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Frege's Commitment to an Infinite Hierarchy of Senses

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Abstract: Though it has been claimed that Frege's commitment to expressions in indirect contexts not having their customary senses commits him to an infinite number of semantic primitives, Terrence Parsons has argued that Frege's explicit commitments are compatible with a two-level theory of senses. In this paper, we argue Frege is committed to some principles Parsons has overlooked, and, from these and other principles to which Frege is committed, give a proof that he is indeed committed to an infinite number of semantic primitives—an intolerable result.

Frege's claim that expressions in indirect contexts express not their customary senses but instead their indirect senses is often supposed to commit his theory to each expression of a natural language like English having infinitely many senses as a result of its possible occurrence within each level of an infinite hierarchical structure of embedded, indirect contexts. We will call this commitment 'the infinite hierarchy of senses'. If Frege is committed to the infinite hierarchy of senses, it is a serious difficulty for his position, for, plausibly, it entails that no natural language like English is learnable, contrary to fact.[1]

It is controversial whether Frege's theory commits him to the infinite hierarchy of senses. Terence Parsons has argued forcefully that, contrary to common belief, Frege need not be committed to the infinite hierarchy of senses and that a two-level theory of sense, which accords with the *literal* interpretation of Frege's theory as set forth in *On Sense and Reference* (hence, *OSR*), is consistent with Fregean doctrine (Parsons 1981, 38). The Two-Level Theory Parsons outlines is clearly consistent;

1 As Davidson (1984) has argued in 'Theories of Meaning and Learnable Languages'.

we even allow that it is consistent with Frege's published claims.[2] However, in this paper, we will argue that there is important, overlooked evidence that Frege's theory includes two principles that Parsons does not consider, which, together with the rest of the theory, entail the infinite hierarchy of senses.

We arrive at this conclusion by establishing the following claims. (i) Frege often states, and is committed to, what we will call 'The Complex Sense Compositionality Principle'. (ii) There is good reason to attribute to Frege what we will call 'The Indirect Referent Compositionality Principle'. (iii) If Frege's theory includes both the Complex Sense Compositionality Principle and the Indirect Referent Compositionality Principle, then the theory entails the infinite hierarchy of senses and, hence, is inconsistent with the Two-Level Theory.

Frege, Parsons and the Literal (Two-Level) Theory

Before offering our argument, we provide a brief summary of the important claims of Frege's theory which give rise to the current controversy. Frege distinguishes between the sense and referent of an expression, the referent being the object, if any, that the expression denotes (*OSR*, 57), and the sense being what a competent speaker of a language 'grasps' when she understands that expression.[3] The sense is said to 'contain' its 'mode of presentation' of the expression's referent (*OSR*, 57). Thus, the sense of 'Hesperus' might be expressed by 'the brightest planet on the horizon in the evening'; the sense of 'Phosphorus' might be expressed by 'the brightest planet on the horizon in the

2 We have in mind only the claims he makes explicit in the works he published. In a posthumously-published letter to Russell, Frege explicitly embraces the infinite hierarchy of senses (*Philosophical and Mathematical Correspondence* (hereafter *PMC*), 153-154). His aim there is to respond to Russell's paradox, so it is not clear from this evidence that his commitment runs any deeper than its utility in this pursuit. We will argue that Frege is committed to the infinite hierarchy of senses and that this commitment is independent of his attempt to use the infinite hierarchy to defend himself against Russell's paradox and is central to his conception of the semantics of indirect contexts.

3 We use the terms 'referent' and 'reference' interchangeably and in place of Frege's 'Bedeutung'. In addition we use the term 'denote' in place of Frege's 'bedeuten'.

morning'; and the referent of both 'Hesperus' and 'Phosphorus' is Venus. Every meaningful expression of a language has a sense that determines its referent, provided there is one (*OSR*, 58).

Frege claims that when an expression occurs in a referentially transparent, or direct, context, the utterer of the sentence containing that expression intends to talk about the referent of that expression (*OSR*, 58). In contrast, when an expression occurs in a referentially opaque, or indirect, context, the utterer intends to talk about its sense (*OSR*, 58).[4] Consequently, Frege introduces the notions of *customary* and *indirect senses* as well as *customary* and *indirect referents*:

> In reported speech, one talks about the sense, e.g., of another person's remarks. It is quite clear that in this way of speaking words do not have their customary reference but designate what is usually their sense. In order to have a short expression, we will say: In reported speech, words are used indirectly or have their indirect reference. We distinguish accordingly the customary from the indirect reference of a word; and its customary sense from its indirect sense. The indirect reference of a word is accordingly its customary sense. (OSR, 59)

Thus, in direct discourse, an expression expresses its customary sense and designates its customary referent, while in indirect discourse an expression expresses its indirect sense and designates its indirect referent, which is its customary sense.[5] In (0), 'Hesperus' refers to Venus and expresses its customary sense, which we will suppose is the sense of 'the brightest planet on the horizon in the evening'. However, in (1), 'Hesperus' designates its indirect referent and expresses its indirect

4 Frege calls the occurrence of an expression in such a context 'indirect quotation' (Cf., *OSR*, 147, 149).

5 Unless a particular sentential context is specified, 'the sense of' and 'the referent of' should be taken as synonymous with 'the contextual sense of' and 'the contextual referent of', respectively. We define these two expressions as follows. For any n, the contextual sense (referent) of an expression that occurs in an n-indirect context is its n-indirect sense (referent).

sense. So, since the indirect referent of 'Hesperus' is its customary sense, the referent of 'Hesperus' in (1) is the customary sense of 'the brightest heavenly body on the horizon in the evening'.

(0) Hesperus is bright

(1) Anna believes that Hesperus is bright

Unfortunately, Frege does not tell us anything about the content of the indirect sense of 'Hesperus' or of any other expression.

Although Frege deals in *OSR* primarily with the senses and referents of what he calls 'names', he also discusses the senses and referents of declarative sentences. The sense of a sentence he calls a 'thought', and the referent, if there is one, is the sentence's truth-value, either 'the True' or 'the False' (*OSR* 62-63). The rules for direct and indirect contexts apply to sentences as well. Let H be 'Hesperus is bright'. If H occurs in a *direct* context, as in (0), then it expresses a thought[6] and denotes its customary referent, the True. If H occurs in an *indirect* context, as in (1), then it expresses its indirect sense[7] and denotes its indirect referent, which is the thought it expresses in direct contexts.

At this point a natural question arises: What sense is expressed by H when it occurs in *further levels of indirect embedding*, as in (2)?

(2) Bettina believes that Anna believes that Hesperus is bright.

Since Frege does not discuss such cases, his theory must be extended to cover them. However, there is more than one way to extend the theory. One way to do so is to take Frege's above-quoted introduction to the direct/indirect distinctions as perfectly general and definitive. Parsons claims that if that introduction is taken literally, Frege's theory involves only two levels of sense, the customary and the indirect. So taken, Frege's

6 Intuitively, the thought expressed is the thought that Hesperus is bright. Support for this intuition may be derived from Frege's comments on the thoughts expressed by more complex sentences (*OSR* 67, 77).

