PART I (Calculators Not Allowed)

1. Evaluate the integral $\int_0^1 (8x-3)\,dx.$ $= 4x^2 - 3x\Big|_0^1 = 4-3=1$

(a) $\frac{7}{9}$ (b) $\frac{7}{3}$ (c) 1 (d) 2 (e) 3

2. Evaluate the integral $\int_0^2 |3x-3|\,dx.$ $= \int_0^1 (-3x+3)\,dx + \int_1^2 (3x-3)\,dx$

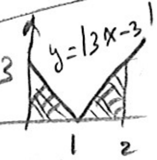
$3x-3>0 \Rightarrow x>1$

$3x-3<0 \Rightarrow x<1$

$= \left(-\frac{3x^2}{2}+3x\right)\Big|_0^1 + \left(\frac{3x^2}{2}-3x\right)\Big|_1^2$

$= \left(-\frac{3}{2}+3\right)-(0)+(6-6)-\left(\frac{3}{2}-3\right)=3$

Or 3



$A=\frac{3}{2}+\frac{3}{2}=3$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

3. Evaluate the integral $\int \frac{1+\cos^2 x}{\cos^2 x}\,dx.$ $= \int\left(\frac{1}{\cos^2 x}+1\right)dx = \int(\sec^2 x+1)\,dx$

$= \tan x + x + C$

(a) $1+\tan x+C$ (b) $x+\tan x+C$ (c) $x\tan x+C$ (d) $x\sec x+C$ (e) $x+\sec x+C$

4. Evaluate the integral $\int_1^2 4x\ln x\,dx.$ $= uv - \int v\,du = (\ln x)2x^2\Big|_1^2 - \int_1^2 2x^2\frac{1}{x}\,dx$

$\boxed{\text{int. by parts}}$

let $u=\ln x \Rightarrow du=\frac{1}{x}dx$

$dv = 4x\,dx \Rightarrow v = 2x^2$

$= (8\ln 2 - 2\ln 1) - \int_1^2 2x\,dx$

$= 8\ln 2 - \left(x^2\Big|_1^2\right) = 8\ln 2 - (4-1) = 8\ln 2 - 3$

(a) $8\ln 2-3$ (b) $8\ln 2-2$ (c) $8\ln 2-1$ (d) $4\ln 2-1$ (e) $4\ln 2-2$

5. Evaluate the integral $\int_{-1}^2 4xe^{x^2}\,dx.$ $= \int 4xe^u\frac{du}{2x} = \int 2e^u\,du = 2e^u\Big| = 2e^{x^2}\Big|_{-1}^2 = 2e^4 - 2e^1$

$\boxed{\text{Sub}}$ let $u=x^2$

$du = 2x\,dx \Rightarrow dx = \frac{du}{2x}$

(a) $2e^2-2e$ (b) $2e^2+2e^{-1}$ (c) $2e^2-2e^{-1}$ (d) $2e^4-2e$ (e) $2e^4-2e^{-1}$

Remark. $u=x^2$, $x=-1 \Rightarrow u=1$

$x=2 \Rightarrow u=4$

or $\int_1^4 2e^u\,du$

Integration by parts

6. Evaluate the definite integral $\int_0^{\pi/2} x\sin(3x)\,dx$ $= \frac{-x\cos(3x)}{3} + \int \frac{+\cos(3x)}{3}dx = -\frac{x\cos(3x)}{3} + \frac{\sin(3x)}{9}\Big|_0^{\frac{\pi}{2}}$

let $u = x \Rightarrow du = dx$

$dv = \sin(3x)dx \Rightarrow v = -\frac{\cos(3x)}{3}$

$= \left(\frac{-\frac{\pi}{2}\cos\frac{3\pi}{2}}{3} + \frac{\sin 3\frac{\pi}{2}}{9}\right) - \left(0 + \frac{\sin 0}{9}\right)$

$= \left(0 + \frac{-1}{9}\right) - 0 = -\frac{1}{9}$

(a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) $-\frac{\pi}{9}$ (d) $\frac{\pi}{9}$ (e) $-\frac{1}{6}$

