Geographic Characteristics of a Network Interdiction Problem
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Abstract

The protection of critical infrastructure from natural and intentional events is a key component of any national security agenda. Protection schemes need to be readily identifiable and adaptable to complex changing environments. In this paper, we identify strategic geographic characteristics that impact the location of detection resources (e.g. sensors) towards the defense of regional critical infrastructure. Specifically, we seek to estimate the relationship between the results of a variation of the traditional shortest path network interdiction problem (SPNIP) and geographical characteristics of the transportation infrastructure and the urban environment. Experiments conducted on three distinct transportation networks of different shapes and granularities (New York City - grid, Houston - radial, Boston - hybrid) underline the importance of geographic characteristics such as the proximity to resource location, attacker entry points as well as network coverage. Insights gained from this work are relevant to policy and decision makers to facilitate the development of analytical and decision-support tools capable of identifying resource allocation strategies. We discuss a heuristic-based framework that prioritizes the selection of detection resources, reflecting the importance of geographic characteristics. The findings underline the importance of geographical characteristics for the allocation of resources in a regional setting.

Keywords: Critical Infrastructure, Geographic Characteristics, GIS, Network Interdiction Problem, Spatial Optimization.

1. INTRODUCTION

Critical infrastructure protection has always been a national security concern especially in large metropolitan areas (George 2008). In the last decade, however, following coordinated interdictions by terrorists (Brown et al. 2006; Church and Scaparra 2007), natural disasters, and an increased reliance on technology, networks have been more interconnected than ever before (Murray 2013); any disruptive event in a network has the potential of producing a cascading failure (Grubesic and Murray 2006). To better protect existing physical infrastructure, we recognize the need to evaluate the threat from a given event.

The allocation of resources is required to support and maximize situational awareness, preparedness, and/or response (Yates et al. 2010). Various models of infrastructure protection have been proposed in the literature, such as the Network Interdiction Problem (NIP) (Wood 1993). The NIP involves two opposing actors engaged in a war-like conflict. Actor 1 uses the network to optimize an objective, while Actor 2 is trying to stop this from happening by interdicting the arcs. As underlined by Murray (2013), the NIP has deep ramifications in spatial sciences; it has a wide range of applications ranging from physical infrastructure (e.g., energy transmission, hazmat transport, communication networks) to non-physical (e.g., cyber security). Specifically, some networks are geographically more vulnerable than others due to their topology, arrangement, structure and morphology. However, aside from Snediker et al. (2008), geographic characteristics that can be incorporated in the formulation have not been systematically considered as they have for other network problems such as the shortest path and p-Median (Church and ReVelle 1976, Church et al. 2004, Reese 2006). Understanding the effects of geographic characteristics in network interdiction problems can guide and provide the basis for developing alternative solutions that would exploit such spatial characteristics (e.g., adding functions and constrains that incorporate such geographic characteristics) and aid in public-policy decisions.

In network infrastructure protection, elements such as the type and extent of the network; the arrangement of links in the network; the relation between the origins, destinations, and allocation of defense resources; and distances between them can be strategic factors. In this paper, we attempt to identify key geographic characteristics of the network that directly impact the location of detection resources towards the defense of regional critical infrastructure. A detailed experimental design embedded within a GIS environment is implemented. A variation of the traditional shortest path network interdiction problem (SPNIP) is used as the basis for experimentation. SPNIP is chosen due to its conceptual simplicity and the available body of knowledge on this problem (Wood 1993, Israeli and Wood 2002, Bayrak and Baily 2008, Yates and Casas 2012). Additionally, its applications to critical infrastructure modeling demonstrate its usefulness and acceptance within the infrastructure protection community (Church et al. 2004, Salmeron et al. 2004, Matisziw et al. 2007, Yates et al. 2010).

Insights gained from this work are important for policy and decision makers to facilitate the development of analytical and decision-support tools capable of quickly identifying strong resource allocation strategies. These tools are useful in their ability to allow policy and decision makers to modify and update geographic parameters and obtain good allocation strategies thereby increasing their situational awareness and response to threats/attacks (Snediker et al 2008).

The remainder of our paper is structured as follows: Section 2 provides an overview of existing literature and underlines the contributions of our work. Section 3 outlines the methodology and introduces the experimental design and the geographic area networks. We present our results in Section 4, followed by a discussion in Section 5, where we present a framework to incorporate our results in a heuristic. Concluding remarks and directions for future research are presented in the last section.

2. LITERATURE REVIEW

The shortest path network interdiction problem (SPNIP) is a discrete optimization model that uses an attacker and defender in competition with the former attempting to reach pre-designated critical infrastructure targets and the later attempting to detect these intrusions. Prior to the introduction of SPNIP, Wood (1993) presented an optimization model simply referred to as the network interdiction problem (NIP). In this model, the attacker is responsible for the interdiction, or destruction (either whole or in-part) of a transportation network being used by the defender to transport goods from given origins to destinations. Dual competing objectives have the defender maximizing throughput on the network while the attacker seeks to minimize this maximum throughput. As the model gained traction in defense/logistics communities, many variations emerged, including the SPNIP. With its first instantiation in Israeli and Wood (2002), the SPNIP maintained its competing objective and attacker-defender format, but the decision variables changed. Whereas network interdiction considers throughput and network flows, SPNIP uses length of the arcs in a network as its metric. Attackers interdict an arc by either destroying or lengthening it. The defender identifies the shortest path through the network between a set of entry points and destinations and the attacker maximizes this minimum path.

Since Israeli and Wood (2002) introduced SPNIP, formulations have emerged in Multi-Commodity Network Interdiction Flow (Lim and Smith 2007), asymmetric information in network interdiction (Bayrak and Baily 2008), stochastic versions of the deterministic SPNIP (Zhang et al. 2005), and alternative objective models such as Zhuang and Bier (2007) which balances protection and risk. Brown et al. (2006) extended the two-level attacker-defender model to a three-level attacker-defender-attacker and defender-attacker-defender formulation (referring to the order of information conveyed between the two-players). Israeli and Wood (2002), Brown et al. (2006), and others have discussed robust solution approaches and approximations for the SPNIP. They motivate standard decomposition approaches based on Benders Decomposition (Bard 1998), an algorithm which is proven to yield a certificate of optimality to such bi-level problems. Yates and Lakshmanan (2011) demonstrate how a knapsack-based formulation can strongly approximate SPNIP solutions.

Geographers have also developed their own formulation while attempting to solve network-based optimization problems (Church et al. 2004, Matisziw et al. 2007, Murray et al. 2007). They underline the importance of location-based and spatial properties and the power that these properties have in formulating constraints to reduce a problems feasible region or to fix its variables for a more concise and simplistic formulation. The notion of an ***r***-interdiction problem was introduced by Church et al. (2004) and expanded upon Church's previous constraint reformulation for the well known *p*-Median problem (Church and ReVelle 1976, Church et al. 2004). Interdiction was also modeled in Murray et al. (2007) using the interdiction of an internet service protocol network as its basis. All of these adaptations are in addition to the large amount of literature focusing on the contributory power of geographic information systems (GIS) to the data input, extraction, overlay, proximity and visualization challenges inherent in such network-based formulations (Ohman and Eriksson 2002, Cova and Conger 2003, Meyer et al. 2009). Yates and Lakshmanan (2011) and Yates and Casas (2012) have proposed an adaption of the SPNIP that incorporates concepts of spatial analysis, GIS and location sciences along with optimization developing a discrete shortest path network interdiction problem (DSPNI).

Regardless of the variety of formulations and the abundance of literature on the NIP, several issues still need to be addressed. In this paper we focus on two of those: First, the different variations of the NIP do not incorporate within their formulation geographic characteristics per-se in spite of these having been proven significant in similar problems such as the shortest path and the p-median problem (Church and ReVelle 1976, Densham and Rushton 1992, Church et al. 2004, Reese 2006). Secondly, scale has been shown to be of considerable importance in a diverse set of spatial optimization problems in urban settings (Miller and Wentz 2003, Delmelle et al. 2012, Tong and Murray 2012, Lin and Ban 2013) and needs to be considered. Addressing these issues with this type of spatial optimization problems that include geographic characteristics is particularly challenging because: (1) geographic elements often involve more complex formulations, constrains, and variables, and (2) data needs (e.g., high resolution) can make problems larger and more difficult to solve (Tong and Murray 2012). Hence, as part of the process, we suggest a framework that can capitalize on the importance of key geographic variables in the allocation of defense resources.