7 Frege claims in *OSR* that the indirect sense of a sentence, j, is just the sense of the expression the thought that φ (66).

theory includes the general principle that any expression that is embedded in an indirect context, regardless of the number of levels of indirect embedding, expresses its indirect sense and denotes its customary sense. Thus, in both (1) and (2), H expresses its indirect sense and denotes its customary sense. Parsons labels this extension of Frege's theory the 'Literal (Two-Level) Theory' and claims that it is not inconsistent with anything to which Frege is committed. Parsons thinks that Frege does not commit himself to such an extension; indeed, Parsons thinks that Frege (eventually) accepted instead the extension Parsons labels the 'Orthodox Theory', the theory to which we now turn (Parsons 1981, 40).

Traditionally, the Two-Level theory has been thought to be inconsistent with other Fregean doctrines and, therefore, has been abandoned in favor of a different extension of Frege's theory, the Orthodox Theory, which holds that an expression has not only its customary and indirect senses, but infinitely many senses.[8,9] That is, an expression occurring in a direct context expresses its customary sense and refers to its customary referent; when it occurs in a singly-indirect context it expresses its indirect sense and refers to its customary sense; in a doubly-indirect context, it expresses its doubly-indirect sense and refers to its indirect sense; in a triply-indirect context, it expresses its triply-indirect sense and refers to its doubly-indirect sense; and so on, *ad infinitum*.[10] The general principle given by the Orthodox Theory is that an expression expresses its n-indirect sense and denotes its (n-1)-indirect sense for any n levels of indirect embedding, where the 0-indirect sense

8 An enumeration of these doctrines begins on p. 6.

9 See, for example, Dummett (1981, 267), Stegmuller (1968, 149), and, on a plausible reading, Carnap (1967, 131).

10 A word occurs in a singly-indirect context just in case it occurs in an indirect context and that context itself does not occur in an indirect context. A word occurs in a doubly-indirect context just in case it occurs in an indirect context and that context itself occurs in a singly-indirect context. In general, we shall say that a word occurs in an n-indirect context just in case it occurs in an indirect context and that context itself occurs in an n-1 indirect context (we will call a 0-indirect context a 'direct context'). We define the n-indirect sense and n-indirect referent of an expression accordingly.

is just its customary sense. Thus, according to the Orthodox Theory, each expression has an infinity of senses, since there is an infinity of contexts requiring distinct senses.

If Frege is committed to the Orthodox Theory, then he, and anyone who holds it, must explain how it is possible for one to learn a natural language. As finite beings, our cognitive capacities for grasping senses must also be finite. However, according to the Orthodox Theory, we would have to grasp infinitely many senses in order to learn each expression (including primitive expressions). So, unless there were some way to grasp higher-level senses on the basis of lower-level ones, i.e., unless each higher-level sense were a function of lower-level ones, we could not learn a natural language containing those expressions, if this theory were correct.[11] We obviously do learn natural languages. Therefore, unless there is some function that allows us to determine higher-level senses from lower-level ones, Frege's theory, if it is the Orthodox Theory, is mistaken. However, there does not seem to be any principled reason to think that a function of that sort is at work.[12]

As mentioned above, the Two-Level Theory has traditionally been thought to be inconsistent with Fregean doctrines. Parsons, however, says that despite several authors' claims that the Two-Level Theory is inconsistent with other Fregean doctrines, he has not found any *proof* of such an inconsistency (1981, 42). He seems to suggest that philosophers who think Frege's view inconsistent have reasoned as follows (Parsons 1981, 42-43). Frege is committed to at least the following doctrines:

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11 We thank Bob Beard and Terence Parsons for bringing to our attention how such a function could make learning a language possible, even given an infinite number of semantic primitives.

12 One suggestion might be that we can grasp a higher-level sense that determines a given sense expressed by φ in L by appeal to our grasp of the sense of the sense of φ in L . There are a number of reasons to reject this suggestion that are too involved to discuss here. It is sufficient, however, to note that Frege is committed to rejecting it, since Frege took it to be fundamental that our grasp of senses is language independent. On his view there would have to be some other, more primary way of grasping the higher-order sense independently of identifying it as something expressed by some expressions in a language.

(3) For any expression φ, the indirect referent of φ is the customary sense of φ (*OSR*, 59);

(4) For any complex expression[13] ψ, the sense of ψ is the value of a function which takes as arguments the senses of its component expressions (Sense Functionality Principle);[14]

(5) For any complex expression ψ, the referent of ψ is the value of a function which takes as arguments the referents of its component expressions (Reference Functionality Principle).[15]

An inconsistency might be thought to arise in the following way. Consider sentence form (6).

(6) Bettina believes that ψ.

Since ψ occurs in a singly-indirect context, the referent of ψ in (6), i.e., the indirect referent of ψ, is the customary sense of ψ. Let ψ be the complex expression ⌜Anna believes that φ⌝ . (6) then yields (7).

(7) Bettina believes that Anna believes that φ.

For all sentences φ, the contextual referent of ⌜Anna believes that φ⌝ in (7) is the customary sense of ⌜Anna believes that φ⌝ . By (4) and (5) we know that, for all sentences φ, the indirect referent of ⌜Anna believes that φ⌝ is a function of the indirect referents of that expression's parts, as is

13 For the purposes of this paper, a complex expression is one that, on Frege's view, is analyzable into sense-expressing parts.

14 Some support is generated for this principle in the following passage: 'If it were a question only of the sense of the sentence, the thought, it would be unnecessary to bother with the reference of a part of the sentence; only the sense, not the reference, of the part is relevant to the sense of the whole sentence' (*OSR* 62-63; see also 66). That the textual evidence for this principle might be considered relatively weak is not an objection to our making use of it, since, as we will show, Frege commits himself to an even stronger principle.

15 This principle is supported by Frege's claims (i) that sentences containing names that do not refer do not *themselves* refer, and (ii) that names occurring in sentences may be replaced *salva veritatae* if and only if the replacing names designate the same object as that designated by the replaced names (*OSR*, 62-64, 67,72-73, 78).

the customary sense of ⌜Anna believes that φ⌝ . We assume the Two-Level Theory and adopt the following notation. For any sentence φ,

R_0 (φ) = the customary referent of φ;

R_i (φ) = the i^{th} indirect referent of φ, for all i ≥ 1;

S_0 (φ) = the customary sense of φ; and

S_i (φ) = the i^{th} indirect sense of φ, for all i ≥ 1

We apply the Complex Customary Sense Function,[16] which we abbreviate as 'F_{S_0}' and the Complex Indirect Referent Function,[17] which we abbreviate as 'F_{R_1}', to the sentence-form ⌜Anna believes that φ⌝ as in (4*) and (5*):[18]

(4*) F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)) = S_0 (⌜Anna believes that φ⌝)[19]

(5*) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 (⌜Anna believes that φ⌝)

Since we are assuming the Two-Level Theory, the doubly-indirect referent of φ and the indirect referent of φ are identical. Thus, the third argument of F_{R_1} is R_1 (φ), the indirect referent of φ. With this notation in place we can express the identity of the indirect referent and customary sense of ⌜Anna believes that φ⌝ as (3*).

