

FAST Prediction of **Protein Thermodynamics**

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Protein Thermodynamic Stability

Dependence on thermodynamic/solvent conditions and sequence

[co-solute] induced denaturation **Temperature induced denaturation** G. Pappenberger, Nature: Struct. Biol. 8, 452-458 (2001) R.M. Ballew, et al., PNAS 93 5759 (1996) cm² dmol⁻¹) x 10⁻³ -8 0.8 fraction native protein examples of how to 0.6 [0]222 (deg denature a protein 0.4 (randomly selected)) 40 60 Temperature, °C 80 20 0.2 1.0 0.8 0.0 в 0.6 2 4 6 0. 0.2 [GdmCl] (M) sequence 20 40 60 Temperature, °C (a.u.) 3000 (a.u.) dependence 60 640 MPa (10 tensity (a.u) 0 tensity (a.u) 2000 fluores pH induced 1 1000 denaturation otal Fraction Unfolded Spector and ο. pressure nce 200 600 ٥ 400 Raleigh. JMB. 276 30 Pressure (MPa) 0 induced 479-489, (1998) 0 denaturation ^b/₂₀ Ô 0 0 Torrent, et. al., 0 Biochemistry, 38, 10 0 15952-15961. 0.1 MPa (1999)0 400 320 360 440 480 10 12 2 Wavelength (nm) pH

Modeling Protein Thermodynamics

Speed versus accuracy tradeoffs



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The problem of "hidden" thermodynamics



Decomposition of the Free Energy of a System in Terms of Specific Interactions A. E. Mark and W. F van Gunsteren, J. Mol. Biol. 240, 167 (1994)
"In regard to the detailed separation of free energy components, we must acknowledge that the hidden thermodynamics of a protein will, unfortunately, remain hidden"

See: Ken A. Dill, "*Additivity Principles in Biochemistry*", The Journal of Biological Chemistry 272, 701-704 (1997)

The Distance Constraint Model (DCM)

Restoring the utility of a free energy decomposition (FED)

A NEW PERSPECTIVE

D.J. Jacobs,et. al., *Network rigidity at finite temperature: Relationships between thermodynamic stability, the nonadditivity of entropy, and cooperativity in molecular systems.* **Physical Reviews E. 68, 061109 1-21 (2003)**

The DCM resolves the problem of nonadditivity

by explicitly regarding network rigidity as a long-range mechanical interaction between components to identify the independent ones.



"I never satisfy myself until I can make a mechanical model of a thing. If I can make a mechanical model I can understand it"! ---- Lord Kelvin

Tao of the DCM: Free Energy Reconstitution

Network rigidity accounts for non-additivity in conformational entropy



Mind your Ps and Qs (P,Q) interdependence G(F) = H(F) - TS(F) $H(F) = \sum h_c p_c(F)$ $S(F) = \sum s_c q_c(F) p_c(F)$ **Regarding NETWORK RIGIDITY as a**

mechanical interaction accounts for NON-ADDITIVITY IN ENTROPY

Linking Molecular Structure to Thermodynamics

The Gibbs ensemble consists of all accessible constraint networks



THERMODYNAMICS

New Modeling Paradigm for Protein Thermodynamics DCM provides high speed and accuracy



Quantified Stability/Flexibility Relationships (QSFR)

Examples of two mechanical response characterizations

Backbone flexibility is dependent upon temperature and pH



Cooperativity correlation quantifies flexibility/rigidity pairwise couplings



For more information, see the following: Livesay, et al. *FEBS Letters* (2004) **576**:468. Jacobs & Dallakyan. *Biophysical J.* (2005) **88**:903. Livesay & Jacobs. *Proteins* (2006) **62**:130. Jacobs, et al. *J. Mol. Biol.* (2006) **358**:882. Livesay, et al. *Chem. Cen. J.* (2008) **2**:17. Mottonen, et al. *Proteins* (2009) **75**:610. Mottonen, et al., *Biophysical J* (2010) **99**:2245.

Distance Constraint Model

The Heart of the DCM Consists of Three Essential Elements



"I have yet to see any problem, however complicated, which, when you looked at it in the right way, did not become still more complicated". --- Poul Anderson in New Scientist (1969)

Protein Stability is Linked to Solvent

Modeling of solvent and conformational interactions



The Free Energy Functional

FAST models all essential enthalpy-entropy mechanisms

FREE ENERGY DECOMPOSITION



Model highlights: Solvent penetration using transfer free energies



Model highlights: Electrostatics determined by optimizing pKa values

$$G_{\text{FEF}} = G_{\text{slv}} + G_{\text{slv}} + G_{\text{slv}} + G_{\text{ion}} + G_{\text{ion}} - TS_{\text{ion}}^{\text{prot}} - TS_{\text{mix}}^{\text{hb}} + G_{\text{enf}}^{\text{res}} + G_{\text{enf}}^{\text{lnk}} + G_{\text{enf}}^{\text{lnk}} + G_{\text{enf}}^{\text{prot}} + G_{\text{enf}} + G_{\text{enf}}^{\text{lnk}} + G_{\text{enf}}^{\text{prot}} + G_{\text{enf}}^{\text{res}} + G_{\text{enf}}^{\text{lnk}} + G_{\text{lnk}}^{\text{lnk}} + G_{\text{lnk}}^{\text$$

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Covalent bond network defines structural template



2D schematic of the process

Solvent penetration defines a heterogeneous local environments



2D schematic of the process

Crosslinking distance constraints couple to solvent penetration

$$G_{\text{FEF}} = \overbrace{G_{\text{slv}}^{\text{res}} + G_{\text{slv}}^{\text{hph}} + G_{\text{slv}}^{\text{shb}}}^{\text{shb}} + \overbrace{G_{\text{ion}}^{\text{base}} + G_{\text{ion}}^{\text{acid}} - TS_{\text{ion}}^{\text{prot}}}^{\text{prot}} - TS_{\text{mix}}^{\text{hb}} + \overbrace{G_{\text{cnf}}^{\text{res}} + G_{\text{cnf}}^{\text{hbb}} + G_{\text{cnf}}^{\text{pck}} + G_{\text{str}}^{\text{pck}} + G_{\text{str}} + G_{\text{vib}}^{\text{pck}}}$$





Based on the solvation state, all crosslinking interactions are added to the covalent bond network to form a constraint network.

