2x^3 + 3x + 20}$

8. A particle is traveling around the circle $x^2 + y^2 = 25$ where x and y are measured in inches. At the instant the particle is at the point $(3, 4)$, $dx/dt = 7\ in/sec$. Find dy/dt at this time.

(a) $-42\ in/sec$
(b) $-21\ in/sec$
(c) $-14\ in/sec$
(d) $-7\ in/sec$
(e) $-5.25\ in/sec$

Related Rates,

$x^2(t) + y^2(t) = 25$

$2x(t)x'(t) + 2y(t)y'(t) = 0$

$y'(t) = -\frac{x(t)x'(t)}{y(t)}$

$y'(t_0) = -\frac{x(t_0)x'(t_0)}{y(t_0)}$

$y'(t) = -\frac{(3)(7)}{(4)}$

$y'(t) = -\frac{21}{4} = -5.25$

9. Which of the following is the derivative of $g(x) = x^3e^{4x+1}$?

(a) $3x^2e^{4x}$
(b) $12x^2e^{4x+1}$
(c) $3x^2e^{4x+1} + x^3e^{4x+1}$
(d) $3x^2e^{4x+1} + 4x^3e^{4x+1}$
(e) $3x^2e^{4x+1} + x^3e^{4x}$

$\frac{d}{dx}\left[x^3e^{4x+1}\right] = x^3\frac{d}{dx}\left[e^{4x+1}\right] + e^{4x+1}\frac{d}{dx}\left[x^3\right]$

$= x^3\cdot e^{4x+1}\frac{d}{dx}\left[4x+1\right] + e^{4x+1}\cdot 3x^2$

$= x^3e^{4x+1}\cdot 4 + 3x^2e^{4x+1} = 3x^2e^{4x+1} + 4x^3e^{4x+1}$

10. Let $f(x) = x^3 + 8x + 2$. Use Newton's method to approximate where $f(x)$ has a zero. Start with $x_1 = 1$ as the first approximation and calculate x_2 and x_3.

(a) $x_2 = 0$ and $x_3 = -0.25$
(b) $x_2 = 0$ and $x_3 = 0.25$
(c) $x_2 = 2$ and $x_3 = -2$
(d) $x_2 = 2$ and $x_3 = 0.7$
(e) $x_2 = 2$ and $x_3 = 3.3$

$f(x) = 0,\ x = ?$ $f'(x) = 3x^2 + 8$

Newton's Method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_{n+1} = x_n - \frac{x_n^3 + 8x_n + 2}{3x_n^2 + 8}$

$x_1 = x_0 - \frac{x_0^3 + 8x_0 + 2}{3x_0^2 + 8}$

2

$x_1 = 1 - \frac{(1)^3 + 8(1) + 2}{3(1)^2 + 8} = 1 - \frac{11}{11} = \boxed{0 = x_1}$

$x_2 = 0 - \frac{(0)^3 + 8(0) + 2}{3(0)^2 + 8} = -\frac{2}{8} = -\frac{1}{4} = \boxed{-0.25}$
$= x_2$

11. Evaluate the limit: $\lim_{x\to 3} \frac{5x+2}{3x-4}$.

$$= \frac{5(3)+2}{3(3)-4} = \frac{17}{5} = \boxed{3.4}$$

denom $\neq 0$ at $x=3$

(a) -0.5
(b) 1.5
(c) 5/3
(d) 3.4
(e) Does not exist

12. Which of the following is the derivative of $f(x) = \sec(x)$?

(a) $-\csc(x)$
(b) $\csc(x)$
(c) $\tan(x)$
(d) $\tan^2(x)$
(e) $\sec(x)\tan(x)$

13. Evaluate the limit: $\lim_{x\to+\infty} \frac{3e^x+8x}{5e^x}$.

$$\overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to+\infty} \frac{3e^x+8}{5e^x} \overset{\left(\frac{\infty}{\infty}\right)}{=} \lim_{x\to+\infty} \frac{3e^x}{5e^x}$$

$$= \lim_{x\to+\infty} \frac{3}{5} = \boxed{\frac{3}{5}}$$

(a) 0
(b) 0.6
(c) 2.2
(d) 8
(e) Does not exist

14. Which of the following is the derivative of $g(x) = x\sin^{-1}(5x)$?

Inverse Sine!