16 The Complex Customary Sense Function is the function referred to in (4).

17 The Complex Indirect Referent Function is the function referred to in (5).

18 For simplicity, we do not explicitly bind 'φ' to the universal quantifier having as its domain the set of all sentences. All subsequent instances of 'φ' should be taken as being so bound.

19 One might read (4*) as follows: 'The value of the Complex Customary Sense Function which takes as arguments the customary sense of "Anna", the customary sense of "believes that" and the indirect sense of φ is the customary sense of Anna believes that φ '. Similarly for (5*).

(3*) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)).

Since the indirect referent of any expression is the customary sense of that expression, we can replace all the arguments of F_{R_1} accordingly. Thus,

(3**) F_{R_1} (S_0 ('Anna'), S_0 ('believes that'), S_0 (φ)) = F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)).

To quote Parsons, this result 'is a surprise' (1981, 42) in that two three-place functions whose arguments are identical with the exception only of the third argument have the same value, *viz*., the customary sense of ⌜Anna believes that φ⌝.[20] Parsons seems to suggest that this has led some to conclude that the Two-Level Theory is inconsistent: '[S]urprises of this sort often suggest that there may be an inconsistency lurking *somewhere* around' (1981, 42). However, though this result may be a surprise, Parsons is surely correct that there is nothing inconsistent about asserting that there are two three-place functions, whether identical or distinct, that differ only by their third arguments and have the same value. For example, the value of the function $f(x, y, z) = x + y + z^2$ with arguments $<1, 1, 1>$ or $<1, 1, -1>$ is the same, namely, 3. Similarly, the two *different* functions $g(x, y, z) = x + y + z$ and $h(x, y, t) = x + y + t/2$ with arguments $<2, 3, 2>$ and $<2, 3, 4>$, respectively, have the same value, namely, 7. Thus, it is not at all yet clear why we should conclude that the Two-Level Theory is inconsistent with Fregean doctrine and, hence, why we should conclude that Frege is committed to the infinite hierarchy of senses.

20 Parsons, making use of the plausible assumption that S_0 ('Anna believes that') is a function taking S_0 (φ) as its argument, puts the result somewhat differently: '[W]henever f is a sense that is also a function, if f is ever applied to the indirect sense of a word in the semantical analysis of a sentence, then f maps that indirect sense to the same thing to which it maps the customary sense of the word'. (1981, 43)

The Compositionality Principles of Sense and Reference

However, there is indeed an inconsistency between the Two-Level theory and other Fregean doctrines 'lurking somewhere around,' and one can begin to discover its whereabouts by taking into account two doctrines, (8) and (9), that Parsons does not consider.[21] Principle (8) says,

(8) A complex expression expresses a complex sense that is wholly composed of the senses of its component expressions

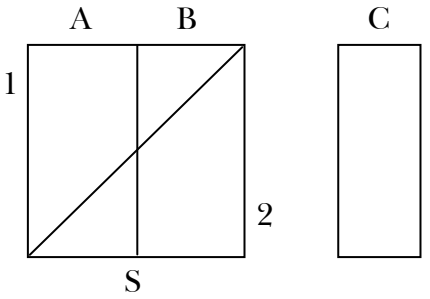

Figure 1

where 'wholly composed' indicates that though a complex sense may be variously decomposed, it cannot have any part that is not composed of parts of the senses of the component expressions.[22] For example, the complex sense S of an expression containing two expressions whose senses are A and B respectively, as shown in Figure 1, can be decomposed into either A and B (the two adjacent rectangles) or into 1 and 2 (the two large triangles), but it cannot contain C as a part, where C is distinct from A and B and their parts.

Principle (9) says,

21 The appendix is a succinct version of the following argument.

22 We leave unexplained *how* the sense's parts compose the whole, since the *method* of composition is irrelevant to our project; it is sufficient for our purposes to claim simply *that* they do so.

(9) For all sentences φ, the indirect referent of ⌜Anna believes that φ⌝ is composed of the indirect referent of 'Anna', the indirect referent of 'believes that' and the doubly-indirect referent of φ,

which we present more formally as (9*).

(9*) R_1 (⌜Anna believes that φ⌝) = R_1 ('Anna') • R_1 ('believes that') • R_2 (φ),

where the '•' signs serve to join the descriptions of the parts of the complex in a way that represents both what parts the complex has and that those parts stand in a composition relation. Of course, (9) is actually an instance of a more general principle, (9**),

(9**) For all sentences φ, if φ occurs in an n-indirect context, where n ≥ 1, then the n-indirect referent of φ is composed of the contextual referents of its component expressions,

but, for simplicity of presentation, we use (9) and (9*) rather than (9**). The reader may notice that (9**) is not as general as (8) and may worry about how to make sense of an application to sentences in direct contexts of a referent compositionality principle more general than (9**), since such a principle would entail the mysterious claim that if 'John is tall' is true, then John is part of the True. However, whether the restricted principle, (9**), can be extended to cover sentences in direct contexts is irrelevant to whether Frege is committed to the restricted principle, and we argue in the next section that he is so committed. Granted, one might argue against Frege's theory of reference on the grounds that he does not provide us with a reference function that is completely general, but (a) it is not clear that he does not provide such a function, since it just is not clear what 'the True' and 'the False' are, and (b) even if one did argue on these grounds, this might just be an *additional* argument against Frege's theory of reference, but it would not be an objection to our argument that Frege is committed to a reference compositionality function for sentences in indirect contexts.

In this section we show that if the best way to make sense of Frege's theory requires including in it these doctrines ((8), (9), (9*), and (9**)), then his theory is inconsistent with the Two-Level Theory and is a version of the Orthodox Theory. In the next section, we show that Frege is committed, and that it is natural for him to be committed, to both (8) and (9). The Two Level Theory *is* strange from a Fregean perspective.

For now, let us suppose that Frege's theory includes (8) and (9). If these principles are part of Frege's theory, then we know more about F_{R_1} and F_{S_0} than we have said above. We know that these functions are *composition* functions and, thus, we know something more about the values they have for certain sorts of arguments. Principle (8) says that the sense of a complex expression is wholly *composed* of the senses of the component parts of the expression. Frege takes this principle to apply to all complex expressions, but we are particularly interested in its application to sentences containing indirectly embedded sentences. An important consequence of this application is that given (4*), (10) must also hold,

(10) F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)) = S_0 ('Anna') ^ S_0 ('believes that') ^ S_1 (φ),

where the '^' signs serve to join the description of the parts of the complex sense in a way that represents what parts the complex has and that those parts *wholly* compose the complex.[23] Thus F_{S_0} simply takes three senses as arguments and yields a complex sense that they wholly compose. From (4*) and (10), (11) follows.

