1. Evaluate $\int (3 + 4x - 2e^x)\, dx$ $= 3x + \frac{4x^2}{2} - 2e^x + C$

(a) $2x^2 - 2e^x + C$
(b) $2x^2 - 2x + \frac{e^{x+1}}{x+1} + C$
(c) $3x + 2x^2 - \frac{1}{2}(e^x)^2 + C$
(d) $3x + 2x^2 - 2ln|x| + C$
(e) $3x + 2x^2 - 2e^x + C$

2. Evaluate $\int \frac{4}{x^2} dx$ $= \int 4x^{-2} dx = \frac{4x^{-1}}{-1} + C = -\frac{4}{x} + C$

(a) $\frac{4x}{\frac{1}{3}x^3} + C$
(b) $4\,ln|x^2| + C$
(c) $-\frac{4}{3}x^{-3} + C$
(d) $\frac{-4}{x} + C$
(e) $\frac{4}{x^2} + C$

3. Evaluate $\int \frac{2x^2+3x-4}{x}\, dx$ $= \int \left(2x + 3 - \frac{4}{x}\right) dx = x^2 + 3x - 4\ln|x| + C$

(a) $x^2 + 3x - 4ln|x| + C$
(b) $\frac{\frac{2}{3}x^3+\frac{3}{2}x^2-4x}{\frac{1}{2}x^2} + C$
(c) $x^2 + 3x + C$
(d) $\frac{5}{2}x^2 - 4x + C$
(e) $(\frac{2}{3}x^3 + \frac{3}{2}x^2 - 4x)ln|x| + C$

4. Evaluate $\int_0^1 (2x - 1)^2\, dx$ $= \int_0^1 (4x^2 - 4x + 1)\, dx = \frac{4x^3}{3} - 2x^2 + x \Big|_0^1$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

$= \frac{4}{3} - 2 + 1 = \frac{1}{3}$

5. Evaluate $\int_0^\infty e^{-2x} dx$ $= \lim_{t\to\infty} \int_0^t e^{-2x} dx = \lim_{t\to\infty} \frac{e^{-2x}}{-2}\Big|_0^t = \lim_{t\to\infty} \left(\frac{e^{-2t}}{-2} - \frac{e^0}{-2}\right)$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

$= 0 - \frac{1}{-2}$

$= \frac{1}{2}$

6. If you use the method of partial fractions to write $\frac{3x-4}{(x-1)(x-2)}$ as $\frac{A}{x-1}+\frac{B}{x-2}$, what is the value of A?

(a) 1 (b) 2 (c) 3 (d) 4 (e) -4

$$\frac{3x-4}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2)+B(x-1)}{(x-1)(x-2)} \Rightarrow 3x-4 = A(x-2)+B(x-1)$$

if $x=1 \Rightarrow 3-4 = A(1-2)+B(1-1)$

$\Rightarrow -1 = -A \Rightarrow A = 1$

7. Let $\int_0^3 f(x)\,dx = 3$ and let $\int_0^3 g(x)\,dx = 4$.
Evaluate $\int_0^3 (2f(x) - 3g(x))\,dx$.

(a) -8 (b) -6 (c) -4(d) 3 (e) 6

$= \int_0^3 2f(x)\,dx - \int_0^3 3g(x)\,dx$

$= 2\int_0^3 f(x)\,dx - 3\int_0^3 g(x)\,dx = 2(3) - 3(4) = 6 - 12 = -6$

8. Make the substitution $u = \sqrt{x}$ to tranform $\int_1^4 e^{\sqrt{x}}dx$ into one of the following integrals. Which of the following is equal to $\int_1^4 e^{\sqrt{x}}dx$?