(a) $\sin^{-1}(5x) + x\cos^{-1}(5)$
(b) $\sin^{-1}(5x) + 5x\cos^{-1}(5x)$
(c) $\sin^{-1}(5x) + \frac{5x}{\sqrt{1-25x^2}}$
(d) $\sin^{-1}(5x) + \frac{5x}{\sin^2(5x)}$
(e) $\sin^{-1}(5x) + \frac{5x}{\cos(5x)}$

$$\frac{d}{dx}\left[x\sin^{-1}(5x)\right]$$

$$= x\frac{d}{dx}\left[\sin^{-1}(5x)\right] + \sin^{-1}(5x)\frac{d}{dx}[x]$$

$$= x\frac{1}{\sqrt{1-(5x)^2}}\frac{d}{dx}[5x] + \sin^{-1}(5x)\cdot 1$$

$$= x\frac{1}{\sqrt{1-25x^2}}\cdot 5 + \sin^{-1}(5x)$$

$$= \boxed{\sin^{-1}(5x) + \frac{5x}{\sqrt{1-25x^2}}}$$

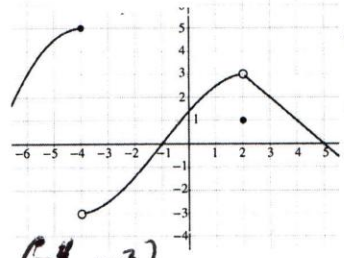
MATH 1241 **FINAL EXAM** **FALL 2011**

Part II, Calculators Allowed

1. Determine the values of A and B (if they exist) using the graph of $f(x)$

$\lim_{x\to -4^+} f(x) = A \qquad \lim_{x\to 2} f(x) = B$

(a) $A = -3$, $B = 3$
(b) $A = -3$, $B = 1$
(c) $A = 5$, $B = 3$
(d) $A = 5$, $B = 1$
(e) $A = 5$, B does not exist



$(-4, -3)$

$\lim_{x\to -4^+} f(x) = -3 = A$

$\lim_{x\to 2} f(x) = 3 = B$

2. Find the x and y coordinates of each point on the graph of $y = x^3 + 7$ where the slope of the tangent line is 12.

(a) $(2, 0)$
(b) $(2, 0)$ and $(-2, 0)$
(c) $(2, 15)$
(d) $(2, 15)$ and $(-2, -1)$
(e) There is no such point

$f'(x) = 12, \quad x = ?$

$f'(x) = 3x^2$

$3x^2 = 12$

$x^2 = 4$

$\boxed{x = \pm 2}$

$f(2) = (2)^3 + 7 = 15$

$f(-2) = (-2)^3 + 7 = -1$

$(2, 15)$ & $(-2, -1)$

3. The function $f(x)$ has a derivative for each value of x and $g(x) = \ln(f(x))$. Find $g'(2)$ given that $f(2) = 5$ and $f'(2) = -3$.

(a) $g'(2) = -1.5$
(b) $g'(2) = -0.6$
(c) $g'(2) = 0.12$
(d) $g'(2) = 0.2$
(e) $g'(2) = 0.5$

$g'(x) = \frac{d}{dx}\left[\ln(f(x))\right] = \frac{1}{f(x)} \frac{d}{dx}\left[f(x)\right]$

$g'(x) = \frac{1}{f(x)} f'(x)$

$g'(2) = \frac{1}{f(2)} \cdot f'(2) = \frac{1}{5} \cdot (-3) = -\frac{3}{5}$

$= \boxed{-0.6}$

4. Find an equation for the line that is tangent to the curve $12x + y^3 = 60y - 52$ at the point $(5, 2)$.

(a) $y = -4x + 22$
(b) $y = -0.4x + 4$
(c) $y = 0.25x + 0.75$
(d) $y = 0.4x$
(e) $y = 4x - 18$

Implicit Differentiation $\boxed{y = f(x)}$

$\frac{d}{dx}\left[12x + y^3\right] = \frac{d}{dx}\left[60y - 52\right]$

$12 + 3y^2 y' = 60y'$

$3y^2 y' - 60y' = -12$

$y'(3y^2 - 60) = -12$

$y' = -\frac{12}{3y^2 - 60}$

$m =$

$y'\big|_{(5,2)} = -\frac{12}{3(2)^2 - 60}$

$= -\frac{12}{-48}$

$= +\frac{1}{4} = m$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{1}{4}(x - 5)$

$y - 2 = \frac{1}{4}x - \frac{5}{4}$

$y = \frac{1}{4}x - \frac{5}{4} + 2$

$\boxed{y = \frac{1}{4}x + \frac{3}{4}}$

5. The graph at right is the graph of the **derivative** of the function $f(x)$ [so it is the graph of $y = f'(x)$]. Which of the following statements is true about the function $f(x)$?

