A multi-period capacitated school location problem with modular equipment and closest assignment considerations

Eric M. Delmelle*†, Jean-Claude Thill*, Dominique Peeters‡ and Isabelle Thomas‡

January 22, 2014

To Appear in the Journal of Geographical Systems.

*Department of Geography and Earth Sciences, University of North Carolina at Charlotte, Charlotte, NC 28223, U.S.A.

†Corresponding author. Tel: (704) 687-5991. Email: Eric.Delmelle@uncc.edu

‡Center for Operations Research and Econometrics, Université catholique de Louvain, Louvain-la-Neuve, Belgium.

Abstract

In rapidly growing urban areas, it is deemed vital to expand (or contract) an existing network of public facilities to meet anticipated changes in the level of demand. We present a multi-period capacitated median model for school network facility location planning that minimizes transportation costs, while functional costs are subject to a budget constraint. The proposed Vintage Flexible Capacitated Location Problem (ViFCLP) has the flexibility to account for a minimum school age closing requirement, while the maximum capacity of each school can be adjusted by the addition of modular units. Non-closest assignments are controlled by the introduction of a parameter penalizing excess travel. The applicability of the ViFCLP is illustrated on a large U.S. school system (Charlotte-Mecklenburg, North Carolina) where high school demand is expected to grow faster with distance to the city center. Higher school capacities and greater penalty on travel impedance parameter reduce the number of non-closest assignments. The proposed model is beneficial to policy makers seeking to improve the provision and efficiency of public services over a multi-period planning horizon.

Keywords: Dynamic school location, non-closest assignment, modular capacity.

1 Introduction

On November 9 2010, North Carolina's Charlotte-Mecklenburg Schools (CMS) district[1], announced its plans to close nearly a dozen schools within the city's urban core while opening new ones in the suburbs[2]. It was the first time that the CMS public school authority faced a substantial budget shortfall, causing a massive overhaul of the entire school system and the reassignment of approximately 25,000 students. Nearly instantly, public outrage broke out on behalf of the community, resulting in the arrest of citizens at school board meetings. Clearly,

[1] CMS provides public education in the fourth fastest growing metropolitean area in the United States (US Census 2010)

[2] http://www.charlotteobserver.com/2010/11/10/1825785/move-to-put-off-cms-vote-fails.html, last accessed: October 11 2013.

school closure is a contentious and emotionally charged issue and one that other rapidly changing metropolitan regions may also face. The purpose of this paper is to propose and develop a modeling approach that reflects educational needs of a community and that is financially sustainable. In order to make school systems more efficient, the current infrastructure must be used strategically and critical decisions must be made on where and when to expand (or contract) an existing network of facilities.

We propose a multi-period capacitated location model, where the number of facilities open in each period reflects anticipated changes in population distribution and is determined through overall cost minimization. The proposed Vintage Flexible Capacitated Location Problem (ViFCLP) keeps travel costs to a minimum, while educational expenses incurred by the school system are constrained by the budget available. In the model, schools represent vintage capital for the school system in that they face different depreciation schedules according to their age. Each facility has a maximum student capacity constraint that can flexibly be raised with the addition of modular equipment. The former is intended to stay within budget, while the latter feature prevents overcrowding and associated educational and disciplinary problems in schools. The model is flexible as it allows for facility closure, facility expansion and downsizing, while the status of any school location can be controlled through an age restriction, preventing recently built facilities from closing until fixed costs have been amortized. The flexibility of the model to handle modular equipment, constraints of facility age, the assignment of students to schools, and uncertainty of student growth are important contributions of this work.

Section 2 reviews the importance of location models in the context of public facilities and public schools in particular, and their associated modeling challenges (capacity, closest assignments, closing requirements). Section 3 presents the **V**intage **F**lexible **C**apacitated **L**ocation **P**roblem (ViFCLP) and its rationale. We demonstrate the applicability of the ViFCLP to the Charlotte-

Mecklenburg Schools (CMS) system in section 4. By letting the available budget in the ViFCLP vary, we obtain the Pareto optimal solutions of a bi-criterion problem whose objectives are the travel costs incurred by the pupils and the educational expenses supported by the system. Unfortunately, proceeding in this way often yields duality gaps and sometimes results in hard computational instances. A way to circumvent the difficulty is to consider a linear combination of the objectives, which allows to generate the convex envelope of the Pareto optimal solutions by solving easier problems. Variations in the weighting schemes allow to generate multiple scenarios minimizing travel impedance and school expenditures, respectively˙ The impact of model parameters on the number of schools, modular units and non-closest assignments is presented. Future enhancements of the model are discussed in section 5.