Within this framework it is expected that the solution to the variation of the SPNIP be affected by key geographic elements, and that this information be usable when developing a heuristic, potentially reducing the computation time of the algorithm. This capability will increase the decision-makers situational awareness and will allow the emergency manager(s) to rapidly and thoroughly assess the region under a variety of scenarios. Since such defense environments are also highly dynamic, it is necessary to enable emergency managers to easily edit/adapt the region to a variety of assumptions/observations, enabling re-analysis of the region and a better understanding of how geographic changes in the network or critical infrastructure impact resource allocation.

3. METHODOLOGY

To determine geographic characteristics that can be significant indicators of defender resource allocation strategies, the discrete shortest path network interdiction problem (DSPNI) (Yates et al. 2010), a variation of the SPNIP, is chosen. In DSPNI, the attacker uses a transportation network to move from pre-defined regional entry points to pre-defined critical infrastructure target locations. Along each arc of the network, the attacker has some non-zero probability of detection associated with its travel, seeking to identify the network path(s) which minimizes its detection probability (path detection being modeled as the product of individual arc detection probabilities for all arcs on the path). The defender, able to locate only a limited number of resources, seeks to identify a geographic location within the region to allocate resources that improve its detection capability. The DSPNI provides the appropriate outcomes to explore the geographic characteristics that directly impact the location of resources towards the defense of critical infrastructure. The formulation of the problem as proposed by Yates et al. (2010) follows.

[Notation]

A = set of all possible resource locations a

Λ = set of network arcs i

N = set of network nodes n

B = total available defense budget

R^{as} = set of all arcs i falling within the range of a type s resource at location a

c_a = cost to locate a resource at location a

k_{ni} = if node n is the of arc i
= 0 otherwise

η_s = power of resource type s

τ = upper bound on resource overlap

$r_i(as)$ = 1 if arc $i \in R^{as}$, 0 otherwise

q_n = {1,0,-1} if n is {entry point, intermediate point, critical infrastructure point}

u_{ist} = detection probability for arc i when covered by t type s resources

[Decision Variables]

$w_i \quad w_i = 1 \text{ if arc } i \in \Lambda \text{ used in the attacker path, otherwise } w_i = 0$

$y_{as} \quad y_{as} = 1 \text{ if resource type } s \in S \text{ located at } a \in A\text{, otherwise } y_{as} = 0$

$x_{ist} \quad \mathrm{x}_{\mathrm{is}} = 1 \text{ if } i \in \Lambda \text{ covered by } t \text{ type } s \in S \text{ resources, otherwise } x_{ist} = 0$

$v_b \quad v_b = 1 \text{ if location } b \in A \text{ is used, otherwise } v_b = 0$

[DSPNI]

$$
\begin{aligned}
z = \max\ \min & \prod_{i,s,t} u_{ist}{}^{w_i x_{ist}} \\
s.t. & \sum_i k_{ni} w_i \leq q_n \quad \forall\, n \\
& x_{ist} - \frac{1}{t}\sum_a r_i^{as} y_{as} \leq 0 \quad \forall\, i,s,t \\
& \sum_{s,t} x_{ist} = 1 \quad \forall\, i \\
& \sum_{a,s} c_a y_{as} \leq B \\
& w, x, y \in \{0,1\}
\end{aligned}
$$

Detection probability for a given arc i in DSPNI is a function of resource power and the number of resources influencing the arc ($u_{ist} = u_{i01} \prod_{\tau} \eta_s$). The objective function of the attacker is to minimize network detection while the defender seeks to maximize this minimum detection value (i.e. the defender wishes to minimize the "worst-case" attack scenario). The constraints of DSPNI ensure that a complete path is chosen by the attacker, guarantee arc coverage and limit defender resource allocation to a fixed budget. Because DPSNI allows resources to be placed within the geographic region (area covered by the network), an additional decision variable x is included with a constraint that connects resource placement with network coverage (this additional constraint ensures that an arc is not influenced by more resources than it is covered by). The objective function guarantees that an arc will always maximize its influence, so this constraint acts as an upper bound. The DSPNI objective is rewriten as its logarithmic transform to remove the non-linearity. This results in the new objective $z^* = \max\ \min \sum_{i,s,t} \log(u_{ist}) w_i x_{ist}$.

We solve the problem using a special case of Bender's decomposition, a standard method to solve such bi-level models (Bard 1998, Brown et al. 2006).

3.1. *Geographic Area Networks*

To gain insight on the importance of the network structure, scale and geographical characteristics, networks of different morphologies and densities are presented. Specifically, three distinct and highly dissimilar large metropolitan areas: New York City, NY, Boston, MA and Houston, TX are used in the experiment. The networks of the three cities – which are displayed in Figure 1 – were chosen for the topological differences among them. This variation allows us to determine whether the geographic characteristics are independent of the network attributes and if so incorporate this knowledge into a pseudo heuristic.

Critical infrastructure for each city is distributed following the original location of airports, hospitals, schools, fire stations, and other significant landmarks. Higher concentrations of critical infrastructure occur in downtown areas where large populations originally settled. Entry points are located at the extreme, or outer, points of a given network. The assumption is that attackers will be entering the designated region from a neighboring region using the designated transportation network. Potential defender resource locations are obtained by covering each network's extent with a uniformly spaced grid. The intersections of this grid become the set of potential defender resource locations (i.e., the set *A* in DSPNI). By changing the density of this grid, the precision with which a defender can locate a resource can be impacted directly (high grid density gives a larger number of potential locations and therefore a higher resolution within the region). Three grid sizes are examined as part of the experimental design and three other key DSPNI parameters are changed (i.e., resource sensitivity η_s, threat level D, and budget B). Table 1 shows the different parameters used. Considering the three regions being tested, the combination of parameters leads to a total number of 720 (3 grid densities *4 resources sensitivity *5 threat level *4 budget * 3 networks) individual DSPNI problem solutions which comprise the case study data for the experiment.

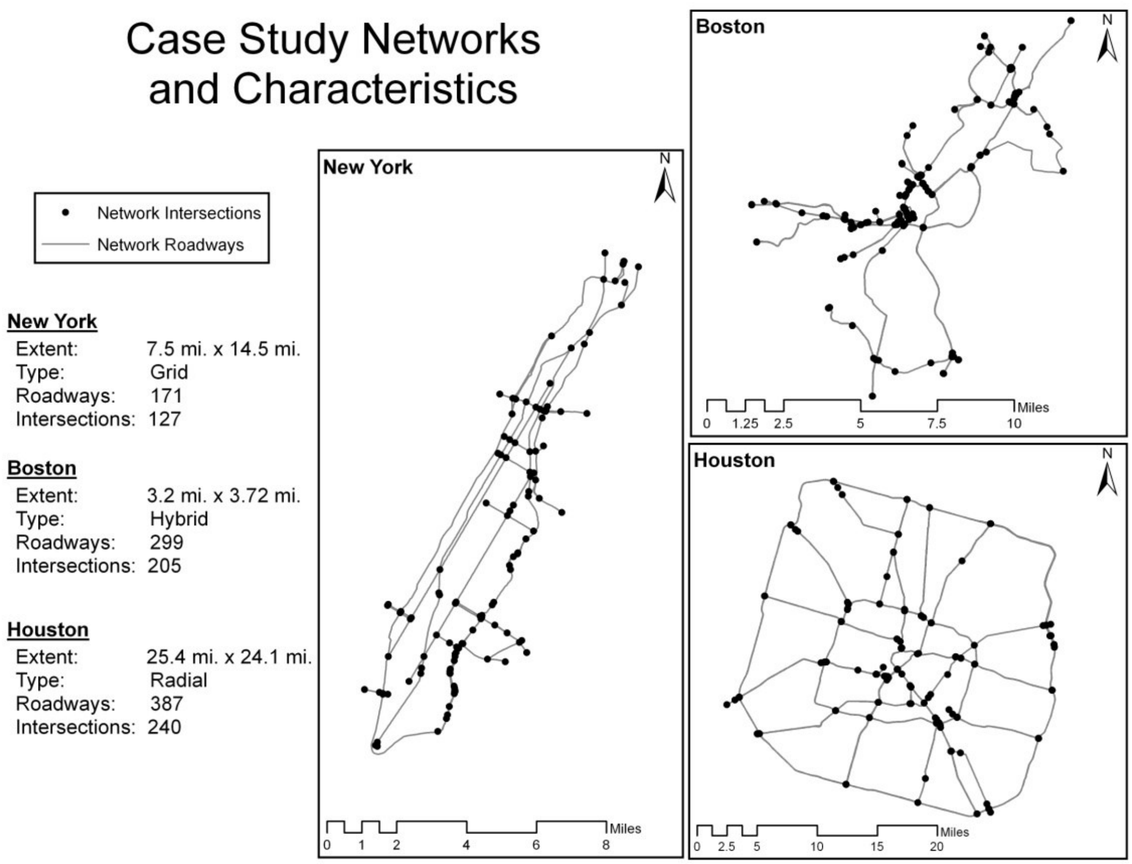

Figure 1: Illustrations of the three test-case networks and their associated characteristics

