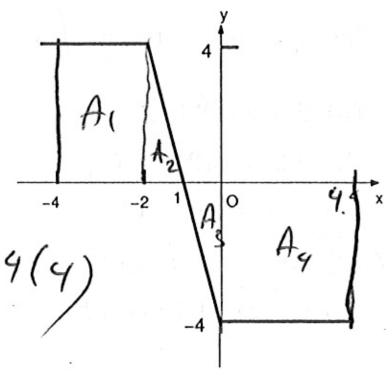
part I

1. The definite integral $\int_0^1 (4x-2)\mathrm{d}x$ equals $= \frac{4x^2}{2} - 2x\Big|_0^1 = 2x^2 - 2x\Big|_0^1 = 2 - 2 = 0$

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. The graph of a function $g(x)$ is shown to the right, which consists of three straight lines. Then the definite integral $\int_{-4}^{4} g(x)\mathrm{d}x$ equals



   (a) -16
   (b) -8
   (c) 8
   (d) 16
   (e) 28

$= A_1 + A_2 - A_3 - A_4$

$= 2(4) + \cancel{\frac{1}{2}(1)(4)} - \cancel{\frac{1}{2}(1)(4)} - 4(4)$

$= 8 - 16 = -8$

3. The indefinite integral $\int \cot x\mathrm{d}x = \int \frac{\cos x}{\sin x}\mathrm{d}x$ equals

   (a) $\ln|\sin x| + C$
   (b) $\ln|\cos x| + C$
   (c) $\ln|\sec x| + C$
   (d) $\ln|\csc x| + C$
   (e) $\tan x + C$

Let $u = \sin x$

$du = \cos x\,dx$

$= \int \frac{du}{u} = \ln|u| + C$

$= \ln|\sin x| + C$

4. The definite integral $\int_0^1 \frac{x^2}{(x^3+1)^2}\mathrm{d}x$ equals $= \int \frac{x^2}{u^2}\frac{du}{3x^2} = \frac{1}{3}\int \frac{du}{u^2} = \frac{1}{3}\int u^{-2}du$

   (a) $-1/2$
   (b) $-1/6$
   (c) $1/6$
   (d) $1/3$
   (e) $1/2$

Let $u = x^3 + 1$

$du = 3x^2\,dx$

$dx = \frac{du}{3x^2}$

$= \frac{1}{3}\frac{u^{-1}}{-1}\Big| = \frac{-1}{3u}\Big|$

$= \frac{-1}{3(x^3+1)}\Big|_0^1 = \frac{-1}{6} - \frac{-1}{3}$

$= -\frac{1}{6} + \frac{1}{3} = \boxed{\frac{1}{6}}$

5. The indefinite integral $\int 5x^4 \ln x dx$ equals

let $u = \ln x \Rightarrow du = \frac{1}{x} dx$
$dv = 5x^4 dx \Rightarrow v = x^5$

(a) $x^5 \ln x - x^5 + C$
(b) $x^5 \ln x + x^5 + C$
(c) $x^5 \ln x - \frac{1}{5}x^5 + C$
(d) $x^5 \ln x + \frac{1}{5}x^5 + C$
(e) $20x^3 \ln x - 5x^3 + C$

$= uv - \int v du$
$= \ln x (x^5) - \int x^5 \cdot \frac{1}{x} dx$
$= x^5 \ln x - \int x^4 dx = x^5 \ln x - \frac{x^5}{5} + C$

6. The indefinite integral $\int 4x \sin(2x) dx$ equals

let $u = 4x \Rightarrow du = 4dx$
$dv = \sin(2x) dx$
$\Rightarrow v = \frac{-\cos(2x)}{2}$

(a) $2\cos(x^2) + C$
(b) $-2x\cos(2x) + C$
(c) $2x\cos(2x) + C$
(d) $\sin(2x) - 2x\cos(2x) + C$
(e) $\sin(2x) + 2x\cos(2x) + C$

