# Applications of the Definite Integral
## ( Areas and Volumes)

In this project, we explore some applications of the definite integral. We use integrals to find the area between the graphs of two functions and the volume of some solids.

## I  *Area between curves*

**1)** We need to find the area of the region enclosed between the graphs of the two functions $f(x) = \cos x$ and $g(x) = x^2 - 2$.

a)  Using your calculator in the **Radian Mode**, $Y_1 = \cos(X)$, $Y_2 = X^2 - 2$, sketch both graphs to the right, using the window $[-5,5] \times [-5,5]$.
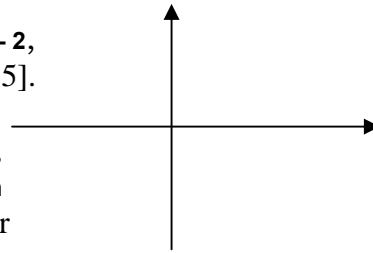
b)  In the home screen, Press **2nd** **PRGM** to get the **DRAW** menu, Choose **Shade** and then get $Y_1$ and $Y_2$ from **VARS ▶ 1:Function** to have **Shade($Y_2$, $Y_1$)** in your home screen. Here $Y_2$ is the lower function and $Y_1$ is the upper function. Copy the shaded region from the calculator on your graph.

c)  To find the area of the shaded region, we need to find first the points of intersections. Press **2nd** **TRACE**, choose **5:Intersection**, answer the questions, and get:

The coordinates of the 1$^{st}$ point of intersection (the one to the left) are (-1.454619,_____)

The coordinates of the 2$^{nd}$ point of intersection (the one to the right) are (_____,_____)

To store these x-values in memory, type -1.454619 on your home screen, press **STO ▶**, **ALPHA**, **A, ENTER**.  Whenever you need the x-coordinate of the first point of intersection, you type **A**. Enter the x-coordinate of the second point of intersection as **B**. Using **MATH 9**, the area between the 2 curves:

$$\text{Area} = \int_{A}^{B} [\cos x - (x^2 - 2)]dx = \textbf{fnInt}(Y_1 - Y_2, X, A, B) = \underline{\hspace{2cm}}$$

d)  Turn off the graphs of $Y_1$ and $Y_2$ by pressing **Y=** , moving the cursor left to the highlighted ▭ sign, and pressing **ENTER**. Do the same for $Y_2$. Now both = signs are not highlighted and the  graphs are turned off. Graph the function $Y_3 = \cos(X) - X^2 + 2$ on the interval (**A, B**) by setting **Xmin = A** and **Xmax = B**. Shade the area with **Shade(0, $Y_3$)**. Copy your graph in the space provided to the right. Is this region the same as the shaded region in parts a) and b)?
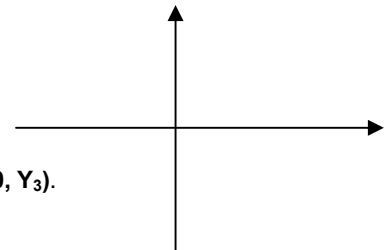
e)  Find the area of the region under the graph of $Y_3$.  Is this area equal to the area in part c)?

**2)** In each of the following 4 cases, use your calculator to sketch the graph of the functions, copy them on this paper, and shade the area of the region(s) enclosed between them. Then set up the integral needed to find the enclosed area(s) and use your calculator to evaluate it.

a) $f(x) = 3x^3 - 3x$ and $g(x) = 3x$ for $0 < x < 1$.

b) $f(x) = 3x^3 - 3x$ and $g(x) = 3x$ for $-1 < x < 1$.

c) $f(x) = 3x^3 - 3x$ and $g(x) = 3x$ (Note that the region enclosed by these two curves consists of two parts. You need to find the points of intersection first)

d) $f(x) = 3x^3 - 3x$ , $g(x) = 3x$ , and $h(x) = 9 - x$ (Note that the region enclosed by these three curves consists of only one part. You need to find the points of intersection first)

# II *Volume of Solids*: To find the volume V of a solid, perform one of the following 2 methods:

- Vertical slicing, find the area A of the slice as a function of x, and integrate with respect to x:
$$V = \int_a^b A(x)\, dx$$

- Horizontal slicing, find the area A of the slice as a function of y, and integrate with respect to y:
$$V = \int_a^b A(y)\, dy$$

3) Consider the region $\mathcal{R}$ bounded by $y = 9 - x^2$, $y = 0$, and $x = 0$ (in the first quadrant).
   (Note: you may use **MATH 9** and your answers should be correct to **4 decimal places**)

   a) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the x-axis.

   Vertical or Horizontal slicing?

   Slice is a Disk or a Washer?  $\qquad\qquad\qquad\qquad V = \int_a^b A(x)\, dx \quad$ or $\quad V = \int_a^b A(y)\, dy$ ?

   Area of a slice A =

   Volume V =

   b) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = 9$.

   Vertical or Horizontal slicing?

   Slice is a Disk or a Washer?  $\qquad\qquad\qquad\qquad V = \int_a^b A(x)\, dx \quad$ or $\quad V = \int_a^b A(y)\, dy$ ?

   Area of a slice A =

   Volume V =

c) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $y = -1$.

  Vertical or Horizontal slicing?

  Slice is a Disk or a Washer?
$$V = \int_a^b A(x)\,dx \quad \text{or} \quad V = \int_a^b A(y)\,dy \ ?$$

Area of a slice A =

Volume V =

d) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the $y$ – axis.

  Vertical or Horizontal slicing?

  Slice is a Disk or a Washer?
$$V = \int_a^b A(x)\,dx \quad \text{or} \quad V = \int_a^b A(y)\,dy \ ?$$

Area of a slice A =

Volume V =

e) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $x = 3$.

  Vertical or Horizontal slicing?

  Slice is a Disk or a Washer?
$$V = \int_a^b A(x)\,dx \quad \text{or} \quad V = \int_a^b A(y)\,dy \ ?$$

Area of a slice A =

Volume V =

f) Sketch the graph and find the volume of the solid obtained by rotating $\mathcal{R}$ about the line $x = 4$.

  Vertical or Horizontal slicing?

  Slice is a Disk or a Washer?
$$V = \int_a^b A(x)\,dx \quad \text{or} \quad V = \int_a^b A(y)\,dy \ ?$$

Area of a slice A =

Volume V =

**4)** Consider the region $\mathcal{R}$ bounded by the curves $y = x^2$ and $y = x$. Sketch the graph, **set up** 2 formulas to find the volume of the solid obtained by rotating $\mathcal{R}$ (**Slicing and Cylindrical shells**), and evaluate these integrals using your calculator:

a) About the x_axis.

b) About the y_axis.

c) About the line $x = 1$.

d) About the line $x = 2$.

e) About the line $x = -2$.

f) About the line $y = 1$.

g) About the line $y = -1$.