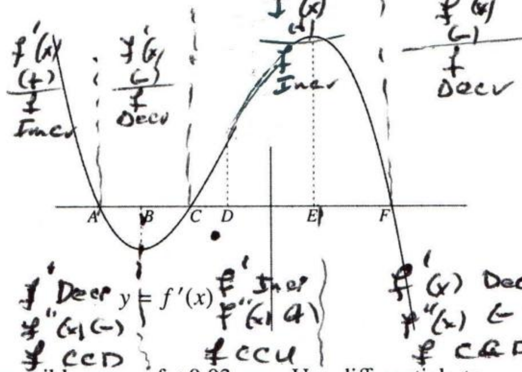
(a) $f(x)$ is increasing when $-\infty < x < A$ and $C < x < F$ and concave down when $-\infty < x < B$ and $E < x < +\infty$

~~(b)~~ $f(x)$ is increasing when $-\infty < x < A$ and $C < x < F$ and concave down when $D < x < F$

~~(c)~~ $f(x)$ is increasing when $B < x < E$ and concave down when $D < x < +\infty$

~~(d)~~ $f(x)$ is increasing when $B < x < E$ and concave down when $-\infty < x < B$ and $E < x < +\infty$

~~(e)~~ $f(x)$ is increasing when $B < x < E$ and concave down when $-\infty < x < D$



6. A cube is measured to have edges of length 10 cm with a possible error of ±0.03 cm. Use differentials to estimate the maximum error in calculating the volume.

(a) ±0.3 cm³
(b) ±0.9 cm³
(c) ±3.0 cm³
(d) ±9.0 cm³
(e) ±30.0 cm³

$V = x^3$ $\quad dx = \pm 0.03$

$dv = 3x^2\,dx$ $\quad x = 10$

$dv = 3(10)^2(\pm 0.03) = 300(\pm 0.03) = \boxed{\pm 9}$

7. Which of the following limits represents the derivative of $f(x) = \sin(2x + 1)$?

(a) $\lim\limits_{h\to 0} \dfrac{\sin(2x + h + 1) - \sin(2x+1)}{h}$

(b) $\lim\limits_{h\to 0} \dfrac{\sin(2x + 2h + 2) - \sin(2x+1)}{h}$

(c) $\lim\limits_{h\to 0} \dfrac{\cos(2x + h + 1)}{h}$

(d) $\lim\limits_{h\to 0} \dfrac{2\sin(x + h + 1/2) - 2\sin(x+1/2)}{h}$

(e) $\lim\limits_{h\to 0} \dfrac{\sin(2x + 2h + 1) - \sin(2x+1)}{h}$

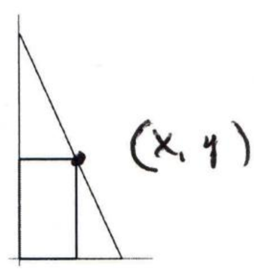
$f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\sin(2(x+h)+1) - \sin(2x+1)}{h}$

$= \lim\limits_{h\to 0} \dfrac{\sin(2x+2h+1) - \sin(2x+1)}{h}$

8. Find the maximum area of a rectangle with the base on the x-axis, one side on the y-axis, and one vertex on the line $y = -5x + 90$ as in the illustration.

(a) 81
(b) 180
(c) 225
(d) 365
(e) 405



$A = xy$

$y = -5x + 90$

$A(x) = x(-5x + 90)$

$A(x) = -5x^2 + 90x$

$A'(x) = -10x + 90 = 0$

$-10x + 90 = 0 \quad -10x = -90 \quad x = 9$

$y = -5x + 90$

$y = -5(9) + 90 = 45$

$A = xy$

$A = (9)(45)$

$\boxed{A = 405}$

9. The second derivative of the function $f(x)$ is $f''(x) = x^3 - 9x$. Find the x-coordinate of each inflection point of the function $f(x)$.