2 Location models for public facilities

Location models for public facilities such as schools, libraries as well as emergency services are critical tools for urban and regional planners as well as decision-makers and a vast body of literature has been dedicated to the subject (ReVelle et al. 1977, Mirchandani and Francis 1990, Daskin 1995, Drezner 1995a). These models generally stipulate that all demand must be served, and when possible, demand must be assigned to a facility so as to incur the least travel impedance, or a facility to be located within an acceptable travel budget from each demand unit, since it is generally recognized that the benefits of a public service decrease with increasing impedance. In fast growing urban regions, it is necessary to build additional public facilities to address increasing demand for service, while closing existing facilities may be needed in areas of population decline (Roodman and Schwartz 1975). Although not restricted to public facilities, Wang et al. (2003) consider both issues of opening and closing facilities to meet anticipated demand, subject to a constraint on the budget (operating and setup). Within the framework of the maximal covering location problem, ReVelle, Murray and Serra (2007) propose a planned shrinkage problem which can integrate closing requirements to decrease operating costs. These models do not integrate constraints on facility capacity.

Urban environments are eminently dynamic, where demand changes rapidly, along with travel and other conditions that affect the operation of a system of public facilities. Dynamic models are designed to optimally locate planned facilities for each time period using forecasted demands and costs (Balou 1968, Wesolowsky 1973, Fong and Srinivasan 1981, Van Roy and Erlenkotter 1982, Drezner 1995b, and Alberada-Sambola et al. 2009). Jacobsen (1990) presents a methodology to determine an optimal sequence of facility capacity expansions to meet time-varying demand at a minimal cost. The multi-period capacitated location (MCL) model attempts to identify where and when new facilities should be added to the current network and their optimal size. Decisions on *when*, *where* and *by how much* are not independent of each other, as also underscored by Manne (1962).

As discussed by Clarke and Surkis (1968), Dost (1968), Maxfield (1972), Viegas (1987), Greenleaf and Harrison (1987), Church and Schoepfle (1993), and Church and Murray (1993), the modeling of a school network is challenging for the following reasons: (i) demand will fluctuate over time, requiring to open new schools and close existing ones; (ii) capacity regulates how much demand can be served at any given time; (iii) each student is assigned to its closest school but capacity constraints may impede this property, thus further affecting the compactness of school districts; (iv) the uneven quality of enrollment forecasts degrades the reliability and efficiency of planning decisions; (v) a certain level of social and ethnic balance must be maintained which may increase total travel time. These critical issues deeply affect school assignment and location decisions. This paper is strictly concerned with issues (i)-(iv).

Modifying an existing school network. Expanding an existing network of schools may be required in regions with high population growth, while school closure may be deemed necessary in areas of population decline. Closing a school may result in a child needing to be bussed further away: student social networks may become more fragile as students from the same neighborhood are assigned to different schools, and as such, the decision to close schools is often a contentious issue (see for instance Witten et al. 2001 in New Zealand). Having an objective and defensible strategy to justify these decisions is therefore crucial. The expansion of an existing school network requires time to plan, obtain approval and to build new facilities (Taylor et al. 1999). Antunes and Peeters (2000, 2001) and Antunes et al. (2009) proposed an extension of the capacitated p-median problem that handles the opening of new educational facilities to address enrollment variations, and the expansion or reduction of existing facilities. Their model cannot force schools to remain open until they reach a certain age nor does it account for the uncertainty associated with future demand forecasts. The former issue is motivated by the high setup costs of schools, which reduces the efficiency of solutions that would close a school before its building costs have been amortized. The latter point is meaningful in that the accuracy of spatial demographic forecasts is higher in the near term.

