MATH 1242 SPRING 2009
COMMON FINAL EXAMINATION
PART I

Name____________________________ Instructor:______________________
Student ID # _____________________ Section/Time____________________

This exam is divided into three parts. **Calculators are not allowed on Part I.** You have three hours for the entire test, but you have only one hour to finish Part I. You may start working on the other two parts of the exam whenever you are done with Part I, but you cannot use your calculator until ALL of the Part I answer sheets are collected. After these answer sheets are collected, your instructor will announce that calculators are allowed on Parts II and III.

These pages contain Part I which consists of 13 multiple choice questions. These questions must be answered without the use of a calculator.

•You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet

•For each question choose the response which best fits the question

•If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

•There is no penalty for guessing

•If you mark more than one answer to a question, the question will be scored as incorrect.

•You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

•**Make sure that your name appears on the answer sheet and that you fill in the circles corresponding to your name.**

After 1 hour, you MUST hand in the answer sheet for Part I.
At the end of the exam, you MUST hand in all remaining test materials including test booklets, the answer sheet for Part II, and scratch paper.

1. Evaluate $\int (3 + 4x - 2e^x)\, dx$

(a) $2x^2 - 2e^x + C$

(b) $2x^2 - 2x + \frac{e^{x+1}}{x+1} + C$

(c) $3x + 2x^2 - \frac{1}{2}(e^x)^2 + C$

(d) $3x + 2x^2 - 2ln|x| + C$

(e) $3x + 2x^2 - 2e^x + C$

2. Evaluate $\int \frac{4}{x^2}\, dx$

(a) $\frac{4x}{\frac{1}{3}x^3} + C$

(b) $4\, ln|x^2| + C$

(c) $-\frac{4}{3}x^{-3} + C$

(d) $\frac{-4}{x} + C$

(e) $\frac{4}{x^2} + C$

3. Evaluate $\int \frac{2x^2+3x-4}{x}\, dx$

(a) $x^2 + 3x - 4ln|x| + C$

(b) $\frac{\frac{2}{3}x^3+\frac{3}{2}x^2-4x}{\frac{1}{2}x^2} + C$

(c) $x^2 + 3x + C$

(d) $\frac{5}{2}x^2 - 4x + C$

(e) $(\frac{2}{3}x^3 + \frac{3}{2}x^2 - 4x)ln|x| + C$

4. Evaluate $\int_0^1 (2x - 1)^2\, dx$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

5. Evaluate $\int_0^\infty e^{-2x} dx$

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$

6. If you use the method of partial fractions to write $\frac{3x-4}{(x-1)(x-2)}$ as $\frac{A}{x-1}+\frac{B}{x-2}$, what is the value of A?

(a) 1 (b) 2 (c) 3 (d) 4 (e) -4

7. Let $\int_0^3 f(x)\,dx = 3$ and let $\int_0^3 g(x)\,dx = 4$.
Evaluate $\int_0^3 (2f(x) - 3g(x))\,dx$.

(a) -8 (b) -6 (c) -4 (d) 3 (e) 6

8. Make the substitution $u = \sqrt{x}$ to tranform $\int_1^4 e^{\sqrt{x}} dx$ into one of the following integrals. Which of the following is equal to $\int_1^4 e^{\sqrt{x}} dx$?

(a) $\frac{1}{2}\int_1^4 \frac{e^u du}{u}$

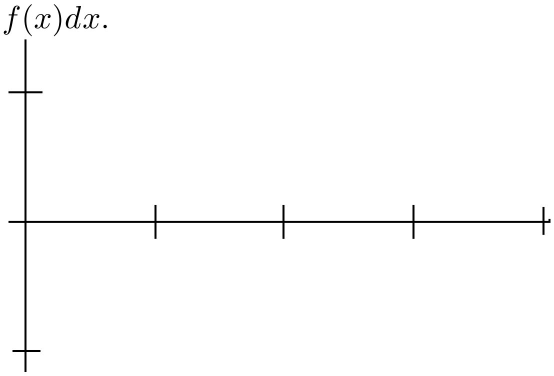
(b) $\frac{1}{2}\int_1^2 \frac{e^u du}{u}$