(11) S_0 ('Anna believes that φ') = S_0 ('Anna') ^ S_0 ('believes that') ^ S_1 (φ).

23 One might read (10) as follows: 'The value of the Complex Customary Sense Function which takes as arguments the customary sense of 'Anna', the customary sense of 'believes that' and the indirect sense of φ is something that is wholly composed of the customary sense of 'Anna', the customary sense of 'believes that' and the indirect sense of φ'

That is, the customary sense of ⌜Anna believes that φ⌝ is wholly composed of the customary sense of 'Anna', the customary sense of 'believes that', and the indirect sense of φ.

Principle (9) is significantly narrower in scope than (8), but, as will become clear, it is broad enough for our purposes. From (9*) and (5*), (12) follows.

(12) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 ('Anna') • R_1 ('believes that') • R_2 (φ).

Recall that we are assuming the Two-Level Theory, and, thus, R_2 (φ) = R_1 (φ). Hence, (13) follows from (12).

(13) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 ('Anna') • R_1 ('believes that') • R_1 (φ).

Fregean principle (3) tells us that the indirect referent of an expression is its customary sense. Thus, from (13) and (3) we can infer (14).

(14) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = S_0 ('Anna') • S_0 ('believes that') • S_0 (φ).

From (14) and (5*), (15) follows.

(15) R_1 (⌜Anna believes that φ⌝) = S_0 ('Anna') • S_0 ('believes that') • S_0 (φ).

Thus, the indirect referent of ⌜Anna believes that φ⌝ has as a part the *customary* sense of φ. We now claim in (16), which we will call the 'No Overlap Principle', that certain kinds of sense overlap that might be thought to invalidate our argument fail to occur. (Because the argument for (16) is rather lengthy and may distract the reader from the present task, we present it only after the conclusion of the present argument.)

(16) S_0 (φ) is neither a part of nor identical to S_1 (φ) and has neither S_0 ('believes that') nor S_0 ('Anna') as a part.

From (11) and (16) we get (17).

(17) S_0 (⌜Anna believes that φ⌝) does not have S_0 (φ) as a part.

So, we know from (15) that R_1 (⌜Anna believes that φ⌝) has the customary sense of φ as a part, but we know from (17) that S_0 (⌜Anna believes that φ⌝) does not. It follows immediately that the two must be distinct, which is expressed in (18).

(18) R_1 (⌜Anna believes that φ⌝) ≠ S_0 (⌜Anna believes that φ⌝).

But this explicitly contradicts (3), which entails that R_1 (⌜Anna believes that φ⌝) = S_0 (⌜Anna believes that φ⌝). Thus, if Frege is committed to (8) and (9), then the Two-Level Theory is inconsistent with Fregean doctrine. In fact, we will show below that any theory with two or more levels of sense is inconsistent with Fregean doctrine. Before doing so, however, we pause to argue for premise (16) above.

We will first explain why (16) is required for the validity of our argument and, then, argue for its truth.

If in (1)

(1) Anna believes that Hesperus is bright,

it were the case that S_0 ('Hesperus is bright') were a proper or improper part of S_1 ('Hesperus is bright') or part of S_0 ('Anna') or S_0 ('believes that'), then we could not on the basis of (11) claim that S_0 ('Hesperus is bright') is not a part of S_0 ('Anna believes that Hesperus is bright'). For though (11), applied to (1), entails that S_0 ('Anna'), S_0 ('believes that') and S_1 ('Hesperus is bright') *wholly* compose S_0 ('Anna believes that Hesperus is bright'), this does not rule out the presence of other senses that are composed of parts of just these three. Thus if S_0 ('Hesperus is bright') is composed of only parts of these three senses, it may yet be a part of S_0 ('Anna believes that Hesperus is bright'), in which case we cannot derive the contradiction in (18). However, as we will now argue, S_0 ('Hesperus is bright') is not composed of only parts of S_0 ('Anna'), S_0 ('believes that') or S_1 ('Hesperus is bright').

If the claim that S_0 *is* so composed is not to be an *ad hoc* objection, then since 'Anna believes that Hesperus is bright' is an arbitrary sentence, the claim would have to be grounded in the following principle:

> Principle: For all φ and for all $n \geq 1$, if S_n (φ) is part of a complex sense, then so is S_{n-1} (φ).

We reject this principle on the following three grounds. First, using this principle to defend Frege is simply *ad hoc*. Second, it is certainly not supported by anything Frege says, nor does it seem to be something to which he is committed. Third, it would make Frege's theory even more mysterious, if not incoherent. For, given the general principle just articulated and our claim that S_0 (⌜Anna believes that φ⌝) is wholly composed of S_0 ('Anna'), S_0 ('believes that') and S_1 (φ), one who holds the general principle must also hold either (i), (ii), or (iii) regarding S_0 (φ).

(i) S_0 (φ) is a proper part of S_1 (φ)

(ii) S_0 (φ) = S_1 (φ)

(iii) S_0 (φ) has as parts (a) a proper or improper part of S_1 (φ) and (b) part of S_0 ('believes that') or part of S_0 ('Anna').

However, none of these options is acceptable for use in an objection to our argument. First, option (i) does not prevent the hierarchy of senses. Second, a theory including (ii)—which, in essence, is the 'One-Level Theory' of senses—has consequences that are simply unacceptable for any semantic theory that posits senses, for it violates both Frege's principle that each sense determines at most a single referent and its sentential correlate that each thought determines at most one truth value.[24] To show the absurdity of these violations, we focus on the disastrous consequence that intuitively univocal sentences, those that on a Fregean theory express only one thought, can, nevertheless, be both

24 Of course, the One-Level Theory is explicitly denied by Frege, but we aim to show that he is committed to rejecting it.

true and false! To see this, first note the consequences for non-sentential expressions. The One-Level Theory entails that, e.g., S_0 ('Hesperus') 'determines' both Venus *and itself*, the former for the obvious reasons, and the latter since (according to (ii)) S_0 ('Hesperus') = S_1 ('Hesperus'), and since the indirect sense of an expression determines that expression's customary sense.[25] Thus, on this theory, each sense of each expression 'determines' two things: itself and what it ordinarily determines. It is now easy to see that, in many cases, intuitively univocal and true sentences will turn out both true and not true! For example, even the simple sentence (0),

(0) Hesperus is bright.

would, under this theory, be both true and false. For S_0 ('Hesperus') would determine both Venus and, since it is identical to S_1 ('Hesperus'), itself, and though Venus is bright, S_0 ('Hesperus') is not. Thus, a single thought expressed by (0) determines both the True and the False, which is just the Fregean way of saying that (0) is both true and false.[26] Therefore, in addition to violating the Fregean rule that each sense determines at most one referent, a theory containing (ii) violates *the much more fundamental* rule that a univocal sentence cannot be both true and not true!