(a) Only inflection point is at $x = 0$
(b) There are two inflection points: at $x = 3$ and at $x = -3$
(c) Only inflection point is at $x = 3$
(d) There are three inflection points: at $x = -3$, at $x = 0$ and at $x = 3$
(e) There are no inflection points

$f''(x) = x(x^2-9) = 0$

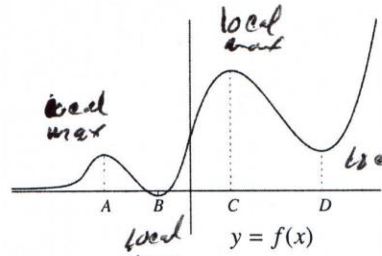
$x = 0$, $x = \pm 3$

$f''(x)$ $(-)$	$f''(x)$ $(+)$	$f''(x)$ $(-)$	$f''(x)$ $(+)$
-3	0	3	
CCD	CCU	CCD	CCU

Change of concavity at $x=0$, $x=3$, $x=-3$

10. The graph of the function $f(x)$ is given below. Find the x-coordinate of each local maximum and each local minimum.

(a) Only local maximum is at $x = C$, only local minimum is at $x = B$
(b) Local maxima at $x = A$ and $x = C$, local minima at $x = B$ and $x = D$
(c) Local maxima at $x = A$ and $x = C$, only local minimum is at $x = B$
(d) Only local maximum is at $x = C$, local minima at $x = B$ and $x = D$
(e) There is no local maximum, only local minimum is at $x = B$



11. Find the absolute maximum and absolute minimum values of the function $f(x) = 2x^3 - 150x^2 + 50{,}000$ on the interval $[-20, 20]$.

(a) Absolute maximum is −26,000 and absolute minimum is −75,000
(b) Absolute maximum is 6,000 and absolute minimum is −75,000
(c) Absolute maximum is 6,000 and absolute minimum is −26,000
(d) Absolute maximum is 50,000 and absolute minimum is −26,000
(e) Absolute maximum is 50,000 and absolute minimum is −75,000

$f'(x) = 6x^2 - 300x = 0$

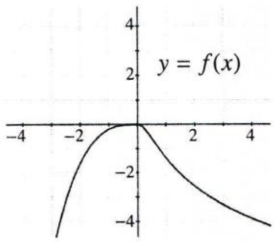
$6x(x-50) = 0$

$\boxed{x=0}$ $\boxed{x=50}$

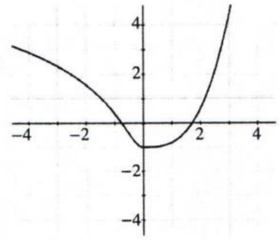
$50 > 2$ not included

x	$f(x) = 2x^3 - 150x^2 + 50000$	
-20	$f(-20) = -26000$	Abs. min
0	$f(0) = 50000$	Abs. max
20	$f(20) = 6000$	

12. The first graph on the left below is the graph of $y = f(x)$. Which of the graphs labeled (a), (b), (c), (d) and (e) best represents the graph of $y = -f(x-1)$? Shift right by 1, flip over x-axis



(a)



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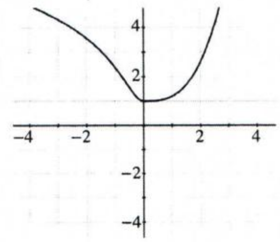
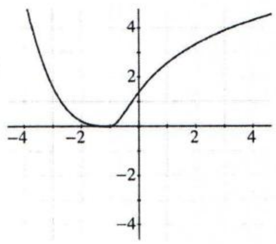
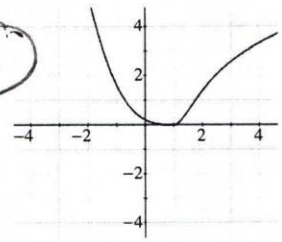


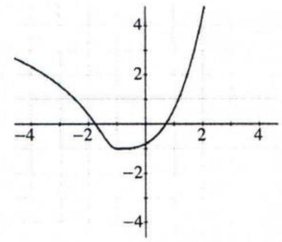
(c)



(d)



(e)



13. The derivative of a function $h(x)$ is given by $h'(x) = -7(x-2)^2(x+3)(x-5)$. Find the x-coordinates [only the x since you don't know what $h(x)$ is] for each local maximum and each local minimum of $h(x)$, if any.