Capacitated problems and modular units. Uncapacitated facility location models have the undesirable property to site facilities with highly uneven workloads (Murray and Gerrard 1997). Although they require a greater computational effort, capacitated facility location models (ReVelle and Laporte 1996) are better suited for locating public facilities as they provide an alternate form of equity, ensuring that utilization remains well balanced across different facilities. For school systems, this translates in a smaller student-teacher ratio or a more even utilization rate (Church and Murray 1993). Capacity expansions may be required when a school authority wishes to reduce average class sizes or when it is faced with enrollment increases.
Rapid fluctuations in student enrollment must be met in a timely manner and can partly be addressed by the addition of modular units -or portable classrooms (Allison 1998, Lyons 2001). Modular units are an inherent part of the school landscape: in 2010, 33% of schools in the

United States had portable classroom buildings. Portable classrooms are more predominant in larger schools (over 700 students), as well as in overcrowded schools in suburban areas (National Center for Education Statistics 2011). Critical advantages of portable classrooms are in the speed of delivery, their flexibility to absorb fluctuations in unstable enrollment and their low-cost (Fleming 1997, Harris 2006). The possibility to relocate them within a school district to meet shifting growth patterns make them very attractive (Buchanan 2003).

Closest assignments. Assignments of students to the facility that minimizes impedance ('closest assignment') is often a desirable outcome of capacitated school location models (Bigotte and Antunes 2007, Teixeira and Antunes 2008). A high rate of closest assignments generally results from greater school capacities. Gerrard and Church (1996) review different constraints specifically tailored to enforce that each demand node is assigned to its least effort facility. Caro et al. (2004) demonstrate that imposing a maximum walking (or travel) impedance to school generates more compact districts. In this paper, we explicitly model the functional relationship between travel cost impedance and physical distance and impose an additional penalty beyond a certain distance in the form of a weight to reduce the incidence of non-closest assignments. A student may still be assigned to a school that is beyond an acceptable travel distance, but this translates into an extra burden to the community.

3 Problem formulation

3.1 Notation

Our model is inspired by the capacitated plant location model discussed in ReVelle and Laporte (1996). The ViFCLP allows to close schools only if they have reached a certain age, to prevent schools critical to the community from closing (for instance under public or political pressure), capacities to be flexible with the addition of modular units and to reduce the number of non-closest assignments. The formulation of the ViFCLP utilizes the following notation:

Indices and sets

i, I: index and set of demand nodes

j, J: index and set of candidate school locations

m, M: index and set of periods (m_0 = base situation)

J^N: set of proposed (new) school locations, $J^N \subset J$, at base time period m_0

J^E set of existing facilities at present, $J^E \subset J$, at base time period m_0

Travel costs

c_{ijm}: travel cost between an individual i and facility j in period m

d_{ijm}: travel distance between an individual i and facility j in period m

d_{im}^{max}: distance threshold; $d_{im}^{max} > \min\limits_{j \in J} d_{ijm}$

Facility costs

c_m^{++}: leasing cost of a mobile unit at m

$c_{f_{jm}}$: fixed cost of operating a school j at m

c_{s_m}: marginal cost per student at m

$c_{N_{jm}}$: cost to open a new school j at m

$c_{E_{jm}}$: cost to close an existing school j at m

Capacities and additional parameters

z_{jm}^+: maximum capacity of facility j at m

K_{jm}: maximum number of additional mobile units at j during time m

z^{++}: capacity of a mobile unit

a_{im}: demand at location i in period m

e_{jm}^E: age of existing facility j at m

$\bar{E}$: minimum age for a facility to close

w_m: discount factor reflecting importance of period m with $\sum_{m \in M \setminus \{0\}} w_m = 1$

A: a very large number

B: total school budget over the planning horizon

Decision variables

$$X_{ijm} = \begin{cases} 1 & \text{if demand } i \text{ is assigned to } j \text{ at } m \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{jm} = \begin{cases} 1 & \text{if location } j \text{ is open at } m \\ 0 & \text{otherwise} \end{cases}$$