Network	Grid Density (ft)		Rows	Columns	|A|	Sensor Range (ft)
New York	*Low*	7218	6	11	66	5280
	Medium	4209	20	12	240	3168
	High	3000	27	15	405	2100
Boston	*Low*	7218	10	9	90	5280
	Medium	4209	17	15	255	3000
	High	3000	22	22	484	2198
Houston	*Low*	14784	9	10	90	13200
	Medium	10560	13	14	182	7329
	High	7218	18	19	342	5280

Experimental Factors and levels for DSPNI

A = Grid Density = {Low, Medium, High}

η_s = Resource Sensitivity = {0.2, 0.4, 0.6, 0.8}

D = {1, 2, 3, 4, 5} threat level

B = {\$800, \$1200, \$1600, \$2000}

Table 1: Parameters used to solve the DSPNI

3.2. *Network and Geographical Characteristics*

We illustrate our approach on a set of numerical examples, identifying which geographic properties from a selected set could potentially be used as coefficients, functions and constraints in the DSPNI formulations to reliably and consistently indicate where defense resources should be located. In other words, we identify geographic properties that can be used in a heuristic to determine strong defense allocation strategies in a limited time frame in lieu of solving the DSPNI problem to optimality. The set of geographic properties are based on distance and containment geographic interactions as well as topological characteristics of the network. They are selected for their simplicity both in calculation and in concept and they represent key spatial relationships between the potential defender resource locations (i.e., the set A) and the transportation network's entry points, targets and roadways. A GIS is used to calculate the different geographic properties in the set.

Topological characteristics of the network include traditional network measures such as: degree, D-matrix (node accessibility based on the shortest path), and T-matrix (number of ways to go from one node to all other nodes).

Distance and containment characteristics include:

1. Minimum Distance to Entry Point (MIN-EP) - *measurement from a defender resource location point to its closest regional entry point*
2. Median Distance to Entry Point (MD-EP) - *measurement from a defender resource location point to its median regional entry point*
3. Minimum Distance to Critical Infrastructure (MD-CI) - *measurement from a defender resource location point to its closest critical infrastructure target*
4. Median Distance to Critical Infrastructure (MD-CI) - *measurement from a defender resource location point to its median critical infrastructure target*
5. Number of Roadways (NRW) - *the number of roadways in the transportation network capable of being covered by a defense resource placed at a given location*
6. Length of Roadways (LRW) - *the total length of all roadways in the transportation network capable of being covered by a defense resource placed at a given location*

7. Coverage of Roadways (CRW) - *the product of NRW and LWR*
8. Count (CNT) - *the number of critical infrastructure targets in the region capable of being covered by a defense resource placed at a given location*

Topological characteristics of the network were excluded from subsequent analysis as these were not significant from preliminary regressions. Hence, our regression model contained eight distance metrics and containment characteristics. Determining the extent to which these eight geographic properties can be used in determining defender resource strategies or analyzing the effects of regional change (e.g., the removal of roadways in the network or the addition of critical infrastructure targets) requires that correlation between the optimal DSPNI solutions and these eight geographic properties be established. In this paper, in addition to correlation matrices, a Negative Binomial Regression is implemented since the dependent variable in the regression (the number of times/frequency that a potential resource location is used by the defender in DSPNI optimal solutions) is a count variable. The Negative Binomial distribution (Hilbe 2007) serves as an alternative to the Poisson distribution for discrete data with the presence of an unbounded positive range whose variance exceeds the sample mean. This property dictates the use of a Poisson-based regression, of which the Negative Binomial is a close variation (Cameron and Trivedi 1998, Hilbe 2007). Additionally, the non-normality of residuals and the fact that most potential defender resource locations will not be used in any single DSPNI solution (this leads to an extreme amount of locations that return a “zero” value in the DPSNI optimal solution, effectively creating an extreme skew in the test data). Correlations and regressions were conducted in the statistical package R. Results (e.g. residuals) were visualized back in a GIS.

4. RESULTS

Recall that the DSPNI solution is a resource allocation strategy for the defender dictating which resource locations are used in an optimal defense configuration. In solving DSPNI to optimality using Benders Decomposition, initial arc detection values, referred to as u_{i01}, were randomly selected using a uniform distribution with bounds {0.3, 0.7}. Entry point and critical infrastructure location sets were chosen randomly a-priori, are mutually exclusive and their union represents a subset of all network intersection points. Figures 2-4 summarize the model solutions for the three networks (NY, Boston and Houston), each at three levels of spatial granularity.

4.1. *Correlation Results*

Correlation runs were conducted for each eight of the explanatory variables and are summarized in Table 2, using as dependent variable the number of times (i.e., frequency) a location was selected in multiple DSPNI runs. The following observations can me made: (1) a sensor is more likely to be selected during the optimization phase when it is located in the close vicinity of entry points, or to critical infrastructure (negative sign indicating that as distance increases, this frequency decreases) and (2) the greater the length of the network a sensor can cover, the more likely the sensor will be chosen as a solution. Different levels of granularities (low, medium and high) confirm the significance of the correlation.

4.2. *Regression Results*

The Negative Binomial regression was run for each of the 3 individual networks at three different density levels, resulting in 9 regression models. The derived output was used to assess the strength of the relationship between the geographic parameters and the location of defender resources. Due to strong multicollinearity between variables (MIN-EP, MED-EP and MIN-CI and MED-CI, respectively), we removed both MED-EP and MED-CI from the regression models. Since the coverage of the roadways (CRW) is the product of the number of roadways (NRW) and the length of the roadways (LRW), we only kept CRW in the regression models. Regression results are presented in Table 3.

Figure 2: **Frequency of use of resource location for New York City, NY with grid density increasing from left to right.**

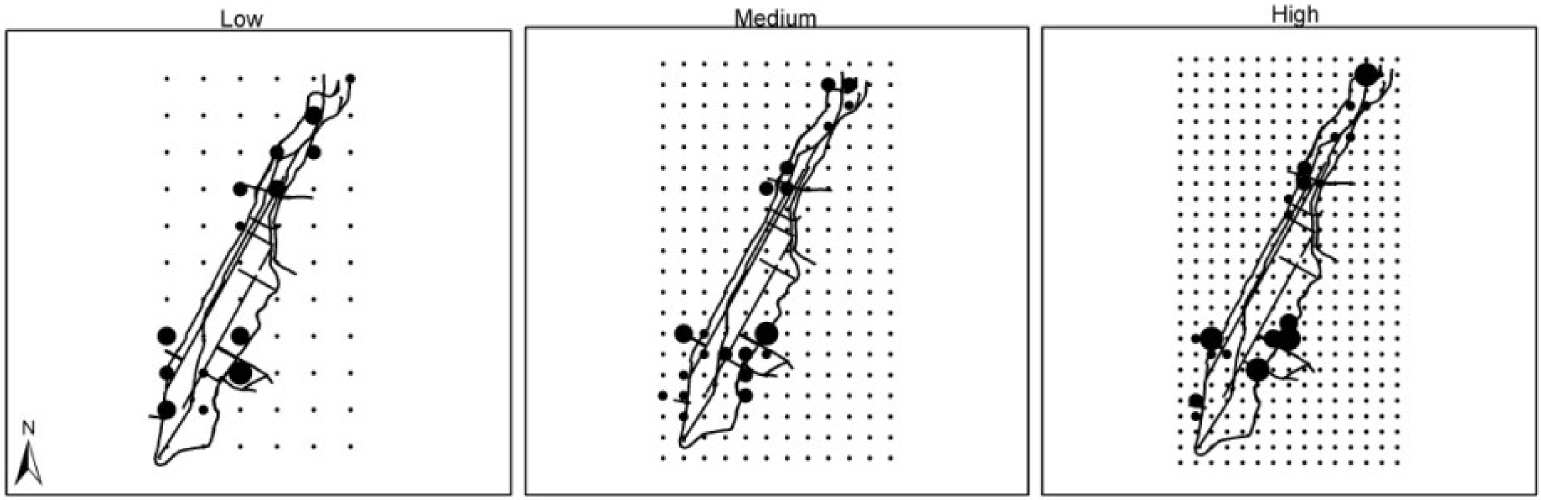

Figure 3: **Frequency of use of resource location for Boston, MA with grid density increasing from left to right.**