(a) $\frac{1}{2}\int_1^4 \frac{e^u du}{u}$

(b) $\frac{1}{2}\int_1^2 \frac{e^u du}{u}$

(c) $\frac{1}{2}\int_1^4 ue^u du$

(d) $2\int_1^4 ue^u du$

(e) $2\int_1^2 ue^u du$

$u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx \Rightarrow du = \frac{1}{2\sqrt{x}}dx$

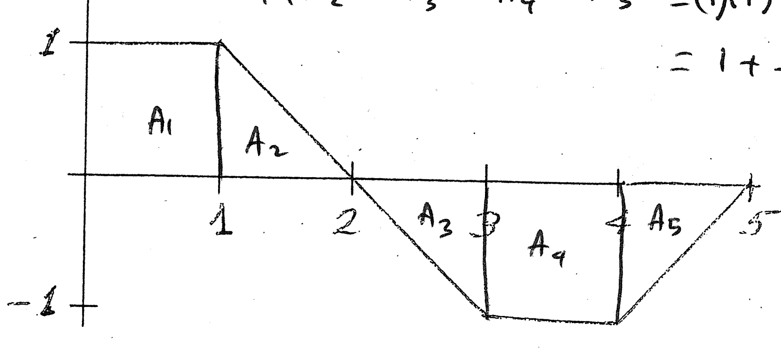
$\Rightarrow 2\sqrt{x}\,du = dx$

$\int_1^4 e^{\sqrt{x}}dx = \int_1^2 e^u \cdot 2\sqrt{x}\,du = \int_1^2 e^u \cdot 2u\,du = 2\int_1^2 ue^u du$

if $x=1 \Rightarrow u = \sqrt{1} = 1$ and if $x=4 \Rightarrow u = \sqrt{4} = 2$

9. The graph of $y = f(x)$ is given below. Use this graph to estimate the value of $\int_0^5 f(x)dx$. $= A_1 + A_2 - A_3 - A_4 - A_5 = (1)(1) + \frac{1}{2}(1)(1) - \frac{1}{2}(1)(1) - (1)(1) - \frac{1}{2}(1)(1)$

$= 1 + \frac{1}{2} - \frac{1}{2} - 1 - \frac{1}{2} = -\frac{1}{2}$



(a) 3.5 (b) 2.0 (c) 1.0 (d) -0.5 (e) -1.5

10. Find a power series representation for the function $\frac{x^2}{1-3x}$

(a) $x^2 + x^3 + x^4 + x^5 + \dots$
(b) $x^2 - x^3 + x^4 - x^5 + \dots$
(c) $1 + 3x + 9z^2 + 27x^3 + \dots$
(d) $x^2 + 3x^3 + 9x^4 + 27x^5 + \dots$
(e) $x^2 - 3x^3 + 9x^4 - 27x^5 + \dots$

$\frac{1}{1-x} = 1+x+x^2+x^3$

$\frac{1}{1-3x} = 1+3x+9x^2+27x^3+\dots$

$\frac{x^2}{1-3x} = x^2(1+3x+9x^2+27x^3\dots)$

$= x^2+3x^3+9x^4+27x^5+\dots$

11. Use integration by parts to evaluate $\int x\, ln\,(x)\, dx$.

(a) $\frac{1}{2}x^2 ln(x) - \frac{1}{4}x^2 + C$
(b) $\frac{1}{2}x^2 ln(x) - \frac{1}{6}x^3 ln(x) + C$
(c) $\frac{1}{2}x(ln\, x)^2 - \frac{1}{6}(ln\, x)^3 + C$
(d) $\frac{1}{2}x^2 ln(x) + \frac{1}{2}x^2 + C$
(e) $\frac{1}{4}x^2(ln\, x)^2 + C$

let $u = \ln x \to du = \frac{1}{x}dx$

$dv = x\,dx \to v = \frac{x^2}{2}$

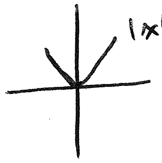
$= uv - \int v\,du$

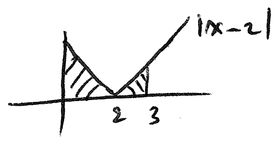
$= \frac{x^2}{2}\ln x - \int \frac{x^2}{2}\cdot\frac{1}{x}dx = \frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx$

$= \frac{x^2}{2}\ln x - \frac{x^2}{4} + C$

12. Evaluate $\int_0^3 |x-2|dx$

(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$ (e) 3





another way

$x-2$ $-$ $+$ (2)