U_{jm} = number of mobile units used at j in period m

3.2 Formulation of the Vintage Flexible Capacitated Location Problem

$$\textsc{Minimize } f_1 = \sum_{i \in I} \sum_{j \in J} \sum_{m \in M \setminus \{0\}} w_m \, c_{ijm} \, a_{im} \, X_{ijm} \tag{1}$$

subject to:

$$\begin{aligned} f_2 = & \Bigg[\Big(\sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_{f_{jm}} \, Y_{jm} \Big) + \Big(\sum_{j \in J^N} \sum_{m \in M \setminus \{0\}} c_{N_{jm}} \, (Y_{jm} - Y_{j,m-1}) \Big) + \\ & \Big(\sum_{j \in J^E} \sum_{m \in M \setminus \{0\}} c_{E_{jm}} \, (Y_{j,m-1} - Y_{jm}) \Big) + \Big(\sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_m^{++} \, U_{jm} \Big) + \\ & \Big(\sum_{i \in I} \sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_{s_m} \, a_{im} \Big) \Bigg] \leq B \end{aligned} \tag{2}$$

$$\sum_{j \in J} X_{ijm} = 1 \qquad \forall i \in I, \forall m \in M \setminus \{0\} \tag{3}$$

$$X_{ijm} \leq Y_{jm} \qquad \forall i \in I, \forall m \in M \setminus \{0\}, \forall j \in J \tag{4}$$

$$\sum_{i \in I} a_{im}\ X_{ijm} \leq (z^{+}_{jm}\ Y_{jm}) + (U_{jm}\ z^{++}) \qquad \forall j \in J, \forall m \in M \setminus \{0\} \tag{5}$$

$$U_{jm} \leq A\ Y_{jm} \qquad \forall j \in J, \forall m \in M \setminus \{0\} \tag{6}$$

$$U_{jm} \leq K_{jm} \qquad \forall j \in J, \forall m \in M \setminus \{0\} \tag{7}$$

$$\bar{E} - e^{E}_{jm} \leq A\ Y_{jm} \qquad \forall j \in J^{E}, \forall m \in M \setminus \{0\} \tag{8}$$

$$Y_{j,m-1} \leq Y_{jm} \qquad \forall j \in J^{N}, \forall m \in M \setminus \{0\} \tag{9}$$

$$Y_{jm} \leq Y_{j,m-1} \qquad \forall j \in J^{E}, \forall m \in M \setminus \{0\} \tag{10}$$

$$X_{ijm}, Y_{jm} \in 0, 1 \qquad \forall i \in I, \forall j \in J, \forall m \in M \setminus \{0\} \tag{11}$$

$$U_{jm} \in \mathbb{Z}^{+} \qquad \forall j \in J, \forall m \in M \setminus \{0\} \tag{12}$$

$$Y_{j,m=0} = \begin{cases} 1 & \text{if } j \in J^{E} \\ 0 & \text{if } j \in J^{N} \end{cases} \tag{13}$$

Objective function (1) minimizes student travel costs to schools, which is the total weighted cost incurred by students traveling to school, summed over all periods, except the base situation m_0, hence $M \setminus \{0\}$. Constraint (2) stipulates that school expenditures must be contained within the overall school budget B, which is an exogenous parameter varying between 0 and ∞. School expenditures include fixed costs (operating schools, opening new schools, closing existing schools, acquiring mobile units) and variable costs (per-student marginal operating cost). J^N denotes the potential school locations at the base time period m_0, with operation starting at the second time period m_1, since it will require some time to get approval, plan and build at the school site. The set J^E contains schools that are currently operational. In constraint (2), the term $\sum_{j \in J} \sum_{m \in M \setminus \{0\}} c^{++}_{m}\ U_{jm}$ represents the leasing costs of mobile units. This equation can be modified if a school plans to acquire these units outright instead. In the ViCFLP model, travel costs c_{ijm} are a linear function of the travel distance d_{ijm} up to a critical threshold d^{max}_{im}:

c_{ijm} : travel impedance from i to j at m; $c_{ijm} = \begin{cases} \alpha d_{ijm} & \text{if } d_{ijm} \leq d_{im}^{max} \\ \alpha d_{ijm} + (d_{ijm} - d_{im}^{max})^{\beta} & \text{if } d_{ijm} > d_{im}^{max} \end{cases}$

Beyond this threshold, an extra impedance penalty is imposed through a weight β, while α governs the relationship between travel distance and travel cost. With this specification, a student i can still be assigned to a school j even if the travel distance from i to j is greater than d_{im}^{max}, but the imposition of an additional impedance makes it more likely to result in assignments to the closest school. Figure 1 illustrates this relationship for an individual i who can be assigned to either of two facilities, one being located nearby and the other at a considerable distance.