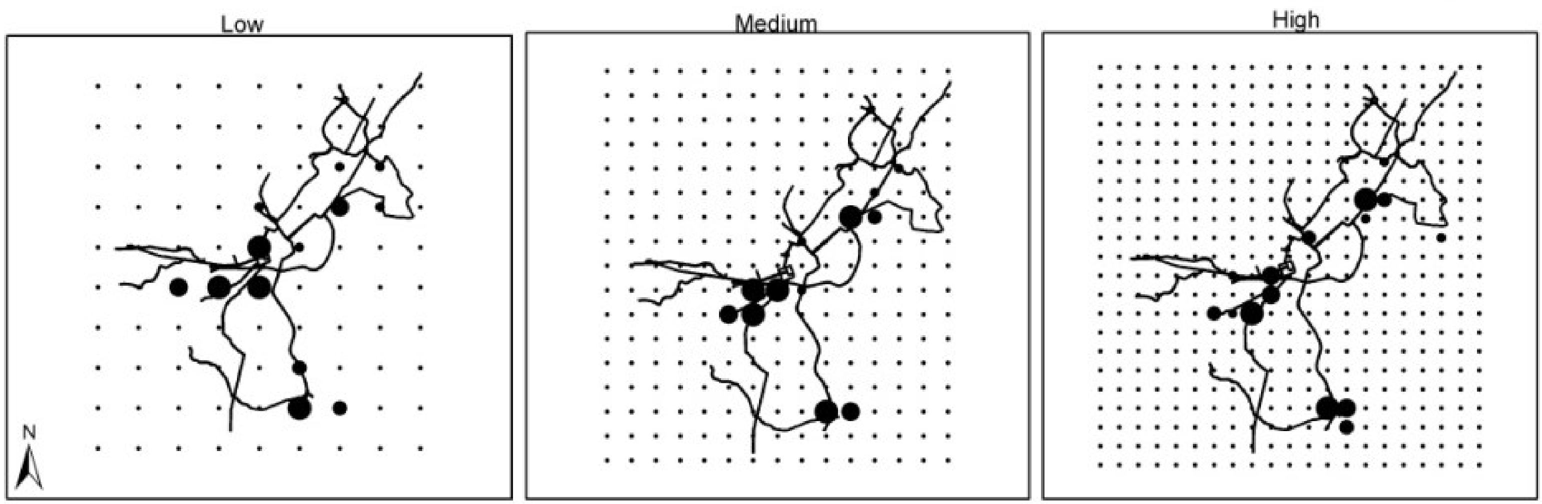

Figure 4: **Frequency of use of resource location for Houston, TX with grid density increasing from left to right.**

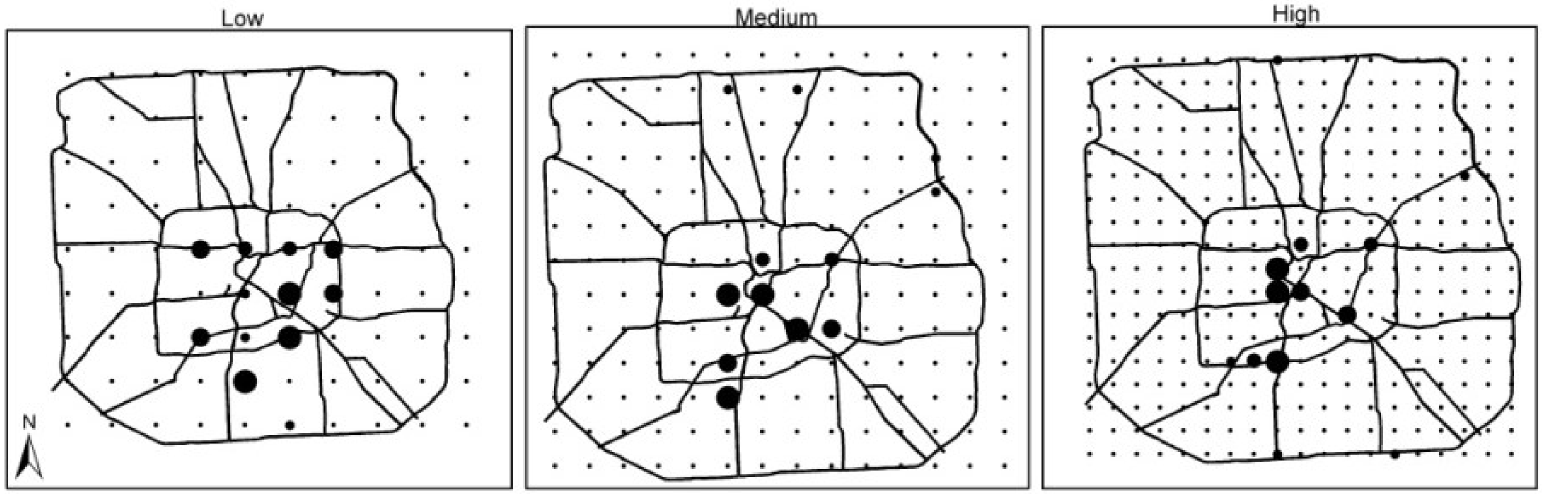

Table 2: Correlation estimates between explanatory variables and frequency of sensor selection

BOSTON	Low	*p*	Medium	*p*	High	*p*
NRW	0.6628	<0.01	0.3887	<0.01	0.2927	<0.01
CRW	0.6512	<0.01	0.4008	<0.01	0.2739	<0.01
CNT	0.6580	<0.01	0.2331	<0.01	0.1044	0.0249
LRW	0.5859	<0.01	0.4489	<0.01	0.3261	<0.01
MIN-EP	-0.1894	0.0737	-0.1210	0.0536	-0.0963	0.0385
MED-EP	-0.2218	0.0356	-0.1640	<0.01	-0.1123	0.0158
MIN-CI	-0.3553	<0.01	-0.2164	<0.01	-0.1392	<0.01
MED-CI	-0.3800	<0.01	-0.2246	<0.01	-0.1514	<0.01
NEW YORK						
NRW	0.6587	<0.01	0.4837	<0.01	0.4314	<0.01
CRW	0.6074	<0.01	0.3948	<0.01	0.4013	<0.01
CNT	0.5334	<0.01	0.2757	<0.01	0.2933	<0.01
LRW	0.6031	<0.01	0.3920	<0.01	0.3336	<0.01
MIN-EP	-0.3317	<0.01	-0.2451	<0.01	-0.1393	<0.01
MED-EP	-0.2137	0.0848	-0.1692	<0.01	-0.1149	0.0207
MIN-CI	-0.4158	<0.01	-0.2605	<0.01	-0.1688	<0.01
MED-CI	-0.1485	0.2340	-0.1425	0.0273	-0.0960	0.0535
HOUSTON						
NRW	0.4722	<0.01	0.3747	<0.01	0.4722	<0.01
CRW	0.4504	<0.01	0.3548	<0.01	0.4504	<0.01
CNT	0.4577	<0.01	0.5244	<0.01	0.4577	<0.01
LRW	0.3925	<0.01	0.3265	<0.01	0.3925	<0.01
MIN-EP	0.4489	<0.01	0.2994	<0.01	0.4489	<0.01
MED-EP	-0.4466	<0.01	-0.2316	<0.01	-0.4466	<0.01
MIN-CI	-0.5262	<0.01	-0.3385	<0.01	-0.5262	<0.01
MED-CI	-0.5383	<0.01	-0.3550	<0.01	-0.5383	<0.01

All regressions converged, and the AIC was lower with finer grid densities. Several variables were significant across all models: for instance the coverage had generally a positive impact in predicting the number of sensors at a particular location, while sensors were also likely to be located closer to entry points (with increasing distance, the sensors were less likely to be selected). From a policy standpoint, entry points and critical infrastructure represent known positions of interest and are used heavily in determining defense allocation schemes (note MD-EP and MD-CI had a negative coefficient in addition to their significance). Intuitively, if all arcs connecting to critical infrastructure are covered, the defender can ensure that any attacker passes through at least one defense resource. In other words, defense strategies attempt to capture the attacker either early in their movement or late in their movement, when the location is more likely to be known. Such observations support intuition, where many facility and regional security models stress the importance of detection early in a security breach and the importance of delay as proximity to a target increases (Przemieniecki 2000, Garcia 2008). Another useful observation from Table 3 is that CRW (coverage) was a significant regional property in connection with the DSPNI optimal defender solutions. This translates to the defender "covering all his/her bases" where the "bases" in this case are potential attacker paths. The more arcs covered and the longer these arcs are the greater likelihood of covering attractive attacker paths.