$A = \frac{1}{2}(2)(2) + \frac{1}{2}(1)(1) = 2.5$

$\int_0^3 |x-2|dx = \int_0^2 -(x-2)dx + \int_2^3 (x-2)dx$

$= \left(-\frac{x^2}{2}+2x\right)\Big|_0^2 + \left(\frac{x^2}{2}-2x\right)\Big|_2^3$

$= \left(-\frac{4}{2}+4\right) + \left(\frac{9}{2}-6\right) - \left(\frac{4}{2}-4\right)$

$= 2 - \frac{3}{2} + 2 = \frac{5}{2}$

13. Given that

i. $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}sin^{-1}\left(\frac{u}{a}\right) + C$.

ii. $\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}ln|u+\sqrt{u^2-a^2}| + C$

iii. $\int \sqrt{u^2+a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}ln\left(u+\sqrt{u^2+a^2}\right) + C$

Complete the square to evaluate the integral $\int \sqrt{x^2+6x}\,dx$.

(a) $\frac{x+3}{2}\sqrt{x^2+6x} + \frac{9}{2}sin^{-1}\left(\frac{x+3}{3}\right) + C$
(b) $\frac{x+3}{2}\sqrt{x^2+6x} - \frac{9}{2}ln|x+3+\sqrt{x^2+6x}| + C$
(c) $\frac{x+3}{2}\sqrt{x^2+6x} + \frac{9}{2}ln\left(x+3+\sqrt{x^2+6x}\right) + C$
(d) $\frac{x}{2}\sqrt{x^2+6x} + 3x\,ln\left(x+\sqrt{x^2+6x}\right) + C$
(e) $\frac{x}{2}\sqrt{x^2+6x} - 3ln|x+\sqrt{x^2+6x}| + C$

$= \int \sqrt{x^2+6x+9-9}\,dx$

$= \int \sqrt{(x+3)^2-9}\,dx$

let $u = x+3 \to du = dx$

$= \int \sqrt{u^2-3^2}\,du$

$= \frac{u}{2}\sqrt{u^2-3^2} - \frac{3^2}{2}\ln|u+\sqrt{u^2-3^2}| + C$

$= \frac{(x+3)}{2}\sqrt{(x+3)^2-9} - \frac{9}{2}\ln|x+3+\sqrt{(x+3)^2-9}| + C$

$= \frac{x+3}{2}\sqrt{x^2+6x} - \frac{9}{2}\ln|x+3+\sqrt{x^2+6x}| + C$

1. Which of the following infinte sequences is both increasing and bounded?

(a) $\{n\}$ (b) $\left\{\frac{(-1)^n}{n+1}\right\}$ (c) $\left\{\frac{n}{n+1}\right\}$ (d) $\left\{\frac{n+1}{n}\right\}$ (e) $\left\{\frac{-n}{n+1}\right\}$

$=\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \dots\right\}$

increasing with $\lim_{n\to\infty} \frac{n}{n+1} = 1$

to prove: increasing

let $f(x) = \frac{x}{x+1}$

$f'(x) = \frac{1(x+1) - 1(x)}{(x+1)^2} = \frac{1}{(x+1)^2} > 0$

2. Use the table below to evaluate the Riemann sum for $f(x)$on the interval [2, 6] using two approximating rectangles of equal width and left endpoints.

2 4 6

$\Delta x = 2$

x	2	3	4	5	6
$f(x)$	3	6	5	7	9

$L_2 = [f(2) + f(4)]\Delta x = [3+5](2) = 16$

(a) 14 (b) 15 (c) 16 (d) 17 (e) 18

3. Use the fundamental theorem of calculus to find the derivative of $f(x) = \int_2^{3x} e^{t^2}\,dt$

(a) $e^{9x^2} - e^4$

(b) $18xe^{9x^2} - e^4$

(c) $18xe^{9x^2}$

(d) $3e^{9x^2} - e^4$

(e) $3e^{9x^2}$

$f'(x) = e^{(3x)^2} \cdot (3x)'$

$= e^{9x^2} \cdot 3 = 3e^{9x^2}$

$\to x = \frac{3}{12}\text{ ft} = .25\text{ ft}$

4. A force of 12 lb is required to stretch a spring 3 inches beyond its natural length. How much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

$\to x = \frac{6}{12} = .5\text{ ft}$

(a) 72 inch-lbs.