$= uv - \int v du$
$= 4x\left(\frac{-\cos(2x)}{2}\right) - \int \frac{-\cos(2x)}{2} \cdot 4dx$
$= -2x\cos(2x) + 2\int \cos(2x) dx$
$= -2x\cos(2x) + \cancel{2}\frac{\sin(2x)}{\cancel{2}} + C$

7. Let $F(x) = \int_1^x \ln(t)\, dt$. Then $F''(2)$ equals

$F'(x) = \ln x$ by the FTC
$F''(x) = \frac{1}{x} \Rightarrow F''(2) = \frac{1}{2}$

(a) 0
(b) 1/2
(c) 1
(d) 2
(e) $\ln(2)$

8. The improper integral $\int_0^\infty e^{-3x} dx$ equals

$= \frac{e^{-3x}}{-3}\Big|_0^\infty = \frac{\cancel{e^{-\infty}}^{0}}{-3} - \frac{e^0}{-3}$ ($e^0 \Rightarrow 1$)
$= 0 + \frac{1}{3} = \frac{1}{3}$

(a) 0
(b) 1/3
(c) 1
(d) 3
(e) ∞

9. The following integral form appears in the Table of Integrals in your text:

$$\int \frac{1}{\sqrt{u^2+a^2}}\,du = \ln\left(u+\sqrt{u^2+a^2}\right)+C.$$

Using this to find the definite integral

$$\int_0^1 \frac{1}{\sqrt{x^2+2x+2}}\,dx. = \int_0^1 \frac{1}{\sqrt{(x+1)^2+1^2}}\,dx, \text{ let } u=x+1$$

$$du=dx$$

(Note that $x^2+2x+2=(x+1)^2+1^2$)

(a) $\ln\left(\sqrt{5}\right)-\ln\left(\sqrt{2}\right)$

(b) $\ln\left(1+\sqrt{5}\right)+\ln\left(\sqrt{2}\right)$

(c) $\ln\left(1+\sqrt{5}\right)-\ln\left(\sqrt{2}\right)$

(d) $\ln\left(2+\sqrt{5}\right)-\ln\left(1+\sqrt{2}\right)$

(e) $\dfrac{1}{\sqrt{5}}-\dfrac{1}{\sqrt{2}}$

$$= \int_{''}^{''} \frac{1}{\sqrt{u^2+1^2}}\,du$$

$$= \ln\left(u+\sqrt{u^2+1^2}\right)\Big|_{''}^{''}$$

$$= \ln\left(x+1+\sqrt{(x+1)^2+1^2}\right)\Big|_0^1$$

$$= \ln\left(1+1+\sqrt{2^2+1}\right)-\ln\left(0+1+\sqrt{1^2+1^2}\right)$$

$$= \ln\left(2+\sqrt{5}\right)-\ln\left(1+\sqrt{2}\right)$$

10. The length of the curve $y=x^2$ from the point $(0,0)$ to the point $(2,4)$ is

(a) $\int_0^2 \sqrt{1+2x}\,dx$

(b) $\int_0^2 \sqrt{1+4x^2}\,dx$

(c) $\int_0^4 \sqrt{1+2x}\,dx$

(d) $\int_0^4 \sqrt{1+4x^2}\,dx$

(e) $\int_0^4 \sqrt{1+x^4}\,dx$

$y'=2x$

$$L=\int_0^2 \sqrt{1+y'^2}\,dx$$

$$=\int_0^2 \sqrt{1+(2x)^2}\,dx$$

$$=\int_0^2 \sqrt{1+4x^2}\,dx$$

11. Let $a_n=\dfrac{2n^2-100n+1}{5-n^2}$ for $n=1,2,3,\cdots$. Which of the following statements is true?

(a) The sequence $\{a_n\}$ converges to 0.

(b) The sequence $\{a_n\}$ converges to 2/5.

(c) The sequence $\{a_n\}$ converges to 1/5.

(d) The sequence $\{a_n\}$ converges to -2.