7. Evaluate the integral $\int e^{-2x+2}\,dx$. $= \frac{e^{-2x+2}}{-2} + C$

u-sub: let $u = -2x+2 \Rightarrow du = -2dx$, $\int e^{-2x+2}dx = \int e^u \cdot \frac{du}{-2} = \frac{-1}{2}e^u + C$

$= \boxed{-\frac{1}{2}e^{-2x+2} + C}$

(a) $2e^{-2x} - 2x + C$

(b) $2e^{-2x} + C$

(c) $\frac{e^{-2x}+2}{2} + C$

(d) $-\frac{e^{-2x+2}}{2} + C$

(e) $-2e^{-2x+2} + C$

8. Evaluate the integral $\int \frac{\sin\left(\frac{\pi}{x}\right)}{x^2}\,dx$. $= \int \frac{\sin u}{x^2} \cdot \frac{x^2}{-\pi}du = -\frac{1}{\pi}\int \sin u\,du = \frac{-1}{\pi}(-\cos u) + C$

$= \boxed{\frac{\cos(\pi/x)}{\pi} + C}$

u-sub: let $u = \frac{\pi}{x} \Rightarrow du = \frac{-\pi}{x^2}dx \Rightarrow dx = \frac{x^2}{-\pi}du$

(a) $\frac{\sin\left(\frac{\pi}{x}\right)}{x} + C$ (b) $\frac{\cos\left(\frac{\pi}{x}\right)}{x} + C$ (c) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi x^2} + C$ (d) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi x} + C$ (e) $\frac{\cos\left(\frac{\pi}{x}\right)}{\pi} + C$

9. Evaluate the integral $\int \frac{3}{(x+7)^{3/2}}dx$ $= \int \frac{3}{u^{3/2}}du = \int 3u^{-3/2}du = 3\frac{u^{-1/2}}{-1/2} + C$

let $u = x+7 \Rightarrow du = dx$

$= -\frac{6}{u^{1/2}} + C$

$= -\frac{6}{\sqrt{x+7}} + C$

(a) $\frac{6}{\sqrt{x+7}} + C$

(b) $-\frac{6}{\sqrt{x+7}} + C$

(c) $\frac{3}{\sqrt{x+7}} + C$

(d) $-\frac{3}{\sqrt{x+7}} + C$

(e) $-\frac{21}{\sqrt{x+7}} + C$

10. The series $\sum_{n=1}^{\infty}(-1)^n\frac{(n+2)!}{2^n n!}$, $\lim\left|\frac{a_{n+1}}{a_n}\right| = \lim\left|\frac{\frac{(n+1+2)!}{2^{n+1}(n+1)!}}{\frac{(n+2)!}{2^n n!}}\right|$

$$= \lim\left|\frac{(n+3)!}{2^{n+1}(n+1)!}\cdot\frac{2^n n!}{(n+2)!}\right| = \lim\left|\frac{(n+3)!}{(n+2)!}\cdot\frac{2^n}{2^{n+1}}\cdot\frac{n!}{(n+1)!}\right|$$

$$= \lim\left|(n+3)\cdot\frac{1}{2}\cdot\frac{1}{n+1}\right| = \lim_{n\to\infty}\left|\frac{n+3}{n+1}\cdot\frac{1}{2}\right| = \frac{1}{2} < 1$$

abs. conv.