(b) 76 inch-lbs.

(c) 80 inch-lbs.

(d) 84 inch-lbs.

(e) 88 inch-lbs.

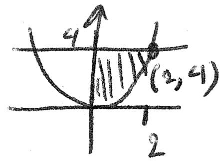
$F = kx$

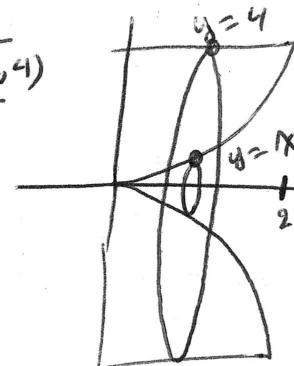
$12 = k(.25) \to k = 48$

$W = \int_0^{.5} 48x\,dx = 24x^2\Big|_0^{.5}$

5. What is the volume of the solid obtained by rotating the region in the first quadrant ($x \geq 0$ and $y \geq 0$) bounded by $y = x^2$, $y = 4$, and, $x = 0$ about the x-axis? (Note that the answers below are given in terms of integrals--you do not need to evaluate them.)

(a) $\pi \int_0^2 (4 - x^2)\, dx$
(b) $\pi \int_0^2 (4^2 - (x^2)^2)\, dx$
(c) $\pi \int_0^4 (4^2 - (x^2)^2)\, dx$
(d) $2\pi \int_0^2 x(4 - x^2) dx$
(e) $2\pi \int_0^4 x(4 - x^2) dx$





$A = \pi R^2 - \pi r^2$

$= \pi (4)^2 - \pi (x^2)^2$

$V = \int_0^2 \pi (4^2 - x^4) dx$

6. Consider an infinite series I and an infinite sequence II.

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ div p series $p = \frac{1}{2} \leq 1$ II. $\left\{ \frac{1}{\sqrt{n}} \right\}$ convergent sequence to 0

Which of the following statements is correct?

(a) Neither I nor II converges.
(b) Both I and II converge.
(c) I converges, but II does not converge.
(d) II converges, but I does not converge.
(e) The sum of the series I is $\sqrt{2}$

7. Which of the following is closest to the sum of the series $\sum_{n=1}^{\infty} \frac{4}{3^{n-2}}$? $= \sum \frac{4}{3^n \cdot 3^{-2}} = \sum_{n=1}^{\infty} 36 \left(\frac{1}{3}\right)^n$

(a) 16
(b) 17
(c) 18
(d) 19
(e) 20

$r = \frac{1}{3}$, $a = 36\left(\frac{1}{3}\right) = 12$

$S = \frac{a}{1-r} = \frac{12}{1 - \frac{1}{3}}$

8. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2(x-3)^n}{4^n}$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3 (e) 4

$\lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \left| \frac{2(x-3)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{2(x-3)^n} \right| = \lim \left| \frac{x-3}{4} \right| = \left| \frac{x-3}{4} \right| < 1$

$\Rightarrow |x-3| < 4 \Rightarrow R = 4$

or $-4 < x - 3 < 4 \Rightarrow -1 < x < 7$

9. In the Taylor series for $f(x) = cos(2x)$ centered at π, what is the coefficient of $(x-\pi)^2$?

(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 2 (e) -2

$\rightarrow f'(x) = -2\sin(2x)$

$\rightarrow f''(x) = -4\cos(2x)$

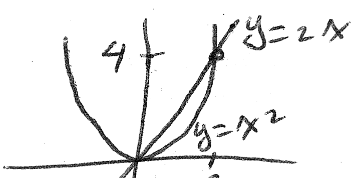
$\rightarrow f''(\pi) = -4\cos 2\pi = -4$

$f(x) = f(\pi) + \frac{f'(\pi)}{1!}(x-\pi) + \frac{f''(\pi)}{2!}(x-\pi)^2 \ldots$

$- - - - - + \frac{-4}{2!}(x-\pi)^2 \ldots \rightarrow$ coefficient $= \frac{-4}{2} = -2$