Though no further justification is needed for rejecting the One-Level Theory, we have seen only a glimpse of its confusion. Similar, and perhaps even more intractable, problems arise from taking into account, first, that the sense of the predicate 'is bright' would also, on this theory, determine two referents, one of which may not even be a function;

25 Clearly, the same applies to indirect sense. S_1 ('Hesperus') 'determines' both S_0 ('Hesperus'), which is identical to S_1 ('Hesperus'), and Venus, the former since the indirect sense of an expression determines that expression's customary sense, which, according to (ii), just is that indirect sense, and the latter since S_1 ('Hesperus') = S_0 ('Hesperus') and S_0 ('Hesperus') determines Venus.

26 If 'S_0 ('Hesperus') is bright' is recognized as a category mistake and thought on those grounds to fail even to be false, a contradiction remains, *viz*., that (0) is both true and not true, or that the thought expressed by (0) both determines the True and fails to determine the True, etc.

second, that, as a consequence, a single thought expressed by a simple sentence like (0) determines four referents, each of which may be either The True or The False or, in cases in which the predicate term does not refer to a function, a mere list of referents; and, third, that the potential number of distinct complex referents determined by each sense of a sentence increases *exponentially* with the complexity of that sense! There could hardly be a more serious indictment of a semantic theory.

The only way to salvage The One-Level Theory of senses, as far as we can see, is to argue that a customary sense determines at most one referent, though the referent that is determined differs from one context to another. According to this 'Context Principle,' a customary sense will determine in direct contexts only what it ordinarily is taken to determine in those contexts and will determine itself in indirect contexts. We reject the Context Principle on the grounds that it is an obvious violation of Frege's fundamental principles that senses are language independent and that sense *alone* determines referent, since, according to the Context Principle, it is sense *plus linguistic context* that determines referent.[27]

Regarding (iii), note that since 'φ' can stand for any sentence, the customary sense of *every* sentence S has to have as a part at least part of the indirect sense of S. Note also that since any sentence S can occur within the scope of any indirect-context-creating operator (e.g., 'believes that', 'it is possible that', 'fears that', 'hopes that', etc.), S_0 (S) has to have as a part at least part of the customary sense of every such operator or, even more absurdly, part of the customary sense of any subject term! Surely, this is not something that one can charitably attribute to Frege. We conclude that (16), the No Overlap Principle, is justified.

We now go on to show that the n-Level Theory, for any $n > 1$, is incompatible with other theses to which Frege is committed. This leaves little room for interpreting Frege as holding anything but the Orthodox

27 For a more detailed discussion of the merits and demerits of the Context Principle, see Dummett (1981, 268), Parsons (1996, 402-403), and Beaney (1996, 181-184).

Theory. To show that the n-Level Theory is incompatible with other Fregean theses, we argue as follows.

Suppose the following n-Level Theory is true for any arbitrary $n > 1$, and then consider (19).

n-Level Theory: For any $i \geq n\text{-}1$, $S_i(\varphi) = S_{n-1}(\varphi)$, and $R_i(\varphi) = R_{n-1}(\varphi)$;

(19) A_1 believes that A_2 believes that ... A_n believes that φ.

From the n-Level Theory and the Compositionality Principles (8) and (9**), we have (20) and (21).

(20) $S_{n-1}(\ulcorner A_n$ believes that $\varphi\urcorner) = S_{n-1}(\text{`}A_n\text{'}) \mathbin{\hat{}} S_{n-1}(\text{`believes that'}) \mathbin{\hat{}} S_n(\varphi)$

(21) $R_n(\ulcorner A_n$ believes that $\varphi\urcorner) = R_n(\text{`}A_n\text{'}) \bullet R_n(\text{`believes that'}) \bullet R_n(\varphi)$,

From (21) and a generalization of (3), (3'), (22) follows.

(3') For any $i \geq 1$ and expression φ, $R_i(\varphi) = S_{i-1}(\varphi)$.[28]

(22) $R_n(\ulcorner A_n$ believes that $\varphi\urcorner) = S_{n-1}(\text{`}A_n\text{'}) \bullet S_{n-1}(\text{`believes that'}) \bullet S_{n-1}(\varphi)$.

Add (23), the n-level version of the No Overlap Principle presented in (16), from which (24) follows.

(23) $S_{n-1}(\varphi)$ is neither part of nor identical to $S_n(\varphi)$ and has neither S_{n-1}('believes that') nor S_{n-1}('Anna') as a part.[29]

(24) $S_{n-1}(\ulcorner A_n$ believes that $\varphi\urcorner)$ does not have $S_{n-1}(\varphi)$ as a part.

28 (3') is a generalization of (3) to which the n-level theorist and the Orthodox theorist are committed.

29 The argument for this premise is analogous to that for (16), but is simply too lengthy to include in the paper.

So, we know from (22) that R_n (⌜A_n believes that φ⌝) has S_{n-1} (φ) as a part, and we know from (24) that S_{n-1} (⌜A_n believes that φ⌝) does not. Therefore, (25) follows.

(25) S_{n-1} (⌜A_n believes that φ⌝) ≠ R_n (⌜A_n believes that φ⌝).

But (25) contradicts (3'). Hence, any n-Level theory, for any arbitrary $n > 1$ is inconsistent with Fregean doctrine.

The Evidence for the Compositionality Principles

We turn now to the arguments for Frege's being committed to Compositionality Principles (8) and (9). The preponderance of evidence shows that Frege does hold (8), the Complex Sense Compositionality Principle. Numerous passages can be found in which Frege either states, explains, or appeals to this principle. We first provide passages showing that Frege holds that the sense of a complex expression is composed of at least the senses of its component expressions. We then provide passages that (i) show that Frege holds that the sense of a complex expression does not have parts that are not already accounted for by the senses of its component expressions,[30] and (ii) show that (8) plays an important role in Frege's account of linguistic communication.

Concerning thoughts, which are the senses of complex sentences, he writes:

> [N]ot all the parts of a thought can be complete; at least one must be 'unsaturated,' or predicative; otherwise they would not hold together. For example, the sense of the phrase 'the number 2' does not hold together with that of the expression 'the concept *prime number*' without a link. We apply such a link in the sentence 'the number 2 falls under the concept *prime number*'; it is contained in the words 'falls under,' which need to be completed in two ways—by a subject and an accusative; and only because their sense

30 Again, this means that though a complex sense may be variously decomposed, it cannot have any part that is not composed of parts of the senses of the component expressions.