One important observation from the regression analysis is the close similarity in geographic properties' significance across all three networks, even though there is diversity in their size, connectivity and geometry. This is a major observation to policy makers and emergency managers as it supports the potential of these properties to be used uniformly in regional defense analysis and without concern to how specific network, entry point and critical infrastructure changes affect decision-making.

Table 3: Coefficient and significance results from the Negative Binomial regression for all three test case networks.

BOSTON	**Low**	**p**	**Medium**	**p**	**High**	**p**
Intercept	3.37	<.01	2.30	<.1	1.72	<.01
CRW	0.0000	<.1	0.0000	<.01	0.0000	<.01
CNT	-0.1167	>.1	-1.3570	<.01	-1.3090	<.01
MIN-EP	-0.0007	>.1	-0.0010	<.01	-0.0008	<.01
MIN-CI	-0.0005	<.01	-0.0005	<.05	-0.0005	<.01
NEW YORK						
Intercept	3.01	<.01	2.86	<.05	-1.05	>.1
CRW	0.0000	<.1	0.0000	<.01	0.0000	<.01
CNT	0.0091	>.1	-1.4260	<.01	2.8970	<.01
MIN-EP	-0.0002	<.01	-0.0004	<.01	-0.0003	<.01
MIN-CI	-0.0002	<.05	-0.0003	<.05	-0.0001	>.1
HOUSTON						
Intercept	7.3900	<.01	3.6650	>.1	3.3450	>.1
CRW	0.0000	>.1	0.0000	<.01	0.0000	<.01
CNT	-0.0449	>.1	0.5866	<.05	-0.1531	>.1
MIN-EP	-0.0001	>.1	-0.0001	<.05	-0.0001	<.05
MIN-CI	-0.0003	<.01	-0.0001	<.01	-0.0001	<.01

AIC	Low	Medium	High
Boston	170.09	195.58	586.5
NYC	158.71	219.9	223.21
Houston	132.46	175.35	221.4

4.3. *Visualizing Regression Results*

While the results indicate that correlation does exist between certain geographic properties and defense allocation strategy, the discrete nature of the resource location set *A* is not conducive to developing true situational awareness at the regional level. In policy terms, observations can only be made on the pre-defined location points and not on the continuous region as a whole. In geospatial terms, observations cannot be made on the continuous region based on a discrete set of vector points directly. To address the scaling issue, we use Kriging (see Goovaerts 1997), an interpolation technique that allows to predict the spatial variation within the region as a whole. Figures 5, 6 and 7 illustrate the six distance and containment properties with applied ordinary Kriging at the high grid density level.

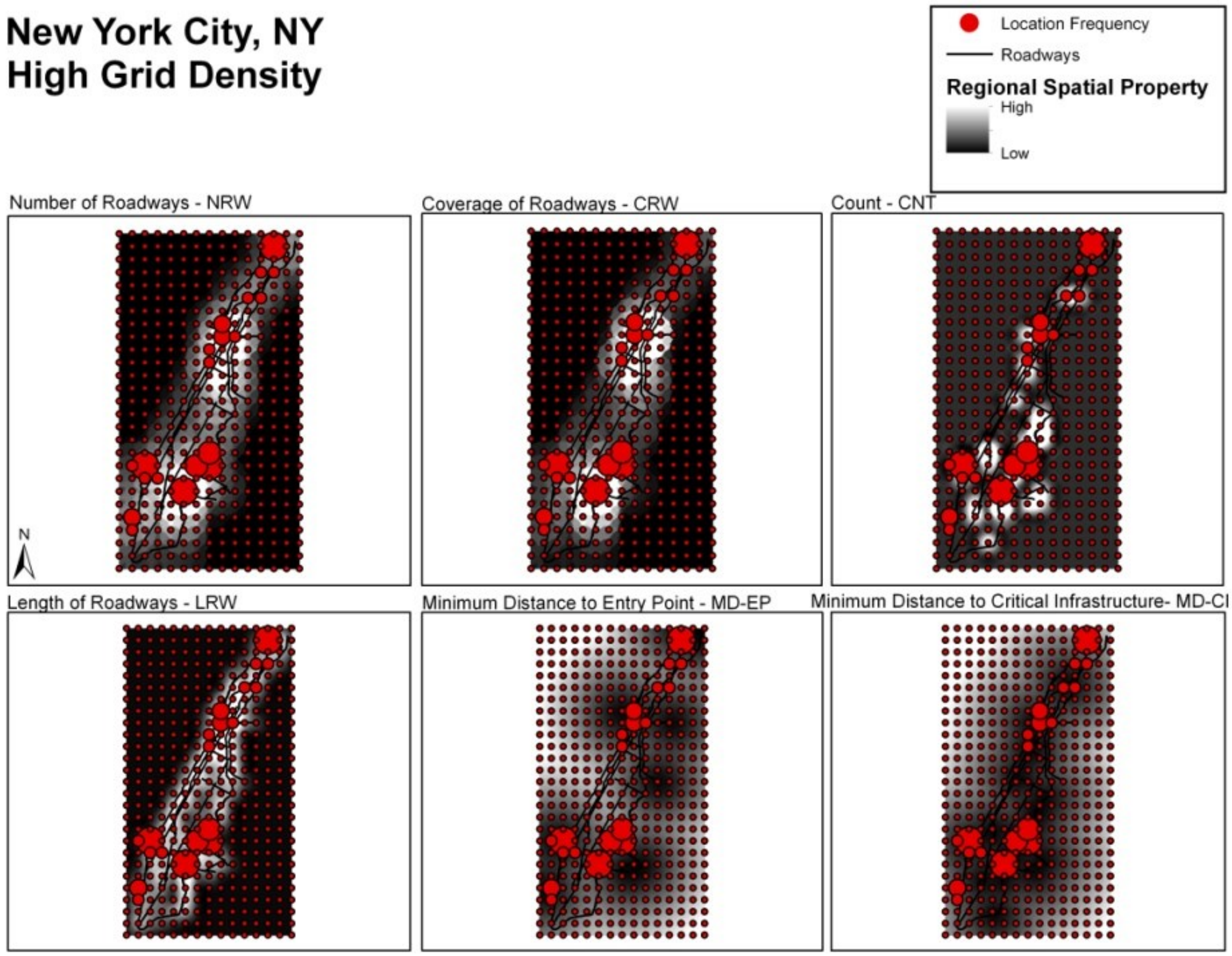

Figure 5: Frequency of use of resource location and regional Kriging for New York City, NY. The figure illustrates frequency in red and the corresponding property value indicated on a color gradient scale.

