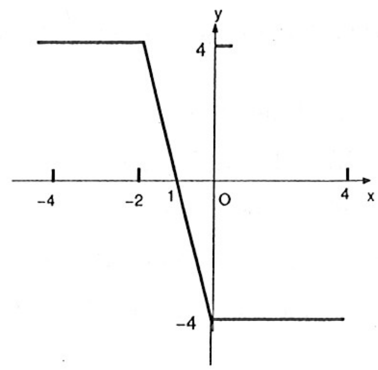
1. The definite integral $\int_0^1 (4x - 2)\mathrm{d}x$ equals
   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4

2. The graph of a function $g(x)$ is shown to the right, which consists of three straight lines. Then the definite integral $\int_{-4}^{4} g(x)\mathrm{d}x$ equals
   (a) -16
   (b) -8
   (c) 8
   (d) 16
   (e) 28



3. The indefinite integral $\int \cot x\mathrm{d}x = \int \frac{\cos x}{\sin x}\mathrm{d}x$ equals
   (a) $\ln|\sin x| + C$
   (b) $\ln|\cos x| + C$
   (c) $\ln|\sec x| + C$
   (d) $\ln|\csc x| + C$
   (e) $\tan x + C$

4. The definite integral $\int_0^1 \frac{x^2}{(x^3+1)^2}\mathrm{d}x$ equals
   (a) $-1/2$
   (b) $-1/6$
   (c) $1/6$
   (d) $1/3$
   (e) $1/2$

5. The indefinite integral $\int 5x^4 \ln x dx$ equals

(a) $x^5 \ln x - x^5 + C$

(b) $x^5 \ln x + x^5 + C$

(c) $x^5 \ln x - \frac{1}{5}x^5 + C$

(d) $x^5 \ln x + \frac{1}{5}x^5 + C$

(e) $20x^3 \ln x - 5x^3 + C$

6. The indefinite integral $\int 4x \sin(2x) dx$ equals

(a) $2\cos(x^2) + C$

(b) $-2x\cos(2x) + C$

(c) $2x\cos(2x) + C$

(d) $\sin(2x) - 2x\cos(2x) + C$

(e) $\sin(2x) + 2x\cos(2x) + C$

7. Let $F(x) = \int_1^x \ln(t)\, dt$. Then $F''(2)$ equals

(a) 0

(b) 1/2

(c) 1

(d) 2

(e) ln(2)

8. The improper integral $\int_0^\infty e^{-3x} dx$ equals

(a) 0

(b) 1/3

(c) 1

(d) 3

(e) ∞

9. The following integral form appears in the Table of Integrals in your text:

$$\int \frac{1}{\sqrt{u^2 + a^2}} \, du = \ln\left(u + \sqrt{u^2 + a^2}\right) + C.$$

Using this to find the definite integral

$$\int_0^1 \frac{1}{\sqrt{x^2 + 2x + 2}} \, dx.$$

(Note that $x^2 + 2x + 2 = (x+1)^2 + 1^2$)

(a) $\ln\left(\sqrt{5}\right) - \ln\left(\sqrt{2}\right)$

(b) $\ln\left(1 + \sqrt{5}\right) + \ln\left(\sqrt{2}\right)$

(c) $\ln\left(1 + \sqrt{5}\right) - \ln\left(\sqrt{2}\right)$

(d) $\ln\left(2 + \sqrt{5}\right) - \ln\left(1 + \sqrt{2}\right)$

(e) $\frac{1}{\sqrt{5}} - \frac{1}{\sqrt{2}}$

10. The length of the curve $y = x^2$ from the point $(0, 0)$ to the point $(2, 4)$ is

(a) $\int_0^2 \sqrt{1 + 2x} \, dx$

(b) $\int_0^2 \sqrt{1 + 4x^2} \, dx$

(c) $\int_0^4 \sqrt{1 + 2x} \, dx$

(d) $\int_0^4 \sqrt{1 + 4x^2} \, dx$

(e) $\int_0^4 \sqrt{1 + x^4} \, dx$

11. Let $a_n = \dfrac{2n^2 - 100n + 1}{5 - n^2}$ for $n = 1, 2, 3, \cdots$. Which of the following statements is true?

(a) The sequence $\{a_n\}$ converges to 0.

(b) The sequence $\{a_n\}$ converges to 2/5.

(c) The sequence $\{a_n\}$ converges to 1/5.

(d) The sequence $\{a_n\}$ converges to -2.

(e) The sequence $\{a_n\}$ diverges.

12. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

(a) is absolutely convergent.

(b) is convergent, but not absolutely convergent.

(c) is divergent.

(d) equals 0.

(e) equals 1.

13. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$

(a) converges by the ratio test.

(b) diverges by the ratio test.

(c) diverges because it is a p-series with $p = 1$.

(d) converges because it is a p-series with $p > 1$.

(e) diverges by comparison of its terms with the terms of the series $\sum_{n=1}^{\infty} \frac{1}{n}$.

1. A table of values of a function f is shown. Find the Riemann sum for f on the interval $[0,8]$, using 4 subintervals of equal width and taking the sample points to be the right endpoints.

x	0	2	4	6	8
$f(x)$	4	1	2	1	2

(a) 6
(b) 8
(c) 12
(d) 16
(e) 20

2. Let $\int_0^3 f(x)\mathrm{d}x = 2$ and $\int_0^3 g(x)\mathrm{d}x = -5$. Then $\int_0^3 (3f(x) - g(x))\,\mathrm{d}x$

(a) equals 11
(b) equals 7
(c) equals 1
(d) equals -3
(e) cannot be calculated from the given information.