INSERT FIGURE 1 ABOUT HERE

Constraint (3) guarantees that each demand node is assigned during each period. Constraint (4) stipulates that a demand node can be allocated to a school only if the school is in operation at that time. Constraint (5) enforces that the demand allocated to a school cannot exceed the permanent capacity of the school expanded by the capacity of installed mobile units. Unlike in Bigotte and Antunes (2007), no minimum capacity requirement is necessary since constraining the overall school budget B will prevent the underutilization of schools. Constraint (6) guarantees that mobile units are added at a site j only if a school exists at this site in period m. In our model, mobile units are leased at the beginning of each period for a fixed amount, if needed. Constraint (7) stipulates that the number of mobile units added to a site j at period m is bound by how many units can be added at this site, K_{jm}. Constraint (8) only allows schools that have reached a certain age $\bar{E}$ to close, while constraint (9) requires that, once a non-existing (candidate) school is opened, it must remain so in subsequent periods, preventing it from closing. The minimum closing age $\bar{E}$ is generally taken to be larger than the planning horizon for the school system. Constraint (10) stipulates that, once an existing school is closed, it cannot be reopened at a later time. Constraints (11) and (12) are integer and single assignment constraints. Since X_{ijm} is binary, split demand assignment cannot occur, that is all the aggregated demand originating from a location is assigned to the same school. Finally, constraint (13) indicates which schools are currently operating (at the base situation, m=0).

The ViFCLP is written in the Python with Geographical Information System (GIS) functionality, interacting with the CPLEX solver, through a Linear Programming form. Results are visualized back into the GIS. The ViFLCP is solved on an IntelCore processor (2.6Ghz) with 4GB of RAM.

4 Real-world case study: Charlotte-Mecklenburg Schools

We illustrate the behavior of the ViCFLP model on a large case study, specifically the CMS system, which serves the city of Charlotte (North Carolina, U.S.A.) and its surrounding county. In the last 30 years, Charlotte has experienced a steady and significant rate of population growth above both North Carolina and the United States at large, which is largely explained by the development of the financial service industry. The city has developed more horizontally in green field locations than vertically by redevelopment or densification of established neighborhoods. Increase in school enrollment has particularly been noticeable during the last two decades. The state of North Carolina as a whole had a projected enrollment growth of 37% from 1999 to 2009; the CMS system is its fastest growing district (U.S. Department of Education 1999).

4.1 Data

Baseline situation. Population increase in the Charlotte area has had a direct impact on the opening/closing of new and existing schools and the ability of CMS to increase school capacity in the short run, for instance through the addition of modular units. In 2008 (m_0), the CMS system operated 16 public high schools (see Fig. 2(a)) with five additional candidate high schools proposed in the periphery [3]. Table 1 reports school capacities, year of construction and number of on-site modular units for the year 2008. From our consultations with school district officials, the schools of Myers Park ($j = 3$), South Mecklenburg ($j = 4$), East Mecklenburg ($j = 5$) and Providence ($j = 6$) (highlighted in Fig. 2(a)) cannot close as these are considered landmarks in Mecklenburg county.

To model high-school demand for the 2008 baseline period, school-age population is generated from population census data and population projections derived from a micro-simulation model of land use and economic dynamics implemented in Mecklenburg County (Ye et al. 2009, Thill et al. 2011). The downtown area of Charlotte (center of the county) is mostly a business district with limited family housing, which explains a low student population. Fig. 2(b) illustrates the actual student allocations derived from school attendance zones in 2008. Blue-colored spider mapping (light gray in hard copy) connects demand nodes to their assigned schools. The width of the line represents the magnitude of the assignment. Graduated symbology is used to reflect the percentage utilization of permanent capacity $\sum_{i \in I} a_{im}\ X_{ijm}/\left(z^{+}_{jm} + U^{+}_{jm}\right)\ Y_{jm}$:]0;1] = green dot (light gray in hard copy) with size increasing with capacity usage,]1,∞] = red (dark gray in hard copy) when enrollment of these schools is above capacity and modular equipment is needed; a white graduated triangle represents the number of mobile units added at each site.