10. Find the area enclosed by the curves $y = 2x$ and $y = x^2$.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{4}{3}$



intersection: $x^2 = 2x \rightarrow x^2 - 2x = 0 \rightarrow x(x-2) = 0$

$x = 0$ or 2

$A = \int (y_{top} - y_{bottom})\,dx = \int_0^2 (2x - x^2)\,dx = x^2 - \frac{x^3}{3}\Big|_0^2$

$= 4 - \frac{8}{3}$

11. Suppose that $f'(x) = 3x^2 + \frac{2}{x^2} - 1$ and $f(1) = 3$. Find a formula for $f(x)$ and then use this formula to evaluate $f(2)$.

(a) $f(2) = 5$
(b) $f(2) = 6$
(c) $f(2) = 8$
(d) $f(2) = 9$
(e) $f(2) = 10$

$f(x) = \int \left(3x^2 + \frac{2}{x^2} - 1\right)dx = x^3 - \frac{2}{x} - x + C$

$f(1) = 1 - \frac{2}{1} - 1 + C = 3 \rightarrow C = 5$

$f(2) = 2^3 - \frac{2}{2} - 2 + 5 = 8 - 1 - 2 + 5 = 10$

12. What is the average value of the function $f(x) = 4x^3 - 1$ on the interval $[0, 2]$?

(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

$f_{average} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 (4x^3 - 1)\,dx$

$= \frac{1}{2}\left(\cancel{4}x^4 - x\right)\Big|_0^2 = \frac{1}{2}(16-2) =$

1. Evaluate the integral $\int \frac{x^3-x+2}{x^2+1}\,dx$. DO NOT USE YOUR CALCULATOR. YOU MUST SHOW YOUR WORK IN ORDER TO GET CREDIT FOR THIS PROBLEM.

$$\begin{array}{r|l} & x \\ \hline x^2+1 & x^3 - x + 2 \\ & x^3 + x \\ \hline & -2x + 2 \end{array}$$

$$\int \frac{x^3-x+2}{x^2+1}\,dx = \int \left(x + \frac{-2x+2}{x^2+1}\right) dx = \int x\,dx - \int \frac{2x}{x^2+1}\,dx + 2\int \frac{1}{x^2+1}\,dx$$

Let $u = x^2+1$

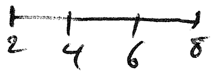
$du = 2x\,dx$

$$= \frac{x^2}{2} - \int \frac{du}{u} + 2\tan^{-1}x + C$$

$$= \frac{x^2}{2} - \ln|u| + 2\tan^{-1}x + C$$

$$= \frac{x^2}{2} - \ln(x^2+1) + 2\tan^{-1}x + C$$

2. (a) Use the trapezoidal rule with $n = 3$ to estimate the value of $\int_2^8 \frac{1}{1+2x}\,dx$. Show your work. (Do not just calculate the exact answer. Be sure to use the Trapezoidal rule.)



$\Delta x = \frac{8-2}{3} = \frac{6}{3} = 2$

$T_3 = \frac{\Delta x}{2}[f(2) + 2f(4) + 2f(6) + f(8)]$

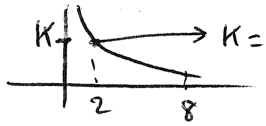
$= \frac{2}{2}\left[\frac{1}{1+4} + 2\frac{1}{1+8} + 2\frac{1}{1+12} + \frac{1}{1+16}\right]$

$= \frac{1}{5} + \frac{2}{9} + \frac{2}{13} + \frac{1}{17} = \boxed{.6348919}$

(b) The maximum possible error when using the Trapezoidal rule to approximate $\int_a^b f(x)\,dx$ is $|E_T| \le \frac{K(b-a)^3}{12n^2}$ where $|f''(x)| \le K$ for $a \le x \le b$. Use this error formula to determine the maximum possible error for the estimation in part (a). Be sure to show how you find K.