3. If $\int_5^0 f(x)\mathrm{d}x = 2$ and $\int_5^{10} f(x)\mathrm{d}x = -5$, then $\int_0^{10} f(x)\mathrm{d}x$

(a) equals 3
(b) equals 7
(c) equals -3
(d) equals -7
(e) cannot be calculated from the given information.

4. The integral $\int_{-1}^1 \frac{2}{x^3}\,\mathrm{d}x$

(a) is a common definite integral and equals 0.
(b) is a common definite integral and equals 2.
(c) is an improper definite integral and equals 1.
(d) is an improper definite integral and equals -1.
(e) is a divergent improper definite integral.

5. The indefinite integral $\int \frac{x+1}{(x+2)(x-3)}dx$ equals

(a) $\ln|x+2| + \ln|x-3| + C$

(b) $\frac{4}{5}\ln|x+2| - \frac{1}{5}\ln|x-3| + C$

(c) $\frac{1}{5}\ln|x+2| - \frac{4}{5}\ln|x-3| + C$

(d) $\frac{4}{5}\ln|x+2| + \frac{1}{5}\ln|x-3| + C$

(e) $\frac{1}{5}\ln|x+2| + \frac{4}{5}\ln|x-3| + C$

6. The area enclosed by the curves $y = x$ and $y = 3x - x^2$ is

(a) 2/3

(b) 4/3

(c) 10/4

(d) 9/2

(e) 2

7. Find the average value of $f(x) = 3x^2 - 12x + 13$ on the interval $[0, 3]$:

(a) 4/3

(b) 2

(c) 4

(d) 6

(e) 12

8. A particle is moved along the x-axis by a force $F(x) = 6x^2 + 4x - 2$. How much work is done in moving the particle from $x = -1$ to $x = 2$?

(a) 12

(b) 14

(c) 16

(d) 18

(e) 20

9. What is the coefficient of $(x+1)^2$ in the Taylor expansion of $f(x) = x^3 + x - 1$ about $a = -1$?

(a) $1/2$
(b) 1
(c) -1
(d) -3
(e) -6

10. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^2}$.

(a) The interval of convergence is $[0, 2]$.
(b) The interval of convergence is $[0, 2)$.
(c) The interval of convergence is $(0, 2)$.
(d) The interval of convergence is $(0, 2]$.
(e) The interval of convergence is $(-\infty, \infty)$.

11. The geometric series $\displaystyle\sum_{n=1}^{\infty} 2^n 5^{1-n} = \sum_{n=1}^{\infty} 2\left(\frac{2}{5}\right)^{n-1}$

(a) converges to 2.
(b) converges to 5/3.
(c) converges to 10/3.
(d) converges to 10.
(e) diverges.

12. Which of the following series **diverges**?

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{100n^3 + 7}$.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$.

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)}$.

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n+1}$.

Part III

1. (a) Use Simpson's rule with $n = 4$,

$$S_4 = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)]$$

to approximate the definite integral $\int_1^5 \ln x dx$.

(b) The error estimate when Simpson's rule is used to approximate $\int_a^b f(x)dx$ is given by

$$|E_n| \leq \frac{K(b-a)^5}{180n^4},$$

where n is the (even) number of subintervals, and K is an upper bound for $\left|f^{(4)}(x)\right|$ on $[a, b]$. Estimate the error of the numerical integration in part (a) above.

2. Consider the region in the first quadrant bounded by the curves $y = x^3, y = 1$, and the y-axis.

   (a) Find the area of the region.

   (b) Find the x-coordinate of the centroid of the region.

   (c) Find the y-coordinate of the centroid of the region.

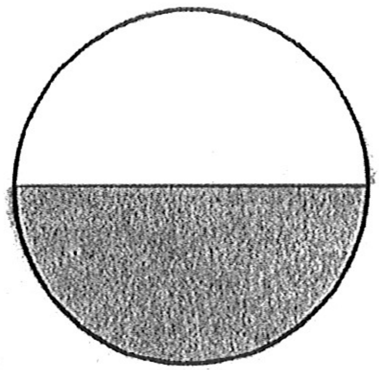
3. Consider the region in the plane bounded by the curves $y = 0, y = x^2 + 1, x = 0$ and $x = 1$.

(a) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the x-axis.

(b) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the line $y = 2$.

(c) Set up (but do not evaluate) a definite integral which gives the volume of the solid formed when the region is rotated about the line $x = 1$. (You may want to use the shell method.)

4. A **spherical** tank with radius 5 meters is partially filled with water, 5 meters deep in the middle. How much work is required to pump all the water out through a hole at the top of the tank? (Use the facts that the mass density of water is 1000 kg/m^3 and the acceleration due to gravity is 9.8 m/s^2.)



5. The Maclaurin power series expansion for $\cos x$ is

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots .$$

(a) Write down the first FIVE nonzero terms of the Maclaurin series for $\cos(t^2)$.

(b) Using part (a), find the first FIVE nonzero terms of the Maclaurin series for the function $f(x) = \int_0^x \cos(t^2)dt$.

(c) Using part (b) with the first THREE nonzero terms to estimate $\int_0^1 \cos(t^2)dt$.