INSERT FIGURE 2 ABOUT HERE

INSERT TABLE 1 ABOUT HERE

Projected school age population. We use projected population of children aged fourteen to seventeen aggregated at the Traffic Analysis Zone (TAZ)-level and assume a county-wide increase of 15% every five years. Increase of school age population is modeled geographically following a *monocentric* growth scenario based on road network travel distance from the

[3]Since 2008, Bailey Road ($j = 20$) and Rocky River ($j = 21$) High Schools have been added to the network, while Garinger ($j = 15$) was threatened of closure and Waddell High School ($j = 14$) closed.

center of the city of Charlotte. Time periods are indexed by $m = 0, \ldots, 3$; each period is separated by five years, and hence facility age increases by five year increments. The projected number of children at the TAZ-level is deflated geographically to account for private and home schooling and children attending special educational programs (total population demand at m_0=33,152; m_1= 38,125; m_2=43,285; m_3=50,504).

Estimating travel, facility and modular capacities. Travel costs, school expenditures and other parameters are estimated through consultation with school planning staff of the Charlotte-Mecklenburg Schools system. Our figures represent conservative estimates for the year 2011. The travel cost per student is for the roundtrip travel of each student to their assigned school, cumulated over the five years of each time period for an average school year of 180 days (rate of \$1 per km). Distances from each TAZ (demand node) to each school site are computed using the road network from the Charlotte Department of Transportation. Information on current physical and modular capacity at each school was received for the year 2008, and it was suggested that a school would not exceed 60 modular units at any given site. Also, local school professionals indicated that a school younger than ten years of age would not be closed, although the impact of this parameter is illustrated later in the paper. School costs and capacities are parameterized as follows in the model:

- Fixed annual cost to operate a school ($c_{f_{jm}}$): \$710,000.
- Cost to build a new school ($c_{N_{jm}}$): \$50,000,000.
- Cost of closing an existing school ($c_{E_{jm}}$): \$1: symbolic cost of one dollar[4].
- Leasing cost per modular unit, per year (c^{++}), including utilities: \$12,500.
- Marginal annual cost of a student (c_{s_m}): \$7,500.

[4] $c_{E_{jm}}$ can also be negative, that is there is a benefit to closing a school. This would be the case if the building or the land can be put back into the market.

- Capacity of a modular unit (z^{++}): 25 students.

Time-dependent weights w_m. Due to the inherent uncertainty of population projections, it seems intuitive that distant periods should be modeled as less influential than near periods in facility location planning. This is all the more so since short-term school decisions are generally more critical than medium- to long-term decisions as the latter can be adjusted later (Antunes and Peeters 2000). Alternatively, a school district may follow a long-term investment strategy, and anticipate the rising cost of new school construction (especially land purchase) in areas of future growth by making early location decisions. In this paper, we assume limited long-term planning and use a discount rate approach to model weights w_m associated to each time period: $w_1 = 0.69103$, $w_2 = 0.23193$, $w_3 = 0.07704$.

4.2 ViFCLP modeling results

We illustrate the change in the ViFCLP objective function over three periods, specifically $m_1 = 2013$, $m_2 = 2018$, $m_3 = 2023$, according to variation in the budget value B - constraint(2) - using parameters d_{max}=5km and $\beta = 0.01$, which means that minor travel penalty is incurred beyond the critical distance threshold d_{max}. The allocation at time period $m_0 = 2008$ is not reported in the figures since our model does not optimize at time period m_0 and student assignments at m_0 are directly derived from actual school attendance zones.

The piecewise smooth curve depicted in Fig 3 - represented by gray triangles - summarizes the relationship between travel costs f_1, which is the ViFCLP objective function, and school expenditures f_2. Each piece is a smooth curve associated with a certain number of schools in operation, which increases from the left to the right of the graph. In contrast, incremental changes in travel costs within the domain of each piece reflect adjustments to the number of mobile units, for a given number of schools. It is noteworthy that a similar increase in school expenditures may impact travel costs differently according to the number of schools. Each piece is concave upward, indicating a negative average rate of change, but that rate is different depending on the total number of facilities in operation. The breakpoints separating the pieces represent major modifications in the optimal configurations: a small increase in the allowed budget results in the creation of new schools, which in turn yields a different pattern of accessibility to the selected sites and a steeper decrease in f_1, while the number of modular units is substantially reduced. We came to realize this is an important and interesting feature of the ViFCLP. For instance, in situations (A) and (B), 54 schools are operational (18 at m_1-m_3), but the number of modular units is different: 654 in (A) and 949 in (B). In cases (C) and (D), 60 schools are operational (20 at m_1-m_3); 511 modular units are necessary in (C) and 858 in (D). The difference of 347 modular units between (C) and (D) reduces travel costs only by 0.73%, while a difference of 295 units between (A) and (B) decreases f_1 by 1.85%. In other words, when the school system operates a larger number of schools through the planning horizon, the addition of modular units has a less significant impact on reducing student-to-school travel than if the system operates fewer schools.