(c) $\frac{1}{2}\int_1^4 ue^u du$

(d) $2\int_1^4 ue^u du$

(e) $2\int_1^2 ue^u du$

9. The graph of $y = f(x)$ is given below. Use this graph to estimate the value of $\int_0^5 f(x)dx$.



(a) 3.5 (b) 2.0 (c) 1.0 (d) -0.5 (e) -1.5

10. Find a power series representation for the function $\frac{x^2}{1-3x}$
(a) $x^2 + x^3 + x^4 + x^5 + ...$
(b) $x^2 - x^3 + x^4 - x^5 + ...$
(c) $1 + 3x + 9z^2 + 27x^3 + ...$
(d) $x^2 + 3x^3 + 9x^4 + 27x^5 + ...$
(e) $x^2 - 3x^3 + 9x^4 - 27x^5 + ...$

11. Use integration by parts to evaluate $\int x\, ln\,(x)\, dx$.
(a) $\frac{1}{2}x^2 ln(x) - \frac{1}{4}x^2 + C$
(b) $\frac{1}{2}x^2 ln(x) - \frac{1}{6}x^3 ln(x) + C$
(c) $\frac{1}{2}x(ln\, x)^2 - \frac{1}{6}(ln\, x)^3 + C$
(d) $\frac{1}{2}x^2 ln(x) + \frac{1}{2}x^2 + C$
(e) $\frac{1}{4}x^2(ln\, x)^2 + C$

12. Evaluate $\int_0^3 |x-2| dx$
(a) 1 (b) $\frac{3}{2}$ (c) 2 (d) $\frac{5}{2}$ (e) 3

13. Given that

i. $\int \sqrt{a^2 - u^2}\, du \;=\; \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} sin^{-1}\left(\frac{u}{a}\right) + C$

ii. $\int \sqrt{u^2 - a^2}\, du \;=\; \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2} ln|u + \sqrt{u^2 - a^2}| + C$

iii. $\int \sqrt{u^2 + a^2}\, du = \frac{u}{2}\sqrt{u^2 + a^2} \;+ \frac{a^2}{2} ln\left(u + \sqrt{u^2 + a^2}\right) + C$

Complete the square to evaluate the integral $\int \sqrt{x^2 + 6x}\, dx$.
(a) $\frac{x+3}{2}\sqrt{x^2 + 6x} + \frac{9}{2} sin^{-1}\left(\frac{x+3}{3}\right) + C$
(b) $\frac{x+3}{2}\sqrt{x^2 + 6x} - \frac{9}{2} ln|x + 3 + \sqrt{x^2 + 6x}| + C$
(c) $\frac{x+3}{2}\sqrt{x^2 + 6x} \;+ \frac{9}{2} ln\left(x + 3 + \sqrt{x^2 + 6x}\right) + C$
(d) $\frac{x}{2}\sqrt{x^2 + 6x} \;+ 3x ln\left(x + \sqrt{x^2 + 6x}\right) + C$
(e) $\frac{x}{2}\sqrt{x^2 + 6x} - 3 ln|x + \sqrt{x^2 + 6x}| + C$

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PART II

Name____________________________ Instructor:______________________
Student ID # _____________________ Section/Time____________________

These pages contain Part II which consists of 12 multiple choice questions. After the answer sheets for Part I have all been collected, and your intstructor announces that calculators are OK, you are allowed to use a calculator on this part of the exam.

- You must use a pencil with a soft black lead (#2 or HB) to enter your answers on the answer sheet

- For each questions choose the response which best fits the question

- If you wish to change an answer, make sure that you completely erase your old answer and any other extraneous marks.

- There is no penalty for guessing

- If you mark more than one answer to a question, the question will be scored as incorrect.