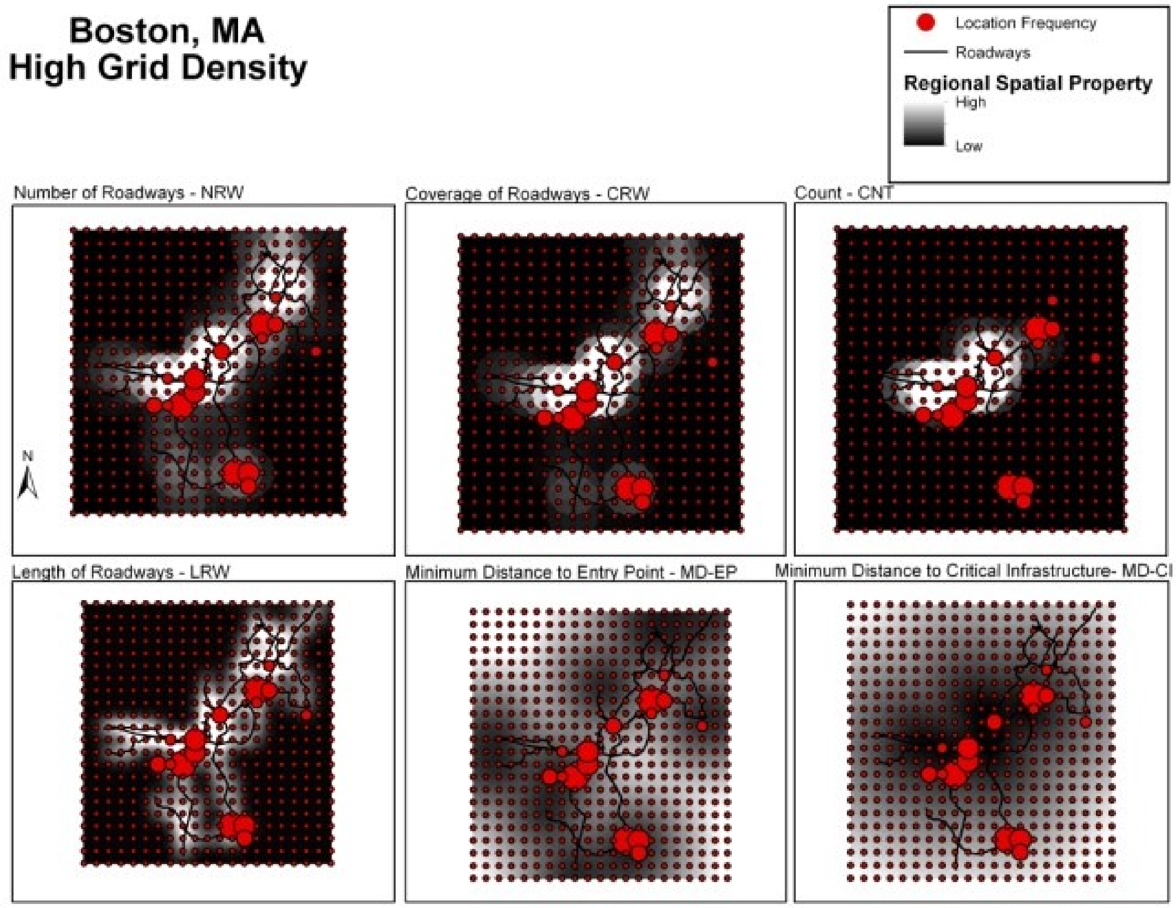

Figure 6: Frequency of use of resource location and regional Kriging for Boston, MA. The figure illustrates frequency in red and the corresponding property value indicated on a color gradient scale.

From the figures, the correlation expressed by the negative binomial regression model is identified. Specifically, figures show the positive correlation between defense resource location and NRW, CRW and CNT by noting how regions of high aggregated frequency (i.e., large red circles) are in close proximity to regions of where the geographic properties exhibit high values (i.e., the white regions). Similarly, an inverse relationship between aggregate frequency and MD-EP and MD-CI is noticed by observing usage clusters in darker areas where the geographic properties exhibit low values. It is also clear from the figures which properties experience higher correlation with defense resource allocation as evident by the presence of outliers, or locations within the uncorrelated "gray area".

By obtaining and visualizing interpolated results, policy decision makers or emergency managers are able to distinguish areas of high and low values of the geographic properties and can easily begin to reason over this regional information in coming to a defense allocation decision. Equally as important is the impact that these visuals lend to the decision-makers policy decisions, which is imperative in obtaining "buy-in" and securing either funding or additional support for proposed regional strategy.

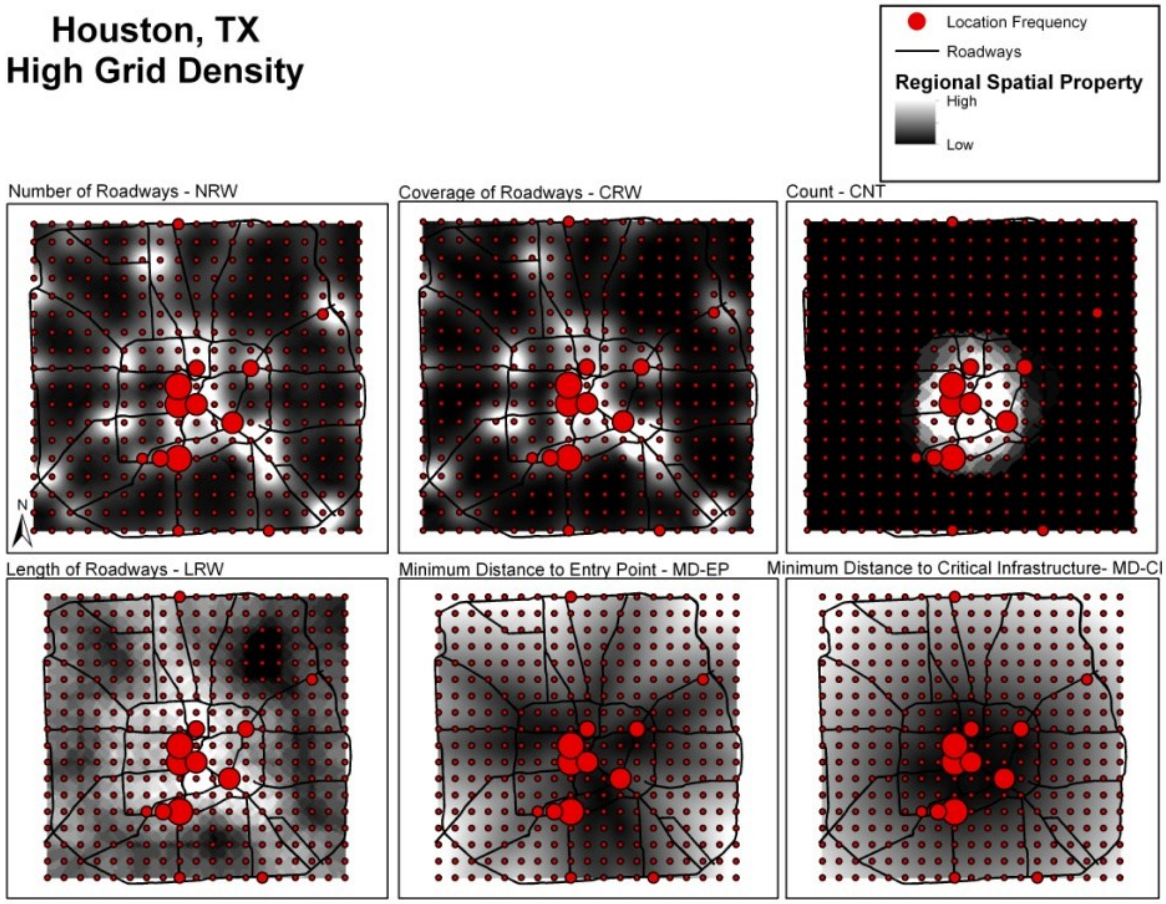

Figure 7: Frequency of use of resource location and regional Kriging for Houston, TX. The figure illustrates frequency in red and the corresponding property value indicated on a color gradient scale.

5. DISCUSSION

Knowledge obtained from the regression can be incorporated in a heuristic, for instance to determine which sensors are part of a first, good solution. Algorithms embedded in optimization solvers may take a long time for large scale and complex problems, and in that respect heuristics provide an alternative (Delmelle et al. 2012, Tong and Murray 2012). Integration of these geographic variables into a heuristic structure (i.e. simulated annealing, genetic algorithm, tabu search) is the next logical step. Successful development and application of such spatially-based heuristics is necessary as real-world problems in policy and defense continue to become more complex. Once the impact of exogenous variables (network, location of critical infrastructure, attacker origins) on the optimization problem has been quantified, it is possible to develop heuristics which capitalize on this information, for instance by using an improved solution to the DSPNI algorithm rather than a random starting solution.

We motivate the need for a heuristic approach by first introducing some computational results for the DSPNI problem sets previously discussed. Table 4 provides processing times for the individual components of the DSPNI Benders Decomposition process as well as the total computation time in CPU seconds.