(a) converges absolutely
(b) converges conditionally
(c) diverges by the comparison test
(d) diverges by the ratio test
(e) diverges by the root test

11. What is the coefficient of $(x+2)^2$ in the Taylor expansion of $f(x) = (x+3)^3$ about $a = -2$?

$$\text{Coeff} = \frac{f''(a)}{2!} = \frac{6}{2!} = \boxed{3}$$

$f'(x) = 3(x+3)^2$
$f''(x) = 6(x+3)$
$f''(-2) = 6(-2+3) = 6$

(a) 1 (b) 2 (c) 3 (d) 4 (e) 6

12. Determine whether the series $\sum_{n=0}^{\infty}\left(\frac{2}{5}\right)^n$ is convergent or divergent. If convergent, find the sum.

geo, $r = \frac{2}{5} < 1$ conv.

$$S = \frac{a}{1-r} = \frac{1}{1-\frac{2}{5}} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

(a) $\frac{5}{7}$ (b) $\frac{2}{7}$ (c) $\frac{2}{3}$ (d) $\frac{5}{3}$ (e) diverges

13. The coefficient of x^4 in the power series for the function $f(x) = \frac{x^2}{1-2x}$ equals

$\boxed{\frac{1}{1-x} = 1 + x + x^2 + \cdots}$

memory

replace x by $2x$

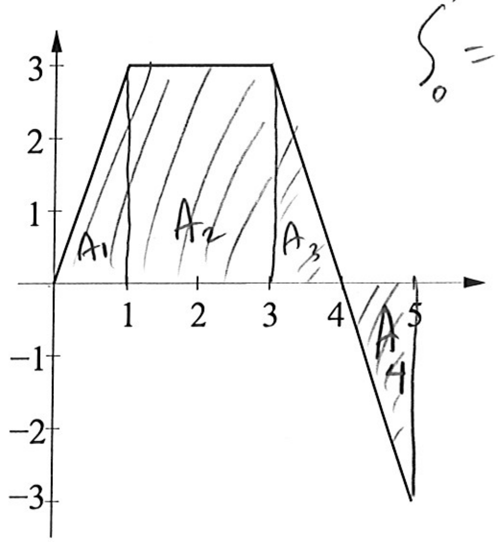
$$= x^2\left(\frac{1}{1-2x}\right) = x^2(1 + 2x + 4x^2 + \cdots)$$
$$= x^2 + 2x^3 + 4x^4 + \cdots$$

Coeff = 4

(a) -1 (b) 2 (c) -2 (d) 4 (c) -4

PART II (Calculators Allowed)

1. The graph of $y = f(x)$ is shown below. Estimate the value of the integral $\int_0^5 f(x)\,dx$.



$\int_0^5 = \frac{1}{2}(1)(3) + (2)(3) + \cancel{\frac{1}{2}(1)(3)} - \cancel{\frac{1}{2}(1)(3)}$

$= 1.5 + 6 = 7.5$

(a) 6 (b) 7.5 (c) 9 (d) 11.5 (e) 13.5

2. If $\int_2^8 f(x)\,dx = 6$, $\int_2^4 f(x)\,dx = 7$, and $\int_6^8 f(x)\,dx = 3$, find $\int_4^6 f(x)\,dx$.

$\int_2^8 = \int_2^4 + \int_4^6 + \int_6^8 \Longrightarrow 6 = 10 + \int_4^6 f(x)\,dx \Rightarrow \int_4^6 f(x)\,dx = -4$

$6 = 7 + {}?? + 3$

(a) 16 (b) 10 (c) 4 (d) 2 (e) -4

3. Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^3}{n^4}$ as a definite integral and evaluate it.

$\Delta x = \frac{b-a}{n}$ $\quad 0 = a \quad 1$ $\quad x_i = a + i\Delta x \Rightarrow x_i = \frac{i}{n}$

$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum f(x_i)\Delta x = \lim_{n\to\infty} \sum \frac{i^3}{n^3}\cdot\frac{1}{n} = \lim \sum \left(\frac{i}{n}\right)^3 \cdot \frac{1}{n}$

$f(x) = x^3$

$\int_0^1 x^3\,dx = \frac{x^4}{4}\Big|_0^1 = \frac{1}{4}$

(a) $\frac{1}{5}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) 1

4. Find the average value of the function $f(x) = 3x^2 - 2$ on the interval $[-1, 2]$.

f average $= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-(-1)}\int_{-1}^{2}(3x^2-2)\,dx$