$f(x) = \frac{1}{1+2x} \Rightarrow f'(x) = -(1+2x)^{-2}(2) = -2(1+2x)^{-2} \rightarrow f''(x) = 4(1+2x)^{-3}(2)$

$f''(x) = \frac{8}{(1+2x)^3}$



$K = \frac{8}{(1+2\cdot 2)^3} = \frac{8}{125} = .064$

$|E_T| \le \frac{.064(8-2)^3}{12(3)^2} = \frac{.064(6)^3}{12(3)^2} = \boxed{.128}$

(c) Use the error formula above to determine how large n should be if we want to use the Trapezoidal rule to estimate the value of $\int_2^8 \frac{1}{1+2x}\,dx$ with an error less than 0.001.

$|E| \le 0.001 \Rightarrow \frac{.064(8-2)^3}{12n^2} \le 0.001 \Rightarrow \frac{.064(6)^3}{\cancel{12}_{2}\, n^2} \le 0.001$

$\Rightarrow \frac{.064(36)}{2(0.001)} \le n^2$

$\Rightarrow 1152 \le n^2 \Rightarrow n \geq 33.9411$

$\Rightarrow \boxed{n = 34}$

3. For each series below, determine whether the series converges or diverges. You must show your work and indicate which test you use to determine convergence or divergence.

(a) $\sum_{n=1}^{\infty} \frac{n+2}{n^2+3n}$ limit comparison test with $\Sigma \frac{1}{n}$

$$\lim_{n\to\infty} \left| \frac{\frac{n+2}{n^2+3n}}{\frac{1}{n}} \right| = \lim \left| \frac{n^2+2n}{n^2+3n} \right| = 1 > 0$$ both series behave the same

Having $\Sigma \frac{1}{n}$ divergent harmonic series

$\Rightarrow \Sigma \frac{n+2}{n^2+3n}$ is also $\boxed{\text{divergent}}$

(b) $\sum_{n=1}^{\infty} \frac{n}{1+e^{-n}}$

$$\lim \frac{n}{1+e^{-n}} = \frac{\infty}{1+0} = \infty \neq 0$$

The series $\boxed{\text{diverges}}$ by the Test for divergence

(c) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$

ratio test: $\lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \left| \frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} \right| = \lim \left| \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2} \right|$

$$= \lim \left| \frac{1}{2} \cdot \underbrace{\frac{(n+1)^2}{n^2}}_{1} \right| = \frac{1}{2} < 1$$

Thus the series is $\boxed{\text{Convergent}}$ by the ratio test

4. In this problem, we wish to find the arclength of the parabola $y = x^2$ from the point $(0, 0)$ to the point (2,4).

(a) Write an integral formula for the arclength, but do not evaluate this integral yet. Be sure to indicate the limits of integration.

$$L = \int_0^2 \sqrt{1+(y')^2}\,dx = \int_0^2 \sqrt{1+(2x)^2}\,dx = \int_0^2 \sqrt{1+4x^2}\,dx$$

1

1

(b) Evaluate the integral in part (a). One of the following formulas may be useful:

i. $\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}sin^{-1}(\frac{u}{a}) + C$

ii. $\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}ln|u + \sqrt{u^2-a^2}| + C$

iii. $\int \sqrt{u^2+a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}ln\left(u + \sqrt{u^2+a^2}\right) + C$

Let $u = 2x \rightarrow du = 2dx$

$$L = \int_0^2 \sqrt{1+4x^2}\,dx = \int \sqrt{1+u^2}\,\frac{du}{2} = \frac{1}{2}\left(\frac{u}{2}\sqrt{u^2+1} + \frac{1}{2}\ln\left(u+\sqrt{u^2+1}\right)\right)\Big/$$

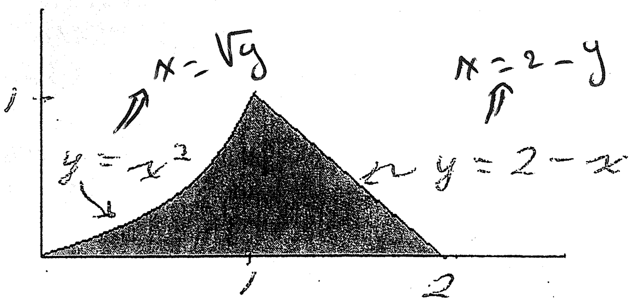
$$= \frac{1}{2}\left(\frac{2x}{2}\sqrt{4x^2+1} + \frac{1}{2}\ln\left(2x+\sqrt{4x^2+1}\right)\right)\Big/_0^2$$

$$= \frac{1}{2}\left(\frac{4}{2}\sqrt{17} + \frac{1}{2}\ln(4+\sqrt{17})\right) - \frac{1}{2}\left(0+\frac{1}{2}\ln(0+\sqrt{1})\right)$$

