**Math 1241 Project 1 - Limits and Asymptotes Name:**

In this project, Limits will be calculated and the asymptotic behavior of rational functions will be studied using graphs and tables.

**Problem I:** Consider the rational function $f(x) = \frac{x^3 - x^2 - 3}{x^2 - x - 2} = \frac{x^3 - x^2 - 3}{(x+1)(x-2)}$.

It is known that this function is not defined when $x$ = -1 and $x$ = 2 because these values are the zeros of the denominator.

Let's study the behavior of this function near $x$ = 2 for example. From the numerical point of view, we can plug the numbers close to the discontinuity from the left and from the right to see what happens to the value of *f*(*x*).

First type $Y_1 = (x^3 - x^2 - 3)/(x^2 - x - 2)$ , press 2nd WINDOW , in the TABLE SETUP, set

Indpnt on Ask and Depend on Auto. Press 2nd GRAPH and start entering values for x to get the values of y automatically.

For example enter 1 in the first column, press enter and you'll get the value of y for x = 1. Do the same for x = 1.5, 1.9, 1.99,…… Copy your answers from the calculator and fill the two tables below.

| *x* | 1.00 | 1.50 | 1.90 | 1.99 | 1.999 | 1.9999 |
|---|---|---|---|---|---|---|
| *f*(*x*) | | | | | | |

From this table, we deduce that the one sided limit of *f*(*x*) from the left is $\lim_{x \to 2^-} f(x) =$ ____________

| *x* | 3.00 | 2.50 | 2.10 | 2.01 | 2.001 | 2.0001 |
|---|---|---|---|---|---|---|
| *f*(*x*) | | | | | | |

From this table, we deduce that the one sided limit of *f*(*x*) from the right is $\lim_{x \to 2^+} f(x) =$ ___________

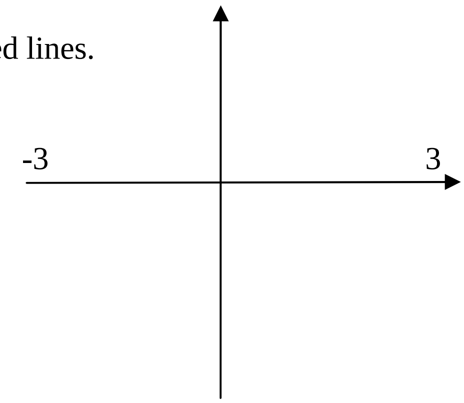
The vertical line $x$ = 2 is one of the vertical asymptotes. The other vertical asymptote is the line :_____

$\lim_{x \to -1^-} f(x) =$ _______ and $\lim_{x \to -1^+} f(x) =$ ________

Now let's verify these results graphically.
Sketch the graph of the function $Y_1 = (x^3 - x^2 - 3)/(x^2 - x - 2)$

Using ZOOM 6 (Zstandard). To study the behavior of the function near the vertical asymptotes, change the WINDOW( for example $X_{\min} = -3, X_{\max} = 3$,.....)

Copy the graph and the two vertical asymptotes represented with dashed lines. Use the TRACE button and move the cursor along the curve from $x = 0$ to get so close to 2 from the left and notice the values of $y$. Do the same to check the values of $y$ when $x$ approaches 2 from the right.



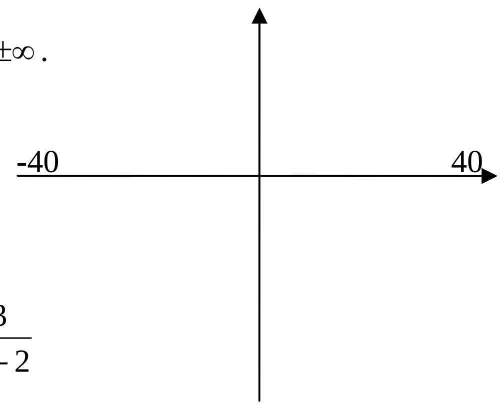

$\lim_{x \to 2^-} f(x) =$ $\lim_{x \to 2^+} f(x) =$

Now let's study the **asymptotic behavior** of the function

$f(x) = \dfrac{x^3 - x^2 - 3}{x^2 - x - 2}$ when $x \to \pm\infty$ .

Back to ZOOM standard. How does the graph look like when $x \to \pm\infty$.

Press ZOOM out and use the TRACE button to see if you can guess the equation of the line (called the slant asymptote) that describes the end behavior of this function in this large view.____________




Another way to find the equation of the slant asymptote is by using long division and writing $f(x) = \dfrac{x^3 - x^2 - 3}{x^2 - x - 2} = x + \dfrac{2x - 3}{x^2 - x - 2}$

Since $\lim_{x \to \pm\infty} \dfrac{2x - 3}{x^2 - x - 2} =$_____, the function $f(x)$ behaves like the line $y = x$ when $x \to \pm\infty$.

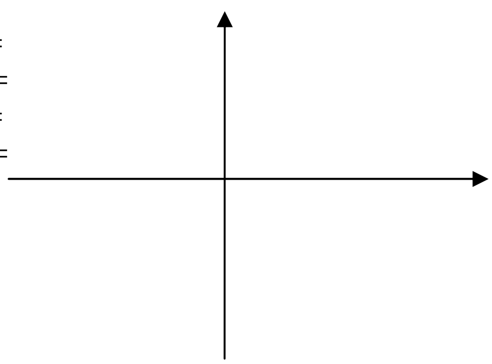
So the equation of the slant asymptote is _______ .

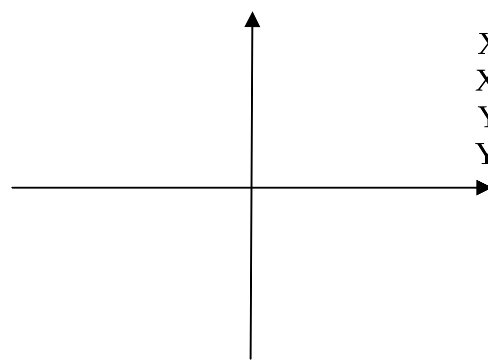
**Problem II:** Let $f(x) = \dfrac{x^3 - 2x^2 - 3}{x - 2}$. Sketch the graph of this function in 2 windows settings:

A small window setting, to show the behavior of the function near the vertical asymptote.

A large window setting, to show the asymptotic behavior of the function and the limits at infinity.

Xmin=
Xmax=
Ymin=
Ymax=





Xmin=
Xmax=
Ymin=
Ymax=

The equation of the vertical Asymptote is___________

A reasonable symbolic formula for the asymptotic function is________

$\lim_{x \to 2^-} f(x) =$_______ , $\lim_{x \to 2^+} f(x) =$________ , $\lim_{x \to -\infty} f(x) =$ ________ , $\lim_{x \to +\infty} f(x) =$ _________ .