Math 1241 Project 2 - Difference quotient and Derivative Name:

In this project, numerical values of the derivative will be calculated using the limit of the difference quotient and a calculator program will be created to compute the slope of the secant line.

Problem I: In this problem, we will find the slope of the tangent line to a curve. First, the slope of the secant line joining the points corresponding to $x = a$ and $x = a + h$ is given by the **difference quotient** $m_{\text{sec}} = \dfrac{f(a+h) - f(a)}{h}$.

If h is small enough, we get a good approximation for the slope of the tangent line at $x = a$. The slope of the tangent line is $f'(a) = \lim_{h \to 0} m_{\text{sec}} = \lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$.

Let $f(x) = \sin x$ and $a = 1$. The slope of the secant line joining the points corresponding to $x = 1$ and $x = 1.1$ is equal to $m_{\text{sec}} = \dfrac{f(1.1) - f(1)}{0.1} = \dfrac{\sin 1.1 - \sin 1}{0.1} = 0.49736$ (here $h = 0.1$).

To find the slope of the tangent line at $a = 1$, take smaller and smaller values of h.

This is easily done using TABLE in the calculator. Take $Y_1 = \dfrac{\sin(1+X) - \sin(1)}{X}$ and plug in smaller and smaller values of X. In the TABLE SETUP, set Indpnt on Ask and Depend on Auto. Press 2nd GRAPH and start entering values for x to get the values of y automatically.
Fill in the following tables (Also remember Radians Mode).

X	.1	.01	.001	.0001	.00001
Y1					

X	-.1	-.01	-.001	-.0001	-.00001
Y1					

Thus the slope of the tangent line to the curve $f(x) = \sin x$ at $x = 1$ is equal to

$f'(1) = \lim_{h \to 0} m_{\text{sec}} = \lim_{h \to 0} \dfrac{\sin(1+h) - \sin(1)}{h} =$ _____ .

Problem II: An easier way to compute the slope of secant lines is by using a program on the calculator. Follow the steps below to create this program. More instructions are on the back page.

PROGRAM:MSEC	Name of the program
:Disp " ENTER H"	Prompt a value of H
:Input H	Accept a value of H
:Disp " ENTER A"	Prompt for the point at which to find the slope
:Input A	
:A→X	Store the value of A in X
:Y_1 →Y	Compute *f*(A) and store it in Y
:A+H→X	Store the value of A+H in X
:(Y_1-Y)/H	Compute the slope of the secant
:Disp Ans	Display the slope

To create this program on your calculator:

- Press PRGM ▶ ▶ to display the **PRGM NEW** menu
- Press ENTER to select **1: Create New**. The **Name =** prompt is displayed (and the A-LOCK is on with the TI 83 Plus), type in **MSEC** the name of the program
- The ALPHA key lets you enter the alphabetic characters . To enter T, for example, press ALPHA , release it, and then press 4 . If you have to enter several alphabetic characters, press 2nd ALPHA to get the **A-lock** so that you avoid pressing the ALPHA key many times.
- After typing the program's name, press ENTER . The program editor is displayed.
- The commands **Disp** and **Input** are in the **PRGM I/O**(input/output) menu. Press PRGM ▶ .
- To get the Y_1, press VARS ▶ to display the **Y-VARS** menu, select **1: Function** to display the Y_n functions.
- " is ALPHA + , : is ALPHA . , **Ans** is 2nd (-), and → is the STO ▶ key.
- When done typing the program, Press 2nd QUIT to leave the program editor and return to the home screen.

To use the **MSEC** program

- Press Y = and set $Y_1 = \sin X$. Press 2nd QUIT to go to the home screen.
- Press PRGM to display the **PRGM EXEC** menu. Select the program's name and press ENTER ENTER to execute the program.

For example let $a = 1$ and $h = 0.1$. The **MSEC** program will give the slope of the secant line joining the points corresponding to x = 1 and x = 1.1.
Use the **MSEC** program to find the slope of the tangent at $a = 1$ by using smaller and smaller values of h.

h	.1	.01	.001	.0001	.00001
MSEC					

h	-.1	-.01	-.001	-.0001	-.00001
MSEC					

Thus the slope of the tangent line to the curve $f(x) = \sin x$ at $x = 1$ is equal to $\lim_{h \to 0} m_{sec} =$ _____ .

Problem III: Use the **MSEC** program to find the slope of the tangent line to the curve $f(x) = \sqrt{3x+1}$ at $a = 2$.

h	.1	.01	.001	.0001	.00001
MSEC					

h	-.1	-.01	-.001	-.0001	-.00001
MSEC					

Thus the slope of the tangent line = _____