(e) The sequence $\{a_n\}$ diverges.

$$\lim \frac{2n^2-100n+1}{5-n^2} = -2$$

by looking at the leading terms

or by l'Hopital's rule

$$\lim \frac{2x^2-100x+1}{5-x^2} = \lim \frac{4x-100}{-2x} = \lim \frac{4}{-2}$$

$$=-2$$

12. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$ is conv. by the Alt. Series test because terms $b_n = \frac{1}{n^2}$ are decreasing with $\lim b_n = 0$

(a) is absolutely convergent.

(b) is convergent, but not absolutely convergent.

(c) is divergent.

(d) equals 0.

(e) equals 1.

$\sum \left|\frac{(-1)^{n+1}}{n^2}\right| = \sum \frac{1}{n^2}$ is a conv. p-series $p = 2 > 1$

so the series is absolutely conv.

13. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$

(a) converges by the ratio test.

(b) diverges by the ratio test.

(c) diverges because it is ~~a p-series~~ with $p = 1$.

(d) converges because it is ~~a p-series~~ with $p > 1$.

(e) diverges by comparison of its terms with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

$\sum \frac{1}{n!} < \sum \frac{1}{n}$ div harmonic

so the comparison doesn't work

Ratio test: $\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \left|\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}}\right| = \lim \left|\frac{n!}{(n+1)!}\right|$

$= \lim \left|\frac{1}{n+1}\right| = 0 < 1$

Series is abs. conv.

1. A table of values of a function f is shown. Find the Riemann sum for f on the interval $[0,8]$, using 4 subintervals of equal width and taking the sample points to be the right endpoints.

x	0	2	4	6	8
$f(x)$	4	1	2	1	2

$R_4 = [f(2)+f(4)+f(6)+f(8)]\Delta x$

$= [1+2+1+2](2)$

$= 6(2) = 12$

(a) 6
(b) 8
(c) 12
(d) 16
(e) 20

2. Let $\int_0^3 f(x)\mathrm{d}x = 2$ and $\int_0^3 g(x)\mathrm{d}x = -5$. Then $\int_0^3 (3f(x) - g(x))\,\mathrm{d}x$

$= 3\int_0^3 f(x)dx - \int_0^3 g(x)dx$

$= 3(2) - (-5) = 11$

(a) equals 11
(b) equals 7
(c) equals 1
(d) equals -3
(e) cannot be calculated from the given information.

3. If $\int_5^0 f(x)\mathrm{d}x = 2$ and $\int_5^{10} f(x)\mathrm{d}x = -5$, then $\int_0^{10} f(x)\mathrm{d}x$ $= \int_0^5 f(x)dx + \int_5^{10} f(x)dx$

$= -2 + (-5) = -7$

(a) equals 3
(b) equals 7
(c) equals -3
(d) equals -7
(e) cannot be calculated from the given information.

$\frac{2}{x^3}$ discontinuous function at $x=0$

4. The integral $\int_{-1}^1 \frac{2}{x^3}\,\mathrm{d}x$ $= \int_{-1}^0 \frac{2}{x^3}dx + \int_0^1 \frac{2}{x^3}dx$

$\int_{-1}^0 2x^{-3}dx = \frac{2x^{-2}}{-2}\Big|_{-1}^0$

$= \frac{-1}{x^2}\Big|_{-1}^0 = \frac{-1}{0} - \frac{-1}{1}$

$= -\infty$

(a) is a ~~common~~ definite integral and equals 0.
(b) is a ~~common~~ definite integral and equals 2.
(c) is an improper definite integral and equals 1.
(d) is an improper definite integral and equals -1.
(e) is a divergent improper definite integral.