	Computation Time			Shortest Path Time			Create Defender			Solve Defender			Create Attacker			Solve Attacker		
	NYC	*Boston*	*Houston*	*NYC*	*Boston*	*Houston*	*NYC*	*Boston*	*Houston*	*NYC*	*Boston*	*Houston*	*NYC*	*Boston*	*Houston*	*NYC*	*Boston*	*Houston*
Low	4.791	27.994	3.312	0.090	0.040	0.236	0.953	4.243	0.611	0.775	3.185	0.204	2.707	20.080	2.240	0.356	0.486	0.257
Medium	6.769	25.578	45.962	0.083	0.047	0.192	1.402	3.664	5.116	1.769	4.008	18.732	3.184	17.433	20.774	0.414	0.473	1.340
High	6.093	27.315	86.406	0.085	0.050	0.151	1.312	3.892	6.912	1.537	5.554	52.996	2.870	17.381	25.048	0.374	0.488	1.449

Table 4: CPU solution times (second) for the various DSPNI Benders Decomposition components. 'Computation Time' refers to total computation time. 'Shortest Path Time' refers to the time to solve the shortest path problem. 'Create Attacker/Defender' refers to the time required to create the attacker/defender optimization problem. 'Solve Attacker/Defender' refers to the time to solve the attacker/defender optimization problem.

From table 4, we can observe significant disparity in solution times depending on the network and the density of the potential resource allocation locations (e.g., low, medium, high). While these problems are built upon real-world networks, they do not represent the size/scope of the real-world region itself (i.e., it would take significantly more than the 387 roadways used to adequately model the complexity of Houston, TX). Given the direct positive correlation between network size and computation time, using Benders Decomposition (or similar approaches as employed by the previously discussed SPNIP and NIP papers) is not a reasonable solution to solving large-scale DSPNI problems. In lieu of solving to optimality, we suggest to capitalize on the information around the spatial parameters – identified through regression – to build a greedy heuristic that quickly locates sensor resources within a given region. We use the same DSPNI network test cases and equation (1) where CT_b represents the total contribution of a sensor located at b in the set A and where w_{NRW} represents the weight assigned to NRW (similarly for the other spatial parameters).

$$CT_b = w_{NRW}*NRW_b + w_{CRW}*CRW_b + w_{CNT}*CNT_b + w_{LRW}*LRW_b - w_{MIN\text{-}EP}*MIN\text{-}EP_b - w_{MIN\text{-}CI}*MIN\text{-}CI_b \tag{1}$$

Given a set of known weights w and the known spatial parameter values for each location b, selecting resource locations takes on the objective (2), where v_b is 1 if a sensor is located at b and zero otherwise. This equates to the selection of $\sum_b v_{b=n}$ resources where ‘n’ represents the number of sensors to be located in the region. This solution is equivalent to the selection of the ‘n’ locations with highest CT_b value.

$$Max \sum_b CT_b v_b \tag{2}$$

This heuristic as proposed is conceptually simple and depends only upon the spatial parameters of the problem to locate resources. For the Boston, Houston and NYC networks, we implement this greedy heuristic and evaluate its performance in Tables 5 and 6.

Table 5 and Table 6 represent the aggregation of 4,090 different weight combinations where weights were allowed to take on the values {0.1, 0.35, 0.6, 0.85} for the six spatial parameters used in (1). Each implementation of the Greedy heuristic, which consisted of 4,090 different sensor location solutions (one for each weight combination), was completed in less than 10 CPU seconds for each network and density setting. There was no statistical difference in the CPU run time across either network or density settings.

Table 5: Comparison of normalized and standard spatial parameter values in Greedy heuristic performance. Numbers represent the total percentage of heuristic solutions whose locations are composed of at least 75% of optimal resource locations.

		Boston			*Houston*			*NYC*		
	Resources Located	**Low**	**Med**	**High**	**Low**	**Med**	**High**	**Low**	**Med**	**High**
Normalized	1	100	100	17.09	99.85	78.27	98.71	100	100	48.85
	2	100	62.8	0.37	77.34	10.94	99.61	100	95.58	16.7
	3	99.58	16.11	0	98.9	0.27	61.2	99.8	0.66	0
	4	43.21	1.2	0	92.63	0	0	99.15	0.02	0
	5	99.9	15.21	0	94.92	38.87	3.44	99.97	3.15	81.08
	6	60.3	0	0	91.26	5.59	0.12	99.73	0.51	45.68
	7	12.79	0	0	80.59	0	0	94.21	0	10.64
	8	1.56	0	0	69.11	0	0	81.86	0	0.2
	9	21.48	0	0	79.98	0	0	99.39	0.17	0.9
	10	0.61	0	0	67.38	0	0	89.89	0.04	0.02
Standard	1	100	100	0	100	100	100	100	100	100
	2	100	100	0	100	0	10.55	100	100	100
	3	100	100	0	100	0	7.03	100	0	0
	4	18.75	0	0	100	0	0	100	0	0
	5	100	10.94	0	100	0	0	100	0	100
	6	100	0	0	100	0	0	100	0	42.97
	7	100	0	0	4.3	0	0	100	0	0
	8	100	0	0	0	0	0	0	0	0
	9	100	0	0	100	0	0	12.5	0	0
	10	0	0	0	99.61	0	0	0	0	0

Table 6: Greedy heuristic location quality as measured by the amount of aggregate optimal resource locations captured. Shown are MIN, MAX and AVG values calculated by running 4,090 heuristic weight combinations.

			Number of Resources Located									
			1	2	3	4	5	6	7	8	9	10
Boston	Low	Min	0.125	0.268	0.268	0.304	0.393	0.402	0.429	0.429	0.438	0.464
		Max	0.143	0.268	0.402	0.429	0.536	0.536	0.67	0.67	0.679	0.714
		Avg	0.143	0.268	0.391	0.405	0.427	0.438	0.449	0.465	0.486	0.505
	Med	Min	0.027	0.027	0.027	0.027	0.152	0.17	0.17	0.277	0.277	0.277
		Max	0.027	0.152	0.277	0.295	0.42	0.491	0.491	0.491	0.491	0.509
		Avg	0.027	0.103	0.132	0.171	0.235	0.278	0.292	0.295	0.301	0.314
	High	Min	0	0.009	0.009	0.045	0.045	0.152	0.152	0.152	0.152	0.152
		Max	0.107	0.116	0.143	0.152	0.152	0.152	0.295	0.295	0.295	0.295
		Avg	0.002	0.026	0.092	0.147	0.151	0.152	0.152	0.152	0.153	0.157
Houston	Low	Min	0	0	0	0	0	0	0	0	0	0
		Max	0.134	0.17	0.241	0.366	0.393	0.429	0.518	0.58	0.714	0.813
		Avg	0.031	0.107	0.191	0.243	0.274	0.328	0.377	0.434	0.485	0.526
	Med	Min	0	0	0	0	0	0.009	0.009	0.018	0.027	0.027
		Max	0.08	0.134	0.241	0.33	0.339	0.446	0.446	0.571	0.589	0.589
		Avg	0.021	0.044	0.117	0.193	0.252	0.312	0.362	0.394	0.406	0.415
	High	Min	0	0	0	0	0.009	0.009	0.009	0.009	0.009	0.018
		Max	0.143	0.25	0.259	0.295	0.384	0.429	0.446	0.527	0.554	0.554
		Avg	0.042	0.131	0.213	0.237	0.242	0.246	0.362	0.267	0.29	0.311
NYC	Low	Min	0.027	0.136	0.173	0.209	0.209	0.264	0.327	0.345	0.345	0.445
		Max	0.145	0.255	0.3	0.391	0.491	0.491	0.555	0.609	0.645	0.736
		Avg	0.069	0.173	0.281	0.318	0.33	0.38	0.439	0.493	0.624	0.581
	Med	Min	0.049	0.049	0.049	0.068	0.068	0.107	0.107	0.126	0.165	0.184
		Max	0.058	0.107	0.126	0.165	0.214	0.233	0.272	0.282	0.369	0.398
		Avg	0.055	0.104	0.106	0.112	0.141	0.159	0.181	0.207	0.227	0.252
	High	Min	0	0.018	0.071	0.116	0.125	0.17	0.179	0.268	0.268	0.269
		Max	0.125	0.161	0.268	0.321	0.339	0.384	0.446	0.455	0.464	0.482
		Avg	0.009	0.05	0.16	0.218	0.256	0.282	0.318	0.349	0.377	0.397

From both Tables 5 and 6, we are able to make some important observations on the heuristic's performance. First, it is necessary to normalize the spatial parameter values to obtain higher quality results. Second, the Greedy approach works best on low density settings, suggesting that a more customized Greedy function may be required to replace (1). Third, we see from Table 6 that even this simplistic Greedy approach based on six spatial parameters is capable of capturing 40 – 60% of the aggregate optimal sensor location density periodically (see Max entries) when only allowing 4-6 sensors and reaches the same average performance level with 9-10 sensors (see Avg entries). While this performance is not significant enough to justify the use of this particular Greedy heuristic, it does support the potential for useful and accurate heuristics to be developed using only a small subset of spatial parameters.