- You may perform your calculations on the test itself or on scratch paper, but do not make any stray marks on the answer sheet.

- **Make sure that your name appears on the answer sheet for Part II and that you fill in the circles corresponding to your name.**

At the end of the exam, you MUST hand in all remaining test materials including test booklets, the answer sheet for Part II, and scratch paper.

1. Which of the following infinte sequences is both increasing and bounded?

(a) $\{n\}$ (b) $\left\{\frac{(-1)^n}{n+1}\right\}$ (c) $\left\{\frac{n}{n+1}\right\}$ (d) $\left\{\frac{n+1}{n}\right\}$ (e) $\left\{\frac{-n}{n+1}\right\}$

2. Use the table below to evaluate the Riemann sum for $f(x)$on the interval $[2,6]$ using two approximating rectangles of equal width and left endpoints.

x	2	3	4	5	6
$f(x)$	3	6	5	7	9

(a) 14 (b) 15 (c) 16 (d) 17 (e) 18

3. Use the fundamental theorem of calculus to find the derivative of $f(x) = \int_2^{3x} e^{t^2}\,dt$

(a) $e^{9x^2} - e^4$

(b) $18xe^{9x^2} - e^4$

(c) $18xe^{9x^2}$

(d) $3e^{9x^2} - e^4$

(e) $3e^{9x^2}$

4. A force of 12 lb is required to stretch a spring 3 inches beyond its natural length. How much work is done in stretching the spring from its natural length to 6 inches beyond its natural length?

(a) 72 inch-lbs.

(b) 76 inch-lbs.

(c) 80 inch-lbs.

(d) 84 inch-lbs.

(e) 88 inch-lbs.

5. What is the volume of the solid obtained by rotating the region in the first quadrant ($x \geq 0$ and $y \geq 0$) bounded by $y = x^2,\ y = 4,$ and, $x = 0$ about the x-axis? (Note that the answers below are given in terms of integrals--you do not need to evaluate them.)

(a) $\pi \int_0^2 (4 - x^2)\, dx$

(b)$\pi \int_0^2 (4^2 - (x^2)^2)\, dx$

(c) $\pi \int_0^4 (4^2 - (x^2)^2)\, dx$

(d) $2\pi \int_0^2 x(4 - x^2) dx$

(e) $2\pi \int_0^4 x(4 - x^2) dx$

6. Consider an infinite series I and an infinite sequence II.

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ II. $\left\{ \frac{1}{\sqrt{n}} \right\}$

Which of the following statements is correct?

(a) Neither I nor II converges.

(b) Both I and II converge.

(c) I converges, but II does not converge.

(d) II converges, but I does not converge.

(e) The sum of the series I is $\sqrt{2}$

7. Which of the following is closest to the sum of the series $\sum_{n=1}^{\infty} \frac{4}{3^{n-2}}$?

(a) 16

(b) 17

(c) 18

(d) 19

(e) 20

8. What is the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{2(x-3)^n}{4^n}$?

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3 (e) 4

9. In the Taylor series for $f(x) = cos(2x)$ centered at π, what is the coefficient of $(x-\pi)^2$?
(a) 1 (b) -1 (c) $\frac{1}{2}$ (d) 2 (e) -2

10. Find the area enclosed by the curves $y = 2x$ and $y = x^2$.
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$ (e) $\frac{4}{3}$

11. Suppose that $f'(x) = 3x^2 + \frac{2}{x^2} - 1$ and $f(1) = 3$. Find a formula for $f(x)$ and then use this formula to evaluate $f(2)$.
(a) $f(2) = 5$
(b) $f(2) = 6$
(c) $f(2) = 8$
(d) $f(2) = 9$
(e) $f(2) = 10$

12. What is the average value of the function $f(x) = 4x^3 - 1$ on the interval $[0, 2]$?
(a) 5
(b) 6
(c) 7
(d) 8
(e) 9

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PART III

Name____________________________ Instructor:______________________
Student ID # _____________________ Section/Time____________________

These pages contain Part III which consists of 5 free response questions.