5. The indefinite integral $\int \frac{x+1}{(x+2)(x-3)}dx$ equals

(a) $\ln|x+2| + \ln|x-3| + C$

(b) $\frac{4}{5}\ln|x+2| - \frac{1}{5}\ln|x-3| + C$

(c) $\frac{1}{5}\ln|x+2| - \frac{4}{5}\ln|x-3| + C$

(d) $\frac{4}{5}\ln|x+2| + \frac{1}{5}\ln|x-3| + C$

(e) $\frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| + C$

$\frac{A}{x+2} + \frac{B}{x-3} = \frac{A(x-3)+B(x+2)}{(\ )(\ )}$

$x+1 = A(x-3) + B(x+2)$

if $x=3$: $4 = 0A + 5B \Rightarrow B = \frac{4}{5}$

if $x=-2$: $-1 = -5A + 0B \Rightarrow A = \frac{1}{5}$

get $\int \frac{1/5}{x+2}dx + \int \frac{4/5}{x-3}dx = \frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| + C$

6. The area enclosed by the curves $y = x$ and $y = 3x - x^2$ is

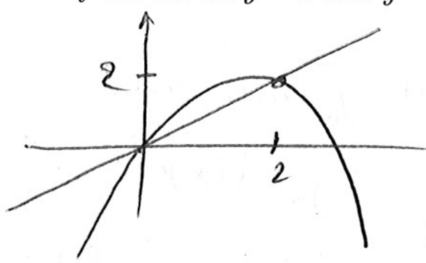
(a) 2/3

(b) 4/3

(c) 10/4

(d) 9/2

(e) 2



~~intersectio~~

$x = 3x - x^2$

$x^2 - 2x = 0 \Rightarrow x(x-2) = 0$

$x = 0, 2$

$A = \int_0^2 [(3x - x^2) - x]dx$

$= \int_0^2 (2x - x^2)dx = x^2 - \frac{x^3}{3}\Big|_0^2$

$= 4 - \frac{8}{3} = \frac{4}{3}$

7. Find the average value of $f(x) = 3x^2 - 12x + 13$ on the interval $[0, 3]$:

~~(a) 4/3~~

(b) 2

(c) 4

(d) 6

(e) 12

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{3-0}\int_0^3 (3x^2 - 12x + 13)\cdot dx$

$= \frac{1}{3}(x^3 - 6x^2 + 13x)\Big|_0^3 = \frac{1}{3}(3^3 - 6*9 + 13*3) = \frac{1}{3}(12) = 4$

8. A particle is moved along the x-axis by a force $F(x) = 6x^2 + 4x - 2$. How much work is done in moving the particle from $x = -1$ to $x = 2$?

(a) 12

(b) 14

(c) 16

(d) 18

(e) 20

$W = \int_{-1}^2 (6x^2 + 4x - 2)dx$

$= 2x^3 + 2x^2 - 2x\Big|_{-1}^2$

$= (2*8 + 2*4 - 4) - (-2 + 2 + 2)$

$= (16 + 8 - 4) - 2 = 18$

9. What is the coefficient of $(x+1)^2$ in the Taylor expansion of $f(x) = x^3 + x - 1$ about $a = -1$?

(a) 1/2
(b) 1
(c) −1
(d) −3
(e) −6

$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 \ldots$

$f(x) = f(-1) + \frac{f'(-1)}{1!}(x+1) + \frac{f''(-1)}{2!}(x+1)^2$

$f(x) = x^3 + x - 1 \to f'(x) = 3x^2 + 1 \to f''(x) = 6x$

$f''(-1) = -6$

Cofficient $= \frac{f''(-1)}{2!} = \frac{-6}{2} = -3$

10. Consider the series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$.

If $x=2$: $\sum \frac{1}{n^2}$ conv

(a) The interval of convergence is $[0, 2]$.
(b) The interval of convergence is $[0, 2)$.
(c) The interval of convergence is $(0, 2)$.
(d) The interval of convergence is $(0, 2]$.
(e) The interval of convergence is $(-\infty, \infty)$.

$\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \left| \frac{\frac{(x-1)^{n+1}}{(n+1)^2}}{\frac{(x-1)^n}{n^2}} \right| = \lim \left| \frac{(x-1)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(x-1)^n} \right|$

$= \lim \left| (x-1) \cdot \frac{n^2}{(n+1)^2} \right| = |x-1| < 1$

$-1 < x-1 < 1 \to 0 < x < 2$ If $x=0$: $\sum \frac{(-1)^n}{n^2}$ conv.