(a) Local maxima at $x = 2$ and $x = 5$, local minimum at $x = -3$

(b) Local maximum at $x = 5$, local minimum at $x = -3$

(c) Local maximum at $x = 2$, local minima at $x = -3$ and $x = 5$

(d) Local maxima at $x = -3$ and $x = 5$, local minimum at $x = 2$

(e) Local maximum at $x = -3$, local minimum at $x = 5$

$h'(x) = 0$, $x = 2$, $x = -3$, $x = 5$

$h'(x)(-)$ h Decr | $h'(x)(+)$ h Incr | $h'(x)(+)$ h Incr | $h'(x)(-)$ h Decr

-3 0 2 5

x	$h'(x)$
-4	$(-)(+)(-)(-) = (-)$
0	$(-)(+)(+)(-) = (+)$
3	$(-)(+)(+)(-) = (+)$
6	$(-)(+)(+)(+) = (-)$

local min (at -3)   local max (at 5)

Part III, Calculators Allowed

1. Answer the questions below based on the following information about the function f. You must justify your answers.

(i) The function f is continuous and differentiable for all values of x.

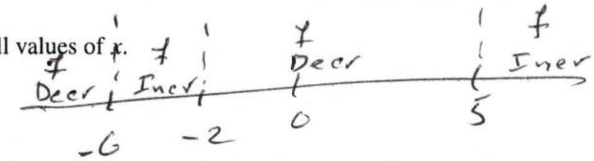
(ii) $f(x) > 0$ for $x < 0$; $f(x) < 0$ for $0 < x$.

(iii) $f'(x) > 0$ for $-6 < x < -2$ and $5 < x$.

(iv) $f'(x) < 0$ for $x < -6$ and $-2 < x < 5$.

(v) $f''(x) > 0$ for $x < -4$ and $3 < x < 7$.

(vi) $f''(x) < 0$ for $-4 < x < 3$ and $7 < x$.



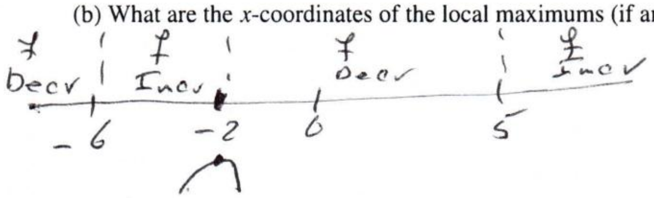
So f is increasing on $\boxed{(-6,-2) + (5,\infty)}$

(a) On which intervals is the function increasing?

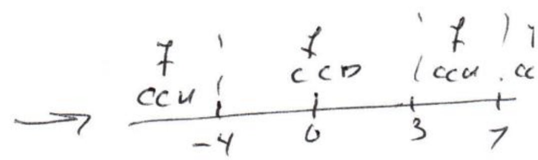
From (iii) + (iv)

(b) What are the x-coordinates of the local maximums (if any)?

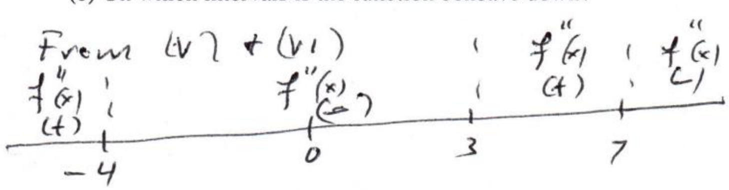
$\boxed{X = -2}$



(c) On which intervals is the function concave down?

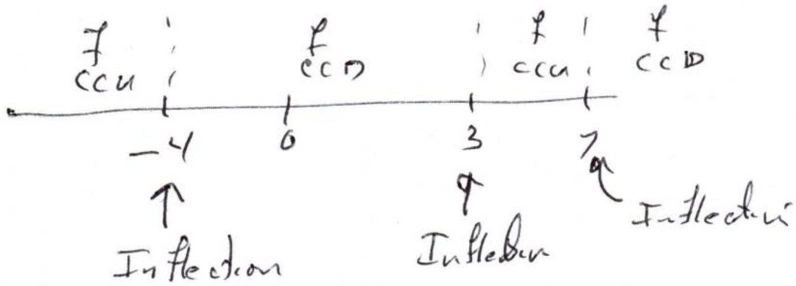


From (v) + (vi)



So f is CCD on $\boxed{(-4,3) + (7,\infty)}$

(d) What are the x-coordinates of the inflection points (if any)?