Please show all your work in this test booklet. Loose paper will not be graded.

•If you are basing your answer on a graph on your calculator, sketch this graph in the answer booklet. Be sure to label your window by putting a scale on each axis.

At the end of the exam, you MUST hand in all remaining test materials including test booklets, answer sheet, and scratch paper.

PROBLEM	**1**	**2**	**3**	**4**	**5**	**6**
GRADE						

FREE RESPONSE SCORE:___________________

1. Evaluate the integral $\int \frac{x^3-x+2}{x^2+1}\,dx$. DO NOT USE YOUR CALCULATOR. YOU MUST SHOW YOUR WORK IN ORDER TO GET CREDIT FOR THIS PROBLEM.

2. (a) Use the trapezoidal rule with $n = 3$ to estimate the value of $\int_2^8 \frac{1}{1+2x}\,dx$. Show your work. (Do not just calculate the exact answer. Be sure to use the Trapezoidal rule.)

(b) The maximum possible error when using the Trapezoidal rule to approximate $\int_a^b f(x)\,dx$ is $|E_T| \le \frac{K(b-a)^3}{12n^2}$ where $|f''(x)| \le K$ for $a \le x \le b$. Use this error formula to determine the maximum possible error for the estimation in part (a). Be sure to show how you find K.

(c) Use the error formula above to determine how large n should be if we want to use the Trapezoidal rule to estimate the value of $\int_2^8 \frac{1}{1+2x}\,dx$ with an error less than 0.001.

3. For each series below, determine whether the series converges or diverges. You must show your work and indicate which test you use to determine convergence or divergence.

(a) $\sum\limits_{n=1}^{\infty} \frac{n+2}{n^2+3n}$

(b) $\sum\limits_{n=1}^{\infty} \frac{n}{1+e^{-n}}$

(c) $\sum\limits_{n=1}^{\infty} \frac{n^2}{2^n}$

4. In this problem, we wish to find the arclength of the parabola $y = x^2$ from the point $(0, 0)$ to the point (2,4).

(a) Write an integral formula for the arclength, but do not evaluate this integral yet. Be sure to indicate the limits of integration.

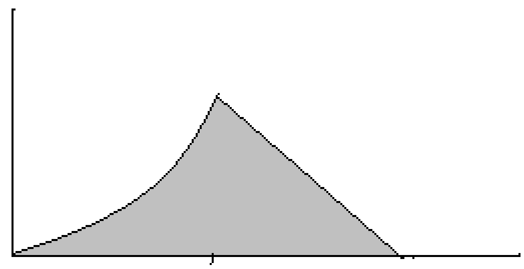
(b) Evaluate the integral in part (a). One of the following formulas may be useful:

i. $\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2} sin^{-1}\left(\frac{u}{a}\right) + C$

ii. $\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2} ln|u + \sqrt{u^2 - a^2}| + C$

iii. $\int \sqrt{u^2 + a^2}\, du = \frac{u}{2}\sqrt{u^2 + a^2} + \frac{a^2}{2} ln\left(u + \sqrt{u^2 + a^2}\right) + C$

5. Consider the shaded region shown in the diagram below:



(a) Find the area of the shaded region.

(b). We wish to find the y-coordinate of the centroid for the shaded region shown above. We have learned that, if a region is bounded above by $y = f(x)$ and below by $y = g(x)$ on an interval $[a, b]$, then

$$\overline{x} = \frac{\int_a^b x(f(x)-g(x))dx}{\int_a^b (f(x)-g(x))dx} \text{ and } \overline{y} = \frac{\frac{1}{2}\int_a^b ([f(x)]^2-[g(x)]^2)dx}{\int_a^b (f(x)-g(x))dx}.$$

Find the y-coordinate of the centroid. You do NOT need to find the x-coordinate.