> is thus 'unsaturated' are they capable of serving as a link. Only when they have been supplemented in this twofold respect do we get a complete sense, a thought. (*OSR*, 54)

In the above passage, Frege suggests that the sense of a sentence is a complex sense that is composed of the senses of the component expressions. He gives further evidence for this claim in *OSR*:

> The case of an abstract noun clause, introduced by 'that,' includes the case of indirect quotation in which we have seen the words to have their indirect reference coinciding with what is customarily their sense. In this case, then, the subordinate clause has for its reference a thought, not a truth value; as sense not a thought, but the sense of the words 'the thought, that ...,' which is only a part of the thought in the entire complex sentence. (*OSR*, 66)

Still clearer statements of the principle are found in Frege's correspondence. In a letter to Russell from October 20, 1902, Frege writes:

> [I]n saying something about the meaning [Bedeutung] of the sign '3 + 5', I express a sense, or thought. And part of this thought is not the meaning of the sign '3 + 5' but its sense. Likewise, the sense of '3', the sense of '+', and the sense of '5' are parts of the sense of '3 + 5'. The object about which I am saying something ... is always the meaning of the sign; but in saying something about it I express a thought, and the sense of the sign is a part of the thought. (*PMC*, 149)
>
> [A] class cannot be a component part of a thought, though the sense of a class name can. (*PMC*, 149)

He says the following in a May 21, 1903 letter to Russell:

> If 'p ε m ⊃ p' expresses a thought, then the sense of 'm' will be a component part of this thought. (*PMC*, 157)

> [W]hile the sense of a number sign can be part of a thought, a number itself cannot. (*PMC*, 158)

In a letter to Russell from November 13, 1904, Frege writes:

> The sense of the word 'moon' is a component part of the thought that the moon is smaller than the earth. (*PMC*, 163)
>
> When I say '7 - 1 = 6', the number 7 does not occupy the same stage as the sense of '7 - 1', any more than it occupies the same stage as the thought that 7 - 1 = 6. On the other hand, the sense of the sign '7' occupies the same stage as this thought; it can be said to be part of this thought, as well as part of the sense of '7 - 1'. (*PMC*, 165)

It should be obvious from these passages that Frege holds the view that (a) the sense of a complex expression is composed of the senses of the expression's component expressions. Still, one might argue that he is not *committed* to (a), i.e., that (a) does not play an important role in his thinking and that its negation is consistent with the rest of his views. In response to such an anticipated objection, we present two passages in which Frege uses not only (a) but also the other conjunct of (8), *viz.*,

(b) the sense of a complex expression has no parts that are not accounted for by the senses of the component expressions,

to explain the possibility of linguistic communication, and, in particular, the possibility of grasping the thoughts expressed by never before encountered sentences, and for expressing a potentially infinite number of thoughts using finite resources. That Frege uses (a) and (b) to explain the possibility of linguistic communication shows that (a) and (b) hold a central place in Frege's theory of sense and reference and, therefore, that Frege is committed to (8), the Complex Sense Compositionality Principle.

In response to a question of Philip E. Jourdain, Frege writes:

> [A] proposition consists of parts which must somehow contribute to the expression of the sense of the proposition, so they themselves must somehow have a sense. Take the proposition 'Etna is higher than Vesuvius'. This contains the name 'Etna', which occurs also in other propositions, e.g., in the proposition 'Etna is in Sicily'. The possibility of our understanding propositions which we have never heard before rests evidently on this, that we construct the sense of a proposition out of parts that correspond to the words. If we find the same word in two propositions, e.g., 'Etna', then we also recognize something common to the corresponding thought, something corresponding to this word. Without this, language in the proper sense would be impossible. We could indeed adopt the convention that certain signs were to express certain thoughts...; but in this way we would always be restricted to a very narrow area, and we could not form a completely new proposition, one which would be understood by another person even though no special convention had been adopted beforehand for this case. (*Letter to Jourdain* hence *LJ*, 319-320)

He gives a very similar argument in his paper *Compound Thoughts* (hence *CP*), part III of a series of articles entitled 'Logical Investigations':

> It is astonishing what language can do. With a few syllables it can express an incalculable number of thoughts, so that even if a thought has been grasped by an inhabitant of the Earth for the very first time, a form of words can be found in which it will be understood by someone else to whom it is entirely new. This would not be possible, if we could not distinguish parts in the thought corresponding to the parts of a sentence, so that the structure of the sentence can serve as a picture of the structure of the thought.[31] (*CP*, 390)

31 In apparent contrast with our position the quotation continues, 'To be sure, we really talk figuratively when we transfer the relation of whole and part to thoughts; yet the

Frege's argument here is that if it were not the case that complex senses had parts corresponding to the parts of the complex expressions that express them, new complex senses could not be expressed by new complex expressions using the same words and be understood by someone who has never before encountered that new complex expression. The argument supposes (i) that a previously unentertained, unexpressed thought could not be expressed by any sentence unless that thought had parts that could be expressed by parts of sentences already in the language, and (ii) that the sense of a complex expression cannot contain any parts that are not already accounted for by the senses of the component expressions, since, if there were such parts, then grasping the senses of the component expressions of a previously unencountered sentence would not suffice for understanding that sentence. We make no claims about whether Frege's argument is sound, nor do we need to, for it is clear that Frege here argues that if not for the Complex Sense Compositionality Principle, we could not express new (complex) thoughts in our language so that others could grasp them. That Frege always employs this principle in order to solve such a fundamental problem in philosophy of language as that of how it is possible to communicate new thoughts in natural language shows that he is committed to its truth and that it holds an important place in his overall theory.

We now argue that we are justified in attributing to Frege principle (9), the Indirect Referent Compositionality Principle. Frege does not articulate (9) explicitly. What we argue is that (9) should be reckoned among the principles to which Frege is rightly committed. For (i) we can show directly that (9) is true when the domain of φ is limited to sentences

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analogy is so ready to hand and so generally appropriate that we are hardly even bothered by the hitches which occur from time to time' (CP, 390). This apparent difficulty is resolved by assuming that Frege's reluctance toward speaking literally of the parts of senses is an instance of a general reluctance to speak of abstract, as opposed to physical, objects' partaking of the part-whole relation. It is clear that Frege thinks the logic of the part-whole relation does apply to any complex sense of a complex expression and the senses of the component expressions. That is all our argument in the previous section requires.

in direct contexts, and the *reason* it is makes it overwhelming plausible that Frege took (9**) to be the general form of the Complex Indirect Referent Function (henceforth, 'CIRF'). Furthermore, (ii) if we take the CIRF to be a composition function, we have a neat and clean account of how the referent of a sentence in an (n>1)-indirect context is a complex sense that is wholly composed of the contextual senses of its component expressions. However, (iii) when we try to find an alternative account of what the CIRF is which respects the Two-Level Theory, we will find, at the least, that the Two-Level theorist is committed to a thesis which there could be no reason to believe, and which it is very doubtful Frege would have accepted. Thus, we present three arguments to show that the CIRF, for any indirect context, is a composition function and, hence, that we are justified in attributing to Frege principle (9). We first present without comment the rough, general form of the three arguments, from which it should be clear that we take Arguments 1 and 2 to be strongly suggestive of, and Argument 3 to be conclusive of, this conclusion. We then discuss the arguments in more detail. Roughly, the three arguments are as follows.