$\boxed{X = -4, \quad x = 3, \quad x = 7}$

2. Use the following table of values for (a), (b) and (c) below

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	2	3	11	-5
2	3	4	-1	9
3	4	2	6	8
4	2	1	-7	3

(a) Find $b'(3)$ for $b(x) = \dfrac{f(x)}{g(x)}$.

$$b'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\text{So } b'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \frac{(2)(6) - (4)(8)}{(2)^2} = \frac{12-32}{4} = \frac{-20}{4} = \boxed{-5}$$

(b) Find $h'(1)$ for $h(x) = f(g(x))$.

$$h'(x) = f'(g(x))\cdot g'(x) \quad \text{So } h'(1) = f'(g(1))\cdot g'(1) = f'(3)g'(1) = (6)(-5) = \boxed{-3}$$

(c) Find $k'(4)$ for $k(x) = (f(x))^3$.

$$k'(x) = 3f^2(x)\cdot f'(x) \quad \text{So } k'(4) = 3f^2(4)\cdot f'(4) = 3(2)^2(-7)$$
$$= (12)(-7) = \boxed{-84}$$

3. The height of a projectile at time t is given by $h(t) = 200t - t^3$.

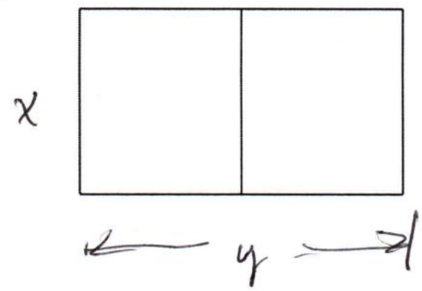
(a) Calculate the average velocity on the time interval [5, 10].

$$\text{Average velocity} = \frac{h(10) - h(5)}{10-5} = \frac{[200(10) - (10)^3] - [200(5) - (5)^3]}{5}$$
$$= \frac{[2000 - 1000] - [1000 - 75]}{5} = \frac{1000 - 925}{5} = \frac{75}{5} = \boxed{25}$$

(b) Use some calculus to find the instantaneous velocity at $t = 5$.

$$v(t) = h'(t) = 200 - 3t^2$$
$$v(5) = 200 - 3(5)^2 = 200 - 75$$
$$\boxed{v(5) = 125}$$

4. A rectangular area of 60,000 square feet is to be fenced off as in the diagram below (a large rectangle divided into two smaller rectangles). What are the dimensions of the area that uses the least amount of fencing? Give the exact answer and then approximate to the nearest foot.



$L = 3x + 2y$

$xy = 60{,}000$

$y = \frac{60{,}000}{x}$

$L(x) = 3x + 2\left(\frac{60000}{x}\right) = 3x + \frac{120000}{x}$

$L'(x) = 3 - \frac{120000}{x^2} = 0$

$\frac{3x^2 - 120000}{x^2} = 0$

$3x^2 - 120000 = 0$

$3x^2 = 120000$

$x^2 = 40000$

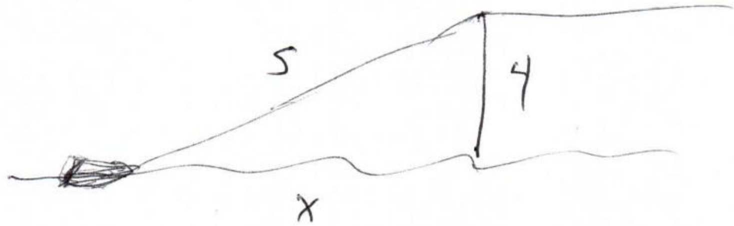
$\boxed{x = 200}$

$y = \frac{60000}{x}$

$y = \frac{60000}{200}$

$\boxed{y = 300}$

5. A boat is pulled into a dock by a rope attached to the bow of the boat. The rope passes through a pulley on the dock that is 4 feet above the bow of the boat. If the rope is pulled in at the rate of 3 feet per second, how fast is the boat approaching the dock when it is 10 feet away? Find the exact answer and then approximate to the nearest 0.01 feet per second.



$s'(t_0) = 3$

$x'(t_0) = ?$

$x(t_0) = 10$

$s^2(t) = x^2(t) + 4^2$

$s^2(t) = x^2(t) + 16$

$2s(t)\,s'(t) = 2x(t)\,x'(t)$

$x'(t) = \dfrac{s(t)\,s'(t)}{x(t)}$

$x'(t_0) = \dfrac{s(t_0)\,s'(t_0)}{x(t_0)}$

$\boxed{x'(t_0) = \dfrac{(\sqrt{116})(3)}{10}}$ exact

$\boxed{x'(t_0) \approx 3.23}$

$s^2(t_0) = x^2(t_0) + 16$

$s^2(t_0) = (10)^2 + 16$

$s^2(t_0) = 100 + 16$

$s^2(t_0) = 116$

$s(t_0) = \sqrt{116}$