11. The geometric series $\sum_{n=1}^{\infty} 2^n 5^{1-n} = \sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1}$

$S = \frac{a}{1-r} = \frac{2\left(\frac{2}{5}\right)^{1-1}}{1-\frac{2}{5}} = \frac{2}{\frac{3}{5}} = \frac{10}{3}$

(a) converges to 2.
(b) converges to 5/3.
(c) converges to 10/3.
(d) converges to 10.
(e) diverges.

12. Which of the following series **diverges**?

(a) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$. conv. by Comparison with $\sum \frac{1}{n^2}$ because $\sum \left|\frac{\cos n}{n^2}\right| < \sum \frac{1}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{n^3}{100n^3 + 7}$. → by the test for divergence because $\lim a_n = \frac{1}{100} \neq 0$

(c) $\sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$. → conv. by comparison with $\sum \frac{1}{n^2}$

(d) $\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$. → conv. by comparison with $\sum \frac{1}{n^2}$

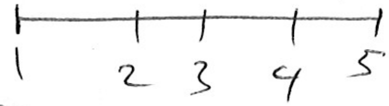
(e) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$. → conv. AST

1. (a) Use Simpson's rule with $n = 4$,

$$S_4 = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

to approximate the definite integral $\int_1^5 \ln x \, dx$.

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{4} = 1$$



$$S_4 = \cancel{\Delta x}\, \frac{1}{3}\left[\ln(1) + 4\ln(2) + 2\ln(3) + 4\ln(4) + \ln(5)\right]$$

$$= 4.041476$$

(b) The error estimate when Simpson's rule is used to approximate $\int_a^b f(x)dx$ is given by

$$|E_n| \leq \frac{K(b-a)^5}{180n^4},$$

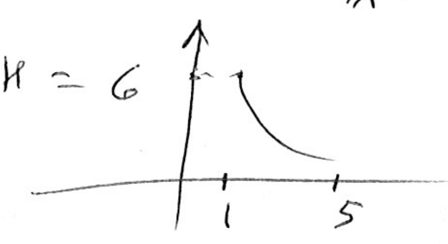
where n is the (even) number of subintervals, and K is an upper bound for $|f^{(4)}(x)|$ on $[a, b]$. Estimate the error of the numerical integration in part (a) above.

$$f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x} \Rightarrow f''(x) = \frac{-1}{x^2} = -x^{-2}$$

$$\Rightarrow f'''(x) = 2x^{-3} \Rightarrow f^{(4)}(x) = 6x^{-4} = \frac{6}{x^4}$$

$$K = \max_{[1,5]} |f^{(4)}(x)| = 6$$

$K = 6$

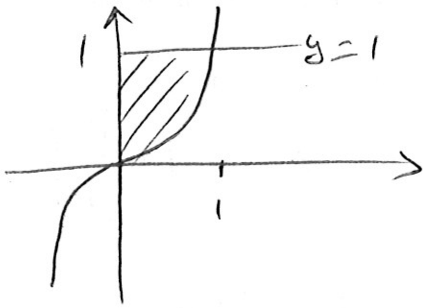


$$|E| \leq \frac{6(5-1)^5}{180(4)^4} = \frac{6(4)^5}{180(4)^4} = \frac{24}{180}$$

$$= \frac{12}{90} = \frac{4}{30} = \frac{2}{15} \approx .1333$$

2. Consider the region in the first quadrant bounded by the curves $y = x^3, y = 1$, and the y-axis.

(a) Find the area of the region.



$$A = \int_0^1 (1 - x^3)\,dx = x - \frac{x^4}{4}\Big|_0^1$$

$$= 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$$

(b) Find the x-coordinate of the centroid of the region.

$$\bar{x} = \frac{1}{A}\int_0^1 x[f(x) - g(x)]\,dx = \frac{1}{\frac{3}{4}}\int_0^1 x\left[1 - x^3\right]dx$$

$$= \frac{4}{3}\int_0^1 (x - x^4)\,dx = \frac{4}{3}\left(\frac{x^2}{2} - \frac{x^5}{5}\right)\Big|_0^1 = \frac{4}{3}\left(\frac{1}{2} - \frac{1}{5}\right)$$

$$= \frac{4}{3}\left(\frac{3}{10}\right) = .4$$

(c) Find the y-coordinate of the centroid of the region.