Argument 1

(26) Frege clearly takes the CIRF for singly-indirect contexts to be a composition function.

(27) Frege does not say anything at all about what the CIRF is for (n>1)-indirect contexts.

(28) The most plausible explanation for the truth of (26) and (27) is that Frege took it to be perfectly obvious that the CIRF for singly-indirect contexts is the same kind of function as the CIRF for (n>1)-indirect contexts; i.e., that the CIRF is, for any indirect context, a composition function.

Therefore,

(29) It is reasonable to conclude that the CIRF is, for any indirect context, a composition function.

Argument 2

(30) Frege clearly takes the CIRF for singly-indirect contexts to be a composition function.

(31) Taking the CIRF for (n>1)-indirect contexts to be a composition function gives a neat and clean account of how the referent of a sentence in an (n>1)-indirect context is a complex sense that is wholly composed of the contextual senses of its component expressions.

(32) If (30) and (31) are true, it is reasonable to conclude that the CIRF is, for any indirect context, a composition function.

Therefore,

(33) It is reasonable to conclude that the CIRF, for any indirect context, is a composition function.

Argument 3

(34) The CIRF, for any context, must be specifiable using an incomplete definite description whose completion is consistent with Frege's views about functions and sense and reference.

(35) The only incomplete definite description whose completion is consistent with Frege's views about functions and sense and reference is this one: 'The sense that is composed of w, x, y, and z, where the values of 'w', 'x', 'y', and 'z' are the contextual senses of the component expressions'.

(36) The function picked out by the incomplete definite description mentioned in (35) is a composition function.

(37) If (34)-(36) are true, then the CIRF, for any indirect context, is a composition function.

Therefore,

(38) The CIRF, for any indirect context, is a composition function.

Let's examine these arguments more closely. Frege is committed to (3), the principle that the indirect referent of an expression is that expression's customary sense, and (3) applies both to complex expressions and to their component expressions. Frege is also committed explicitly to (8), the principle that the sense of a complex expression is wholly composed of the senses of the component expressions. Finally, Frege is committed to the claim that the indirect referent of a complex expression is the value of a function which takes as arguments the indirect referents of the component expressions, since this claim is entailed by (5), to which Frege is also committed. The question that needs to be answered, then, is this: What kind of function can the CIRF be if it is to (i) take as arguments the indirect referents of the component expressions and yield the customary sense of the whole expression and (ii) best account for Frege's commitment to (8) and both applications of (3)? We can infer that if there is an *obvious* answer to this question for the cases that Frege did discuss, that answer is what Frege had in mind. Indeed, if it were *perfectly obvious*, there would have been no reason for Frege to belabor it.

Now Frege did not discuss doubly indirect contexts. Let us see, then, what the answer must be for singly indirect contexts. The answer becomes obvious if we consider an example. According to (3), the contextual referent of (0) in (1) is the customary sense of (0).

(0) Hesperus is bright

(1) Anna believes that Hesperus is bright

(5) entails that there is a function from the indirect referents of 'Hesperus' and 'is bright' to the indirect referent of 'Hesperus is bright'. What is it? Well, it also follows from (3) that the indirect referents of 'Hesperus' and 'is bright' are, respectively, the customary senses of 'Hesperus' and 'is bright'. And we know that the customary sense of 'Hesperus is bright' is *wholly composed* of the customary senses of 'Hesperus' and 'is bright'. Thus, the indirect referent of (0) is *wholly composed* of the indirect referents of its constituents, the customary senses of 'Hesperus' and 'is bright'. Thus, for singly indirect contexts, the CIRF *just is* the composition function. And this would have been *perfectly obvious* to Frege. Moreover, if one thinks about Frege's remarks about the function of indirect discourse—'in indirect speech one talks about the sense, e.g., of another person's remarks' (*OSR*, p. 154)—it seems clear that, given the principle that the sense of a sentence that expresses a thought is composed wholly of the customary senses of the constituent expressions, in using a singly-embedded sentence each of whose words when unembedded refers to its customary sense, he would take the CIRF for singly-indirect contexts simply to be composition. There is no reason *whatsoever* to suppose he thought anything else.

Let us now explicitly consider doubly indirect contexts and our second and third arguments for Frege's commitment to (9). For there are difficulties in the way of seeing what reference function other than composition one could appeal to while respecting the Two-Level Theory, difficulties that are directly connected with Frege's views about functions and the relation between sense and reference. Consider again (2), repeated here.

(2) Bettina believes that Anna believes that Hesperus is bright.

We know that the indirect referent of (1) in (2) is the customary sense of (1) and that the customary sense of (1) is wholly composed of the contextual senses of (1)'s constituent expressions. If each of the expressions in (1) as embedded in (2) refers to the sense it has in (1) when not embedded in (2), then taking the CIRF to be a composition

function gives exactly the right result. It is a neat and general account of how the referent of the embedded sentence is determined from the referents of its constituent expressions. There can be no reason to think Frege had anything else in mind. But—it requires the infinite hierarchy of senses (and the infinite hierarchy of senses clearly requires it).

What could the CIRF be if we wanted to adhere to the Two-Level Theory? The difficulty is this. The CIRF is a function from senses to senses. For Frege, such a function is an unsaturated object (and not, as on the set-theoretic view, a set of ordered pairs of objects). A function which has unsaturated objects as values is denoted by an incomplete expression derived from a subject term that refers to an object. On Frege's account the only complex subject terms that refer to objects are definite descriptions.[32] It follows that a function which has unsaturated objects as values is denoted by a definite description with one or more free variables. Furthermore, the CIRF plays a role in our understanding sentences of indirect discourse and attitude sentences. So, it must be a function that we can grasp, since we use it. Therefore, it must be possible for us to express the function. Given this 'expressibility requirement' and our conclusion above, we can conclude that it must be possible for us to express the CIRF using a definite description with one or more free variables. Our task, then, is to say what definite description with one or more free variables could pick out the relevant function. We can describe schematically what we need, given that we know that the referent of the embedded sentence is a composition of the customary senses of 'Anna' and 'believes that' and the values of some function from the customary senses of 'Hesperus' and 'is bright' to their singly-indirect senses. Thus, we can express the CIRF schematically as:

(39) S_0 ('Anna') ^ S_0 ('believes that') ^ F(S_0 ('Hesperus')) ^ F(S_0 ('is bright')),

32 Recall that, for Frege, noun phrases which serve as subject terms like 'everything', 'something', and the like refer to second-order functions from functions to truth-values, and so do not refer to objects; he treats expressions such as 'x + 2', on the other hand, as equivalent to definite descriptions, such as 'the number which is the sum of x and 2'.

where 'F(x)' stands in for some (other) appropriate function from the customary senses to the appropriate indirect senses. We now need to specify the CIRF using a definite description.