(a) $\frac{5}{3}$ (b) $\frac{1}{3}$ (c) 1 (d) 1.5 (e) 3

1 $= \frac{1}{3}\left(x^3 - 2x\right)\Big|_{-1}^{2} = \frac{1}{3}\left((8-4)-(-1+2)\right)$

$= \frac{1}{3}(4-1) = 1$

5. Which of the following sequences converge?

I. $(-1)^n \dfrac{n}{1+\sqrt{n}}$ II. $\dfrac{n-2}{3n+5}$ III. $\dfrac{\cos 3n}{2n-1} \to 0$ conv.

$\to \sqrt{n} \to \pm\infty$

$\to \frac{1}{3}$ conv.

$-1 \leq \cos 3n \leq 1$

$\frac{-1}{2n-1} \leq \frac{\cos 3n}{2n-1} \leq \frac{1}{2n-1}$

$\to 0 \leftarrow$

(a) I only (b) II only (c) III only (d) I and II only (e) II and III only

6. Find the length of the curve $y = 3 + 2x^{3/2}$, $0 \leqslant x \leqslant 1$. $\Rightarrow y' = 2 \cdot \frac{3}{2} x^{1/2} = 3\sqrt{x}$

$L = \int_0^1 \sqrt{1+y'^2}\,dx = \int_0^1 \sqrt{1+(3\sqrt{x})^2}\,dx = \int_0^1 \sqrt{1+9x}\,dx$ u-substitution $u = 1+9x$

$= \frac{2}{3}\frac{(1+9x)^{3/2}}{9}\Big|_0^1 = \frac{2}{27}\left[(10)^{3/2} - 1^{3/2}\right]$

(a) $\dfrac{2(10^{3/2}-1)}{27}$ (b) $\dfrac{10^{3/2}-1}{27}$ (c) $\dfrac{2(8^{3/2}-1)}{27}$ (d) $\dfrac{2(7^{3/2}-1)}{27}$ (e) $\dfrac{2(5^{3/2}-1)}{27}$

7. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+1}$ II. $\displaystyle\sum_{n=1}^{\infty}\frac{2^n}{3e^n}$ III. $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{4^n}$

$\lim a_n = \lim \frac{n}{n+1} = 1 \neq 0$

Div by Test for Div

geo $r = \frac{2}{e} < 1$ Conv.

ratio test: $\lim \left|\dfrac{\frac{(n+1)!}{4^{n+1}}}{\frac{n!}{4^n}}\right| = \lim \left|\frac{(n+1)!}{4^{n+1}} \cdot \frac{4^n}{n!}\right| = \lim_{n\to\infty}\left|\frac{n+1}{4}\right| = \infty > 1$

Div

(a) I only (b) II only (c) III only (d) I and II only (e) II and III only

8. Find the derivative of the function $\displaystyle\int_1^{x^2} \cos^2 t\,dt$.

FTC

$f'(x) = \cos^2(x^2) \cdot (x^2)'$

$= \cos^2(x^2) \cdot 2x$

(a) $2x\cos^2 x^2$ (b) $2x\cos^2 x$ (c) $-x^2\cos^2 x^2$ (d) $-2x\cos^2 x$ (e) $-2x\cos^2 x^2$

example: $\left(\int_1^{4x} \tan(t^2+1)\,dt\right)' = \tan(16x^2+1) \cdot 4$

9. A particle is moved along the x-axis by a force $F(x) = x^2 + 5x - 6$. How much work is done in moving the particle from $x = -2$ to $x = 4$?

$$W = \int_{-2}^{4} f(x)\,dx = \int_{-2}^{4} (x^2+5x-6)\,dx = \frac{x^3}{3} + \frac{5x^2}{2} - 6x \Big|_{-2}^{4}$$
$$= 18$$

(a) 15 (b) 17 (c) 19 (d) 18 (e) 23

10. Suppose that $f'(x) = 2x^3 - \dfrac{8}{x^3}$ and $f(2) = 5$. Find a formula for $f(x)$ and then use it to evaluate $f(1)$.