6. CONCLUSION

This paper identified the importance of a select set of geographic variables to the spatial pattern of the solutions of a network interdiction problem. An experimental design collected solution data for three sub networks of the New York City, Boston and Houston primary roadways under various problem parameters which included the density of candidate resource locations, resource coverage sensitivity, and budget. While the observed networks were relatively small and could be solved in a short period of time, large problems representative of real-world size and complexity require heuristics to obtain quality solutions in a reasonable amount of time. The work in this paper supports the derivation of such solution techniques through the exploitation of influential geographic variables.

Regression analysis indicates the importance of exogenous variables on the selection of resources in the optimization. Results indicate that the strategic location of a resource, which is how many arcs (and the length of arcs) can be covered from each resource location within its range, is of utmost importance in identifying resource allocation solutions. The distance to regional entry points plays a significant role in the selection process as well. As the distance to entry points increases, the likelihood of the resource location at that point decreases.

The results also show that critical infrastructure location is statistically the least important regional spatial property indicator while the spatial variable that influences results the most is coverage (CRW – which is the hybrid property that accounts for both the number of arcs covered and the length of arcs covered). Simply stated, this result says that network topology (does not mean the topology is significant across the different networks) have a higher impact on resource location than critical infrastructure itself. From the defender's standpoint, results go against an intuitive focus on locating resources near critical infrastructure. Mainly, this is because of reduced efficiency in resource capability. Using critical infrastructure locations as a major input in determining resource locations would significantly change solution stability as the policy maker or emergency manager would constantly be playing a "cat and mouse" game dependent on infrastructure location. In contrast, a single resource located near an entry point or which covers many arcs has the capability to impact every path that uses that entry point regardless of the critical infrastructure target chosen by the attacker. If the defender focuses on covering critical infrastructure independently, there is a "one-to-one" effect while a defender focusing on the coverage of origins experiences a "one-to-many" impact (many destinations are potentially covered or protected by the allocation of a single resource). This situation might not hold in small urban or rural areas where the critical infrastructure is minimal or spread out through the region.

A simple Greedy heurist was proposed to show how the results could be incorporated to help decision makers reach decisions faster with the same level of confidence. The combination of finer-grained data with computational advances continually creates new opportunities for developing solution techniques that seek to shorten solution time and increase solution accuracy. These results can aid in the allocation of resources and policy planning, not only for extreme events management (including natural and man-made events) but also for future planning in locating critical infrastructure. To develop such solution methods, it is necessary that exploration of exploitable characteristics take place and that these characteristics be shown as strong, reliable influencing parameters for a large majority of network cases.

7. REFERENCES

Bard, J. 1998. *Practical bilevel optimization; Algorithms and applications*. Boston: Kluwer Academic Publishers.

Bayrak, H. & M. Baily (2008) Shortest path network interdiction with asymmetric information. *Networks,* 52**,** 133-140.

Brown, G., M. Carlyle, J. Salmeron & K. Wood (2006) Defending critical infrastructure. *Interfaces,* 36**,** 530-544.

Cameron, A. & P. Trivedi. 1998. *Regression analysis of count data*. Cambridge University Press.

Church, R. & C. ReVelle (1976) Theoretical and computational links between the p-median locaiton set-covering and the maximal covering location problem. *Geographical Analysis,* 8**,** 406-415.

Church, R., M. Scaparra & R. Middleton (2004) Identifying critical infrastructure: the median and covering facility interdiction problems. *Annals of the Association of American Geographers,* 94**,** 491-502.

Church, R. L., & Scaparra, M. P. (2007). Protecting Critical Assets: The r- Interdiction Median Problem with Fortification. *Geographical Analysis*, 39(2), 129-146.

Cova, T. & S. Conger. 2003. Transportation hazards. In *Transportation engineer's handbook,* ed. M. Kutz.

Delmelle, E., L. Shuping & A. T. Murray (2012) Identifying bus stop redundancy: a gis-based spatial optimization approach. *Computers, Environment and Urban Systems,* 36**,** 445-455.

Densham, P. & G. Rushton (1992) A more efficient heuristic for solving large p-median problems. *Papers in Regional Science: The Journal of the RSAI,* 71**,** 307-329.

Garcia, M. L. 2008. *Design and evaluation of physical protection systems*. Butterworth-Heinemann.

George, R. (2008). Critical infrastructure protection. *International Journal of Critical Infrastructure Protection*, 1, 4-5.

Goovaerts, P. 1997. *Geostatistics for natural resources evaluation*. Oxford University Press.

Grubesic, Tony H., and Alan T. Murray. "Vital nodes, interconnected infrastructures, and the geographies of network survivability." *Annals of the Association of American Geographers* 96.1 (2006): 64-83.

Hilbe, J. 2007. *Negative binomial regression*. Cambridge, UK: Cambridge University Press.

Israeli, E. & K. Wood (2002) Shortest path network interdiction. *Networks,* 40**,** 97-111.

Lim, C. & J. C. Smith (2007) Algorithms for discrete and continuous multicommodity flow network interdiction problems. *IIE Transactions,* 39**,** 15-26.

Lin, J. & Y. Ban (2013) Complex network topology of transportation systems. *Transport Reviews,* 33**,** 658-685.

Matisziw, T., A. Murray & T. Grubesic (2007) Boudning network interdiction vulnerability through cut-set identification. *Advanced Spatial Science***,** 243-255.

Meyer, B. C., J.-M. Lescot & R. Laplana (2009) Comparison of two spatial optimziation techniques: a framework to solve multiobjective land use distribution problems. *Environmental Management,* 43**,** 264-281.

Miller, H. J. & E. A. Wentz (2003) Representation and spatial analysis in geographic information systems. *Annals of the Association of American Geographers,* 93**,** 574-594.

Murray, Alan T. "An overview of network vulnerability modeling approaches.*GeoJournal* 78.2 (2013): 209-221.

Murray, A., T. Matisziw & T. Grubesic (2007) Critical network infrastructure analysis: interdiciton and system flow. *Journal of Geographical Systems,* 9**,** 103-117.

Ohman, K. & L. O. Eriksson (2002) Allowing for spatial consideration in long-term forest plannign by linking linear programming with simulated annealing. *Forest Ecology and Management,* 161**,** 221-230.

Przemieniecki, J. S. 2000. *Mathematical methods in defense analysis*. Reston, Virginia: AIAA Education Series.

Reese, J. (2006) Solution methods for the p-median proglem: an annotated bibliography. *Networks,* 48**,** 125-142.

Salmeron, J., R. K. Wood & R. Baldick. 2004. Analysis of electric grid security under terrorist threat. ed. D. o. O. Research. Naval Postgraduate School Monterey, CA.

Snediker, Diane E., Alan T. Murray, and Timothy C. Matisziw. Decision support for network disruption mitigation. *Decision Support Systems* 44.4 (2008): 954-969.

Tong, D. & A. Murray (2012) Spatial optimization in geography. *Annals of the Association of American Geographers,* 102**,** 1290-1309.

Wood, K. (1993) Deterministic network interdiction. *Mathematical and Omputer Modelling,* 17**,** 1-18.

Yates, J., R. Batta & M. Karwan (2010) Optimal placement of sensors and interception resource assessment for the protection of regional infrastructure from covert attack. *Journal of Transport Security***,** 1-27.

Yates, J. & I. Casas (2012) Role of spatial data in the protection of critical infrastructure and homeland defense. *Applied Spatial Analysis and Policy,* 5**,** 1-23.

Yates, J. & K. Lakshmanan (2011) A constrained binary knapsack approximation for shortest path interdiction *Computers & Industrial Engineering***,** 1-12.

Zhang, T., S. Madhani & E. van den Berg. 2005. Sensor on patrol (SOP): using mobile sensors to detect potential airborn nuclear, biological and chemical attacks. In *IEEE Military Communications Conference MILCOM*, 2924-2929.

Zhuang, J. & V. Bier (2007) Balancing terrorism and natural disasters - defensive strategy with endogneous attacker effect. *Operations Research,* 55**,** 976-991.