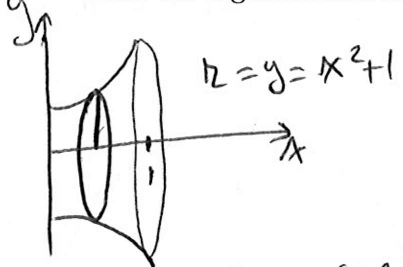
$$\bar{y} = \frac{1}{A}\int_0^1 \frac{1}{2}\left[f(x)^2 - g(x)^2\right]dx$$

$$= \frac{1}{\frac{3}{4}}\int_0^1 \frac{1}{2}\left[1^2 - (x^3)^2\right]dx = \frac{4}{3}\cdot\frac{1}{2}\int_0^1 (1 - x^6)\,dx$$

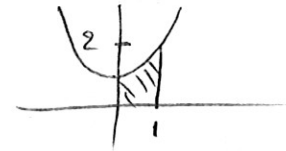
$$= \frac{2}{3}\left(x - \frac{x^7}{7}\right)\Big|_0^1 = \frac{2}{3}\left(1 - \frac{1}{7}\right) = \frac{2}{3}\cdot\frac{6}{7} = \frac{4}{7}$$

3. Consider the region in the plane bounded by the curves $y = 0, y = x^2 + 1, x = 0$ and $x = 1$.

(a) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the x-axis.



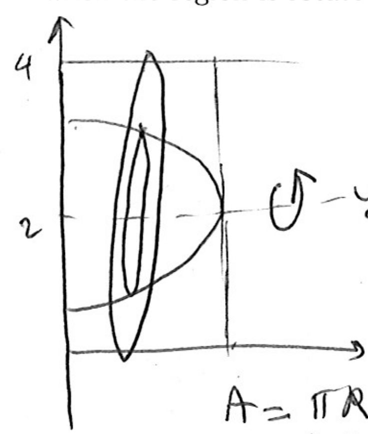
$r = y = x^2+1$



$A(x) = \pi r^2 = \pi(x^2+1)^2$

$V = \int A(x)dx = \int_0^1 \pi(x^2+1)^2 dx$

(b) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the line $y = 2$.



$\circlearrowleft - y = 2$

$R = 2$

$r = 2 - (x^2+1) = 1 - x^2$

$A = \pi R^2 - \pi r^2 = \pi(2)^2 - \pi(1-x^2)^2$

$V = \int_0^1 [\pi(4) - \pi(1-x^2)^2] dx$

(c) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the line $x = 1$. (You may want to use the shell method.)

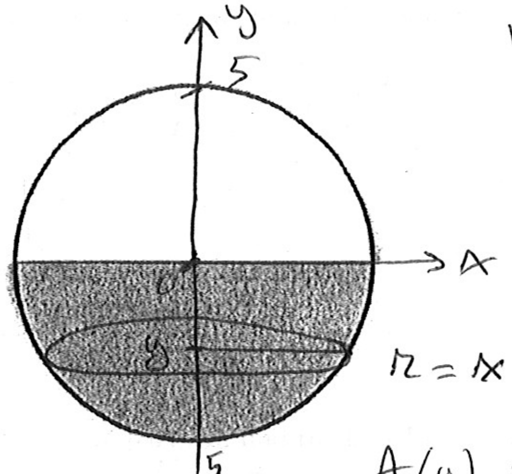
$r = 1 - x$

$x = 1$

$V = \int (\text{circumference})(\text{height}) \, dx$

$V = \int_0^1 2\pi(1-x)(x^2+1) \, dx$

4. A **spherical** tank with radius 5 meters is partially filled with water, 5 meters deep in the middle. How much work is required to pump all the water out through a hole at the top of the tank? (Use the facts that the mass density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2.)