Table 2 provides a summary of different problem instances for the ViFCLP under different budget regimes, as well as the ones that coincide to points (A), (B), (C) and (D). The columns "open schools" and "modular units" are the total number of schools and modular units in operation, respectively. The column "% non-closest" reflects the percentage of students not sent to their closest school, averaged for the three time periods m_1 to m_3, and so is the average student-to-school travel distance. The last column in Table 2 summarizes the computational effort for the solver to reach an optimal solution, expressed in seconds. We observe that a diminishing budget reduces the number of schools in operation, incurring higher travel costs and a higher percentage of non-closest assignments, the average traveled distance per student increases from 5.21km to 5.43km (3.8%) when the number of schools in operation is reduced from 63 to 54 schools. Computing run times sharply increase when the budget for school expenditures becomes smaller, since capacity constraints are tighter, making the allocation procedure much more complex computationally.

INSERT TABLE 2 ABOUT HERE

INSERT FIGURE 3 ABOUT HERE

4.3 A *bi-objective* ViFCLP

By letting the available budget in the ViFCLP vary, we obtain the Pareto optimal solutions of a bi-criterion problem whose objectives are f_1, the travel costs incurred by the pupils and f_2, the educational expenses supported by the system. This gives the trade-off between the investment and the performance and is valuable information to the decision-maker. Unfortunately, when solving the ViFCLP, we are often confronted with a problem of duality gap implied by constraint(2) in the ViFCLP, which results in the non convex shape of the Pareto curve depicted in Fig. 3 and may imply burdensome running times when filling the gap for instance by branch-and-bound techniques. To address this issue, we incorporate this constraint in the objective function and consider a *bi-objective* function $F = \gamma_1 f_1 + \gamma_2 f_2$ with $\gamma_1 + \gamma_2 = 1$, where γ_1 and γ_2 reflect the importance given to travel costs and school expenditures, respectively. On the one hand, this problem is generally easier to solve than the original ViFCLP and on the other hand by varying γ_1 and γ_2 we generate the lower envelope of the Pareto curve. In some sense, this approach is similar to performing a Lagrangian relaxation of constraint (2).

Formulation The *bi-objective* ViFCLP is written as follows:

$$\text{Minimize } F = \gamma_1 f_1 + \gamma_2 f_2 \tag{14}$$

where

$$f_1 = \left[\sum_{i \in I} \sum_{m \in M \setminus \{0\}} \Big(\sum_{j \in J} w_m \; c_{ijm} \; a_{im} \; X_{ijm} \Big) \right] \tag{15}$$

$$\begin{aligned} f_2 = & \left[\Big(\sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_{f_{jm}} \; Y_{jm} \Big) + \sum_{i \in I} \sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_{s_m} \; a_{im} \Big) + \right. \\ & \Big(\sum_{j \in J^N} \sum_{m \in M \setminus \{0\}} c_{N_{jm}} \; (Y_{jm} - Y_{j,m-1}) \Big) + \Big(\sum_{j \in J^E} \sum_{m \in M \setminus \{0\}} c_{E_{jm}} \; (Y_{j,m-1} - Y_{jm}) \Big) + \\ & \left. \Big(\sum_{j \in J} \sum_{m \in M \setminus \{0\}} c_m^{++} \; U_{jm} \Big) \right] \end{aligned} \tag{16}$$

subject to constraints (3)-(12). When parameters $\gamma_1 > 0$ and $\gamma_2 = 0$, the objective function (14) boils down to a weighted capacitated median problem and all schools are opened in each period m, since we only minimize travel costs. The relative magnitude of γ_1 and γ_2 is a matter of public choice, and these values should be set to capture the preponderance of these matters in the community, as perceived by the local school planning agency. In the next paragraphs, we illustrate the changes in school location and student allocation for the *bi-objective* ViFCLP optimized over the three-period planning horizon (m_1 = 2013, m_2 = 2018, m_3 = 2023).