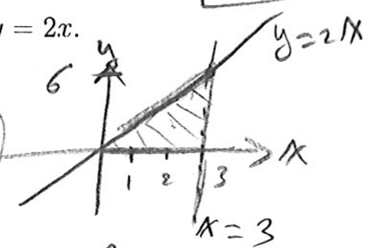
$$f(x) = \int \left(2x^3 - \frac{8}{x^3}\right)dx = \frac{2x^4}{4} - 8\cdot\frac{x^{-2}}{-2} + C = \boxed{\frac{x^4}{2} + \frac{4}{x^2} + C}$$
$$f(2) = 5 = \frac{2^4}{2} + \frac{4}{2^2} + C \Rightarrow C = -4, \quad f(1) = \frac{1}{2} + \frac{4}{1} + (-4)$$

(a) $f(1) = \dfrac{5}{2}$ (b) $f(1) = 2$ (c) $f(1) = \dfrac{3}{2}$ (d) $f(1) = 1$ (e) $f(1) = \dfrac{1}{2}$ $= \boxed{\frac{1}{2}}$

11. Find the x-coordinate of the centroid for the region bounded by $y = 0$, $x = 3$, and $y = 2x$.

x-axis

$$\bar{x} = \frac{1}{A}\int_0^3 x f(x)\,dx = \frac{1}{9}\int_0^3 x(2x)\,dx = \frac{1}{9}\cdot\frac{2x^3}{3}\Big|_0^3 = 2$$



(a) $\dfrac{20}{9}$ (b) $\dfrac{5}{3}$ (c) 2 (d) $\dfrac{22}{9}$ (e) $\dfrac{7}{3}$

$A = \frac{1}{2}(3)(6) = 9$

12. It is known that $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$. Use this formula to evaluate $\displaystyle\int \frac{3dx}{9+4x^2}$.

Let $u = 2x \Rightarrow du = 2\,dx \Rightarrow dx = \frac{du}{2}$

$$= \int \frac{3\,du/2}{9+u^2}$$

(a) $\dfrac{1}{3}\tan^{-1}\dfrac{2x}{3} + C$

(b) $\dfrac{1}{2}\tan^{-1}\dfrac{2x}{3} + C$

(c) $\dfrac{1}{3}\tan^{-1}\dfrac{4x}{3} + C$

(d) $\dfrac{1}{2}\tan^{-1}\dfrac{4x}{3} + C$

(e) $\tan^{-1}\dfrac{2x}{3} + C$

$$= \frac{3}{2}\int \frac{du}{9+u^2}$$
$$= \frac{\cancel{3}}{2}\cdot\frac{1}{\cancel{3}}\cdot\tan^{-1}\frac{u}{3} + C$$
$$= \boxed{\frac{1}{2}\tan^{-1}\left(\frac{2x}{3}\right) + C}$$

PART III (Calculators Allowed)

1. Find the interval of convergence for the power series $\sum_{n=1}^{\infty}(-1)^n\frac{(x+1)^n}{n3^n}$. Be sure to check for convergence or divergence the endpoints of the interval.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{\cancel{(-1)^{n+1}}(x+1)^{n+1}}{(n+1)3^{n+1}}}{\frac{\cancel{(-1)^n}(x+1)^n}{n3^n}}\right| = \lim\left|\frac{(x+1)^{n+1}}{(n+1)3^{n+1}}\cdot\frac{n3^n}{(x+1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{x+1}{3}\cdot\underbrace{\frac{n}{n+1}}_{\to 1}\right| = \left|\frac{x+1}{3}\right|<1 \Rightarrow |x+1|<3 \Rightarrow -3<x+1<3, \quad R=3$$

$$-4<x\le 2$$

If $x=-4$: $\sum(-1)^n\frac{(-3)^n}{n3^n} = \sum\frac{3^n}{n3^n}=\sum\frac{1}{n}$ Div harmonic