area distance

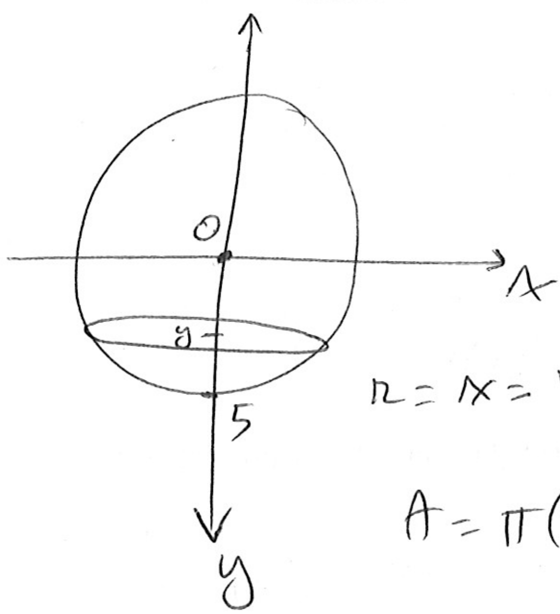
$$W=\int_{-5}^{0}\underbrace{(9.8)(1000)}_{2}\,\underbrace{\pi(25-y^2)}_{3}\,\underbrace{(5-y)}_{3}\,dy$$

3pts

$$r=x=\sqrt{25-y^2}$$

$$A(y)=\pi r^2=\pi(25-y^2)$$

or



$$W=\int_{0}^{5}(9.8)(1000)\pi(25-y^2)(5+y)\,dy$$

$$=5614583.332\,\pi$$

$$=17638733.75$$ 1

$$r=x=\sqrt{25-y^2}$$

$$A=\pi(5-y^2)$$

distance

$$=9800\pi\int_0^5(10-y)(25-(y-5)^2)\,dy$$

$$=9800\pi\int_0^5(10y-y^2)(10-y)\,dy$$

5. The Maclaurin power series expansion for $\cos x$ is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots.$$

(a) Write down the first FIVE nonzero terms of the Maclaurin series for $\cos(t^2)$.

$$\cos(t^2) = 1 - \frac{(t^2)^2}{2!} + \frac{(t^2)^4}{4!} - \frac{(t^2)^6}{6!} + \frac{(t^2)^8}{8!} \cdots$$

$$= 1 - \frac{t^4}{2!} + \frac{t^8}{4!} - \frac{t^{12}}{6!} + \frac{t^{16}}{8!} \cdots$$

(b) Using part (a), find the first FIVE nonzero terms of the Maclaurin series for the function $f(x) = \int_0^x \cos(t^2)dt$.

$$= \int_0^x \left(1 - \frac{t^4}{2!} + \frac{t^8}{4!} - \frac{t^{12}}{6!} + \frac{t^{16}}{8!} \cdots\right) dt$$

$$= t - \frac{t^5}{2!(5)} + \frac{t^9}{4!(9)} - \frac{t^{13}}{6!(13)} + \frac{t^{17}}{8!(17)} \cdots \Big/_0^x$$

$$= x - \frac{x^5}{10} + \frac{x^9}{9(4!)} - \frac{x^{13}}{13(6!)} + \frac{x^{17}}{17(8!)} \cdots$$

(c) Using part (b) with the first THREE nonzero terms to estimate $\int_0^1 \cos(t^2)dt$.

$$\int_0^1 \cos(t^2)dt \approx 1 - \frac{1}{10} + \frac{1}{9(24)}$$

$$= 1 - \frac{1}{10} + \frac{1}{216}$$

$$= .9046296$$