$\ln 1 = 0$

$$= \sqrt{17} + \frac{1}{4}\ln(4+\sqrt{17})$$

5. Consider the shaded region shown in the diagram below:



or

$$A=\int_0^1 (x_R - x_L)\,dy$$

$$=\int_0^1 (2-y-\sqrt{y})\,dy \quad 2$$

$$= 2y-\frac{y^2}{2}-\frac{2}{3}y^{3/2}\Big|_0^1 \quad 2$$

$$= 2-\frac{1}{2}-\frac{2}{3}=\frac{12-3-4}{6}=\frac{5}{6} \quad 1$$

(a) Find the area of the shaded region.

$$A=\int_0^1 x^2\,dx+\int_1^2 (2-x)\,dx \quad 2$$

$$=\frac{x^3}{3}\Big|_0^1+\left(2x-\frac{x^2}{2}\right)\Big|_1^2 \quad 2$$

$$=\frac{1}{3}-0+\left(4-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)=\frac{1}{3}+2-\frac{3}{2}=\frac{2+12-9}{6}=\frac{5}{6}=.833\ldots \quad 1$$

(b). We wish to find the y-coordinate of the centroid for the shaded region shown above. We have learned that, if a region is bounded above by $y=f(x)$ and below by $y=g(x)$ on an interval $[a,b]$, then

$$\overline{x}=\frac{\int_a^b x(f(x)-g(x))dx}{\int_a^b (f(x)-g(x))dx} \text{ and } \overline{y}=\frac{\frac{1}{2}\int_a^b ([f(x)]^2-[g(x)]^2)dx}{\int_a^b (f(x)-g(x))dx}.$$

Find the y-coordinate of the centroid. You do NOT need to find the x-coordinate.

give 3 points if only mistake is only 1st part

$$\overline{y}=\frac{\frac{1}{2}\int_0^1 ((x^2)^2-0)\,dx+\frac{1}{2}\int_1^2 ((2-x)^2-0)\,dx}{\int_0^1 (x^2-0)\,dx+\int_1^2 ((2-x)-0)\,dx} \quad \begin{matrix}1.5\\1.5\end{matrix}$$

$$=\frac{\frac{1}{2}\int_0^1 x^4\,dx+\frac{1}{2}\int_1^2 (2-x)^2\,dx}{\int_0^1 x^2\,dx+\int_1^2 (2-x)\,dx}=\frac{\frac{1}{2}\int_0^1 x^4\,dx+\frac{1}{2}\int_1^2 (4-4x+x^2)\,dx}{\frac{x^3}{3}\Big|_0^1+\left(2x-\frac{x^2}{2}\right)\Big|_1^2} \quad 1.5$$

1.5

$$=\frac{\frac{1}{2}\left(\frac{x^5}{5}\right)\Big|_0^1+\frac{1}{2}\left(4x-2x^2+\frac{x^3}{3}\right)\Big|_1^2}{\left(\frac{1}{3}-0\right)+\left(4-\frac{4}{2}\right)-\left(2-\frac{1}{2}\right)}=\frac{\frac{1}{2}\left(\frac{1}{5}-0\right)+\frac{1}{2}\left(8-8+\frac{8}{3}\right)-\frac{1}{2}\left(4-2+\frac{1}{3}\right)}{\frac{1}{3}+2-\frac{3}{2}}$$

$$=\frac{\frac{1}{10}+\frac{4}{3}-\frac{7}{6}}{\frac{1}{3}+2-\frac{3}{2}}=\frac{.2666\ldots}{.8333\ldots}=.32 \quad 1$$

or $\frac{8}{25}$