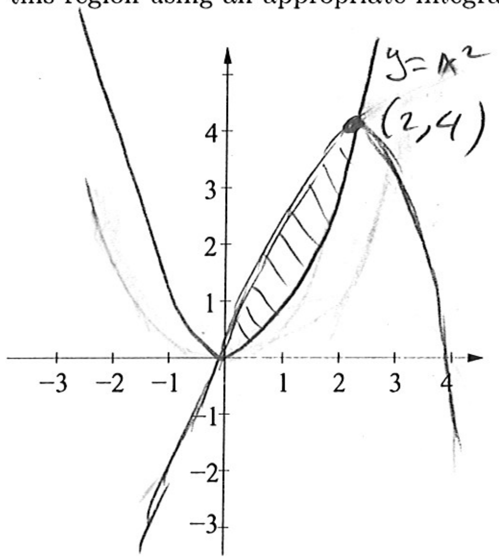
If $x=2$: $\sum(-1)^n\frac{\cancel{3^n}}{n\cancel{3^n}} = \sum\frac{(-1)^n}{n}$ convergent by Alternating series test

The terms $b_n=\frac{1}{n}$ are decreasing

$\lim\frac{1}{n}=0$

Answer: $\boxed{I=(-4,2]}$

2. Sketch the region enclosed by the curves $y=x^2$ and $y=4x-x^2$. Then find the area A of this region using an appropriate integral.



intersection: $x^2=4x-x^2$

$2x^2=4x$

$x=2$

$$A=\int(y_{top}-y_{bottom})$$

$$A=\int_0^2\left[(4x-x^2)-x^2\right]dx$$

$$=\int_0^2(4x-2x^2)\,dx$$

$$=2x^2-\frac{2x^3}{3}\Big|_0^2$$

$$=8-\frac{16}{3}=\frac{24-16}{3}=\frac{8}{3}$$

Answer: $\boxed{A=8/3}$

3. (a) Find the partial fraction expansion of $R = \dfrac{3x+2}{x^3+x}$. Show your work.

$$\frac{3x+2}{x^3+x} = \frac{3x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+x(Bx+C)}{x(x^2+1)}$$

$$3x+2 = Ax^2 + A + Bx^2 + Cx$$

If $x=0$: $\boxed{2 = A}$

If $x=1$: $5 = \underbrace{A}_{=2} + \underbrace{A}_{=2} + B + C \Rightarrow 5 = 4 + B + C \to \boxed{B + C = 1}$

If $x=-1$: $-1 = \underbrace{A(1)}_{=2} + \underbrace{A}_{=2} + B(1) + C(-1) \Rightarrow -1 = 4 + B - C \Rightarrow \boxed{B - C = -5}$

$+$

$2B = -4$

$B = -2$

$C = 3$

Answer: $\boxed{R = \frac{2}{x} + \frac{-2x+3}{x^2+1}}$

(b) Use the answer in part (a) to evaluate $I = \displaystyle\int \frac{3x+2}{x^3+x}\,dx$. Show your work.

$$I = \int \frac{2}{x}\,dx + \int \frac{-2x+3}{x^2+1}\,dx$$

let $u = x^2+1$
$du = 2x\,dx$

$$= 2\ln|x| + \int \frac{-2x}{x^2+1}\,dx + \int \frac{3}{x^2+1}\,dx$$

$$= 2\ln|x| + \int -\frac{2x}{u}\,\frac{du}{2x} + 3\arctan x + C$$

$$= 2\ln|x| - \ln|u| + 3\arctan x + C$$

$$= \boxed{2\ln|x| - \ln|x^2+1| + 3\arctan x + C}$$

Answer: $I =$

4. (a) Use the trapezoidal rule with $n = 3$ to estimate the value of $I = \int_0^3 \frac{dx}{1+3x}$. Show your work.