2D schematic of the process

Modeling Essential Mechanisms

MOLECULAR PARTITION FUNCTION (MPF) $Z(T) = e^{\sum_{o}/R - \beta_{0}U_{0}} \int P(E \mid T_{o})e^{-(\beta - \beta_{0})E} dE$ sampled mean energy $S_{o} = R \sum_{k} \sigma_{k}$ $S_{o} =$ of atomic displacements in XYZ-coordinates. $\sigma_{k} = \frac{1}{2} \ln \left| \left(k_{B} T e^{2} / \hbar \right) \lambda_{k} + 1 \right|$ The { σ_{k} } define a "pure entropy spectrum" for which there will be 3N - 6 finite values.

Modeling Essential Mechanisms



Modeling Essential Mechanisms



Modeling Essential Mechanisms



Modeling Essential Mechanisms

FED
$$G_{\text{cnf}} = G_{\text{cnf}}^{\text{res}} + G_{\text{cnf}}^{\text{lnk}} + G_{\text{cnf}}^{\text{ihb}} + G_{\text{cnf}}^{\text{pck}}$$

Conformational components define a distance constraint network

generic form:

$$G_{\Psi}^{\text{int}} = \sum_{j=1}^{N_{\text{states}}} \left[g_{\Psi j}^{\text{int}} - T \left(R \sum_{k=1}^{3N^{\text{int}}-6} \sigma_{\Psi jk}^{\text{int}} \sum_{i=1}^{N_{k}^{\text{int}}} W_{\Psi ki}^{\text{int}} q_{\Psi jki}^{\text{int}} \right) + TR \ln(p_{\Psi j}^{\text{int}}) \right] p_{\Psi j}^{\text{int}}$$

Accounts for the many-body mechanical interactions Accounts for nonadditivity in conformational entropy

Modeling Essential Mechanisms

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$$G_{\text{cnf}} = G_{\text{cnf}}^{\text{res}} + G_{\text{cnf}}^{\text{lnk}} + G_{\text{cnf}}^{\text{ihb}} + G_{\text{cnf}}^{\text{pck}}$$

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A set of distance constraints are used to model each entropy mode.
normalization condition:
$$\sum_{i=1}^{N_k^{\text{int}}} W_{\Psi k i}^{\text{int}} = 1$$

The probability for a distance constraint to be independent as determined by graph-rigidity calculations.

Modeling Essential Mechanisms

FED
$$G_{enf} = G_{enf}^{res} + G_{enf}^{lnk} + G_{enf}^{ihb} + G_{enf}^{pck}$$

Conformational components define a distance constraint network

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$$G_{\Psi}^{\text{int}} = \sum_{j=1}^{N_{\text{states}}} \left[g_{\Psi j}^{\text{int}} - T \left(R \sum_{k=1}^{3N^{\text{int}}-6} \sigma_{\Psi jk}^{\text{int}} \sum_{i=1}^{N_k^{\text{int}}} W_{\Psi ki}^{\text{int}} q_{\Psi jki}^{\text{int}} \right) + TR \ln(p_{\Psi j}^{\text{int}}) \right] p_{\Psi j}^{\text{int}}$$
$$p_{\Psi j}^{\text{int}} = \frac{\exp\left(-\beta g_{\Psi j}^{\text{int}} + \sum_k \sigma_{\Psi jk}^{\text{int}} \sum_i w_{\Psi ki}^{\text{int}} q_{\Psi jki}^{\text{int}}\right)}{Z_{\Psi}^{\text{int}}}$$

The probability for an interaction to form depends on all other interactions

Free Energy Landscape

Constraint Networks are Defined by Solvent Macrostates

The Gibbs Triangle



Free Energy Reconstruction (FER)

Self-consistent process is used to solve the free energy functional



Network Rigidity Calculations Accounts for non-additivity in conformational entropy

conformational entropies renormalize



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Jacobs, US Patent # 8244504

Self-consistent Network Rigidity Calculation Local environments and nonadditivity in conformational entropy

conformational entropies renormalize



Self-consistent Network Rigidity Calculation Local environments and nonadditivity in conformational entropy

conformational entropies renormalize



Self-consistent Network Rigidity Calculation Local environments and nonadditivity in conformational entropy

conformational entropies renormalize



Flexibility And Stability Test (FAST) DCM = FED + FER



Stability Curves

Results for three typical macrostates



Scalability of FAST

Efficient parallelized sparse-hierarchical-adaptive grid methods

FAST calculation of multi-dimensional free energy landscape

FAST performance characteristics

Example:

Protein size: 150 residues

domain: (150 K ≤ T ≤ 400 K) @ ∆T=1K

(2 ≤ pH ≤ 12) @ ∆pH=0.1

wall time: < 6hrs using 50×(2.3 GHz CPUs)

Scales nearly linear with # of atoms





Conclusions and Acknowledgements



Newtonian Mechanics

Network rigidity is a fundamental mechanical property that directly links protein stability, flexibility and dynamics

Boltzmann **Statistics**

Principal Investigators Don Jacobs Dennis Livesay

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Collaborators

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