On our interpretation of the CIRF, the function is easily specifiable using a definite description, as in (40):

(40) The sense composed of w, x, y, and z,

where the values of 'w', 'x', 'y', and 'z', are the contextual senses of the embedded expressions. But there are difficulties in trying to specify the function using a definite description while also respecting the Two-Level Theory, difficulties that seems to us to be insurmountable. Because the relation of referent to sense is one-many, the CIRF cannot be specified using descriptions such as (41), since (41) fails to denote a unique object.

(41) The mode of presentation of x.

Moreover, since the CIRF takes only senses as arguments, we cannot describe it by making use of the connection of sense with any particular linguistic expression, for doing so would turn the function into one that takes linguistic expressions rather than senses as arguments. Thus, the CIRF cannot be specified using descriptions such as (42),

(42) The sense of x in English,

where the value of 'x' is a linguistic expression. To avoid these problems, one might try to exploit talk about the senses of expressions in, say, English, which are modes of presentation of a given sense, as in (43).

(43) The sense of any expression in English that is singly embedded in an indirect context and which is a mode of presentation of x.

Here, the values that 'x' takes on will be senses, and we may also, arguably, avoid the problem of trying to take the backward road from referents to senses by including additional requirements to get the sense we want. However, this suggestion is not workable in its present form because it makes essential parochial reference to a particular language,

namely, English. Clearly, Frege would not wish to say that understanding sentences in (say) German requires grasping senses that involve reference to English, and equally clearly he would have wanted a translation of a German sentence of indirect discourse into English to express the same thought as the German sentence. Whatever function is involved, then, cannot rely upon any particular language. So far as we can tell, the only remaining kind of definite description to which a Two-Level Theorist could appeal commits one to a thesis that there is no good reason to believe, and which would make the correctness of Frege's entire theory of sense and reference hostage to an empirical issue, something Frege surely would not have wanted. The only route the defender of the Two-Level Theory has left is to try to argue that we can describe the CIRF using descriptions such as (44), since such descriptions would avoid the aforementioned difficulties with (42) and (43).

(44) The sense of any expression (in any language) that is in a singly-indirect context and which refers to x.

Notice that (44) avoids both the difficulties associated with descriptions like (42) and (43), since the values of the variable are senses, and there is no essential parochial reference to any particular language. However, it seems very unlikely that Frege had descriptions like (44) in mind. The difficulty with these descriptions is that there are *many different senses that could be attached by convention* to words that, in a singly-indirect context, would refer to any given sense. There could be no *a priori* reason to think that every actual language in fact uses the same indirect senses, and it would be a coincidence of spectacular proportions if they did; moreover, there are clearly many *possible* languages that use different indirect senses. But surely Frege would not have wanted the function of a language in communicating what others have thought to be contingent on what practices speakers of *other* languages engage in. Thus, this account—one to which the Two-Level Theorist is committed—is wholly implausible both as an account of how indirect contexts work, and of what Frege would have had in mind. The only reasonable thing to

conclude is that Frege was in fact committed to the CIRF being just straightforwardly the composition function, which is what it has to be for the only contexts he discusses, and whose generalization is the only sensible thing when considering further indirect contexts. We therefore conclude that Frege is committed to (9), the Indirect Referent Compositionality Principle.

Conclusion

In this paper we have argued (i) that Frege is explicitly committed to the Complex Sense Compositionality Principle and that it is overwhelmingly plausible that he was committed to the Indirect Referent Compositionality Principle, and (ii) that if Frege's theory includes both the Complex Sense Compositionality Principle and the Indirect Referent Compositionality Principle, then it is inconsistent with the Two-Level Theory. We conclude that Frege's theory is inconsistent with the Two-Level Theory and, thus, entails the infinite hierarchy of senses.

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Appendix

For any sentence φ,

R_0 (φ) = the customary referent of φ;

R_i (φ) = the i^{th} indirect referent of φ, for all $i \geq 1$;

S_0 (φ) = the customary sense of φ; and

S_i (φ) = the i^{th} indirect sense of φ, for all $i \geq 1$.

Fregean Doctrine:

(3) For any expression φ, the indirect referent of φ is the customary sense of φ. [*OSR, 59*]

(4) For any complex expression ψ, the sense of ψ is the value of a function of the senses of its parts. [*Sense Functionality Principle*]

(5) For any complex expression ψ, the referent of ψ is the value of a function of the referents of its parts. [*Reference Functionality Principle*]

(8) A complex expression expresses a complex sense that is wholly composed of the senses of its component expressions. [*Complex Sense Compositionality Principle*]

(9**) For all sentences φ, if φ occurs in an n-indirect context, where n ≥ 1, then the n-indirect referent of φ is composed of the contextual referents of its component expressions. [*Complex Referent Compositionality Principle*]

(9*) R_1 (⌜Anna believes that φ⌝) = R_1 ('Anna') • R_1 ('believes that') • R_2 (φ). [*Application of (9**) to ⌜Anna believes that φ⌝*]

Assume the Two-Level Theory.

(4*) F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)) = S_0 (⌜Anna believes that φ⌝). [*Application of (4) to ⌜Anna believes that φ⌝*]

(5*) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 (⌜Anna believes that φ⌝). [*Application of (5) to ⌜Anna believes that φ⌝*]

(3*) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)). [*(3), (4*), (5*)*]

(3**) F_{R_1} (S_0 ('Anna'), S_0 ('believes that'), S_0 (φ)) = F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)). [*(3), (3*)*]

(10) F_{S_0} (S_0 ('Anna'), S_0 ('believes that'), S_1 (φ)) = S_0 ('Anna') ^ S_0 ('believes that') ^ S_1 (φ), [*Application of the Complex Customary Sense Function to ⌜Anna believes that φ⌝*]

(11) S_0 (⌜Anna believes that φ⌝) = S_0 ('Anna') ^ S_0 ('believes that') ^ S_1 (φ). [*(4*), (10)*]

(12) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 ('Anna') • R_1 ('believes that') • R_2 (φ). [*(9*), (5*)*]

(13) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = R_1 ('Anna') • R_1 ('believes that') • R_1 (φ). [*(12), Two-Level Theory*]

(14) F_{R_1} (R_1 ('Anna'), R_1 ('believes that'), R_1 (φ)) = S_0 ('Anna') • S_0 ('believes that') • S_0 (φ). [*(13), (3)*]

(15) R_1 (⌜Anna believes that φ⌝) = S_0 ('Anna') • S_0 ('believes that') • S_0 (φ). [*(14), (5*)*]

(16) S_0 (φ) is neither a part of nor identical to S_1 (φ) and has neither S_0 ('believes that') nor S_0 ('Anna') as a part. [*No Overlap Principle*]

(17) S_0 (⌜Anna believes that φ⌝) does not have S_0 (φ) as a part. [*(11, 16)*]

(18) R_1 (⌜Anna believes that φ⌝) ≠ S_0 (⌜Anna believes that φ⌝). [*(15), (17)*]

The Two-Level Theory is inconsistent with Fregean doctrine. [*Contradiction: (3) and (18)*]