0 1 2 3 $\Delta x = 1$ $f(x) = \frac{1}{1+3x}$

$T_3 = \frac{\Delta x}{2}\left[f(0) + 2f(1) + 2f(2) + f(3) \right]$

$= \frac{1}{2}\left[1 + 2 \cdot \frac{1}{4} + 2 \cdot \frac{1}{7} + \frac{1}{10} \right]$

$= \frac{1}{2}\left(1 + \frac{1}{2} + \frac{2}{7} + \frac{1}{10} \right) =$

Answer: $\boxed{I \approx 0.942857}$

(b) The error formula for estimating the value of $\int_a^b f(x)\,dx$ using the trapezoidal rule is $|E_T| \leqslant \frac{K(b-a)^3}{12n^2}$ where $K \geqslant |f''(x)|$ for all $a \leqslant x \leqslant b$. Use this formula to estimate the value of $|E_T|$ in part (a) using the smallest possible value of K. Be sure to show how you find K.

$f(x) = \frac{1}{1+3x} = (1+3x)^{-1} \Rightarrow f'(x) = -(1+3x)^{-2} \cdot 3$

$f''(x) = 3 \cdot 2 (1+3x)^{-3} \cdot 3 = \frac{18}{(1+3x)^3}$

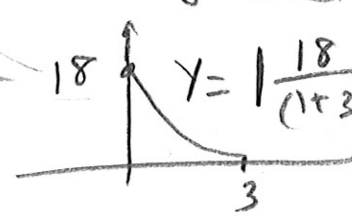
$K = 18$

$|E_T| \leq \frac{18(3-0)^3}{12 \cdot 3^2}$

$= \frac{18 \cdot 27}{12 \cdot 9} = 4.5$

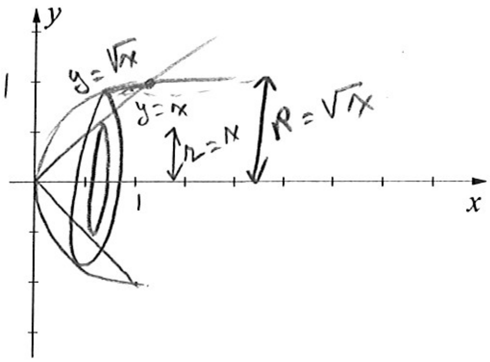
graph it or find the max of $|f''(x)|$ by hand.

$K = 18$ $Y = \left|\frac{18}{(1+3x)^3}\right|$



Answer: $\boxed{K = 18}$ **Answer:** $\boxed{|E_T| \leqslant 4.5}$

5. Find the volume V of the solid obtained by rotating the region bounded by the curves $y = x$ and $y = \sqrt{x}$ about the x-axis. Sketch a graph of the region.



slicing

Washer: $A = \pi R^2 - \pi r^2 = \pi(\sqrt{x})^2 - \pi(x)^2$

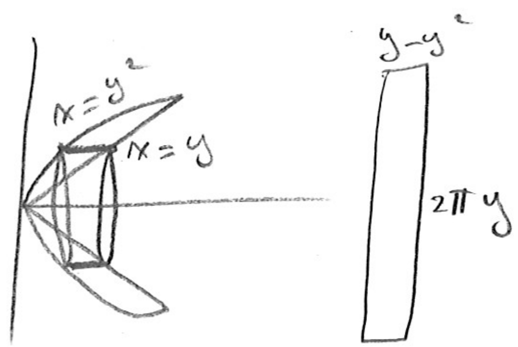
$= \pi(x - x^2)$

$V = \int A(x)\,dx$

$V = \int_0^1 \pi(x - x^2)\,dx = \pi\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^1$

$= \pi\left(\frac{1}{2} - \frac{1}{3}\right) = \boxed{\frac{\pi}{6}}$

Cylindrical shells



$V = \int A(y)\,dy$

$V = \int \text{circumference} \cdot \text{height}\,dy$

$V = \int_0^1 2\pi y(y - y^2)\,dy$

$= 2\pi \int (y^2 - y^3)\,dy$

$= 2\pi\left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Big|_0^1$

$= 2\pi\left(\frac{1}{3} - \frac{1}{4}\right) = 2\pi\,\frac{1}{12}$

$\boxed{= \frac{\pi}{6}}$

Answer: $\boxed{V = \pi/6}$