4.4 *Bi-objective* ViFCLP modeling results.

The behavior of the *bi-objective* ViFCLP model is illustrated for different values of parameters d_{max}, β and γ_1, underlining the impact of those parameters to student-to-school travel.
Table 3 lists a summary of different problem instances, according to variation in the values of γ_1, d_{max}=1, 2 and 5km, and $\beta = 0.01, 0.5, 1$ and 1.5. In Fig. 4(a), the values of f_1 and f_2 are plotted for different values of γ_1 using black circles. When zoomed in Fig. 4(b), it is interesting to note that connecting these dots approximates the curve from the *single-objective* ViFCLP presented in Fig. 3. General observations indicate that a larger value of γ_1 forces the addition of new schools over the planning horizon, reduces student-to-school travel and causes an increased number of closest assignments. Moreover, the addition of modular units helps to

alleviate excess demand for high school and intuitively smaller student-to-school travel distances are incurred with greater β-values and smaller d_{max} thresholds. Solving time of the *bi-objective* ViFCLP is less than one minute, compared to much higher numbers for the *single-objective* ViFCLP). Greater run times are incurred when γ_1 becomes smaller, since the *bi-objective* ViFCLP resembles a fixed-charge capacitated location problem. An important contribution of the bi-objective ViFCLP is that similar f_2-values can be obtained in a much smaller time frame using a *bi-objective* approach than a *single-objective*, budget-constraint model.

INSERT FIGURE 4 ABOUT HERE

INSERT TABLE 3 ABOUT HERE

Impact of travel weight γ_1, impedance parameter β and distance threshold d_{max}. An inherent property of capacitated facility location problems is that assignments may not always be to the closest facility. In the ViFCLP, we resort to inflating travel costs by incorporating a distance-based penalty to limit the number of such instances. This distance penalty is controlled by two parameters, namely the distance threshold d^{max} that determines the assignment distance beyond which an extra travel penalty will occur, and β, regulating the magnitude of this penalty. With this approach, when students are assigned to schools beyond a certain distance threshold d_{max}, marginal travel costs increase non-linearly with distance. In Fig. 4(c), we illustrate the sensitivity of the non-closest assignments property with variation in γ_1 and β using a maximum travel distance d_{max}=5km. The value of β is increased from 0.01 (limited travel penalty) to 0.5 (travel penalty weighted heavily beyond d_{max}), but d_{max} remains fixed at 5km. We observe that non-closest assignments are at a minimum when $\gamma_1 = 1$ but that a small decrease in γ_1 causes an increase in non-closest assignments. Also, the number of non-closest assignments is systematically lower when using parameter $\beta = 0.5$ rather than $\beta = 0.01$. These parameters γ_1 and β have a critical influence on the number of schools and modular units in the network and therefore the prevalence of non-closest assignments. In Fig. 4(d), we study the impact of d_{max} on the number of non-closest assignments, keeping β fixed at 1.0. A d_{max}-value of 1km keeps non-closest assignments systematically lower than using a d_{max}-value of 5km. In summary, a high β-value and a low d_{max}-threshold guarantee short travel distances and keep non-closest assignments to low levels, however a increase in β is more likely to cause the opening of new schools than an increase of d_{max}.

Spatial behavior of the *bi-objective* ViFCLP. Four different scenarios (I, II, III, IV) illustrate the spatial behavior of the *bi-objective* ViFCLP, where γ_1 and β values are varied. With the exception of scenario I, we choose $\bar{E} = 30$ years to illustrate the property of the minimum age closing requirement. We keep $d_{max} = 5$km per consultation with local school officials. High school locations and student-to-school assignments are presented in Fig. 5.

Scenario I: $\gamma_1 = 0.5$, $d_{max} = 5$km, $\beta = 0.01$, $\bar{E} = 10$ years.

The first scenario ($\gamma_1 = 0.5$) forces the ViFCLP model to give more importance to the minimization of school expenditures, and gives the school system the flexibility to close schools which are relatively young (ten years). As a result, eight schools are running well above physical capacity. Through the planning horizon, Waddell High School ($j = 14$) is closed, and students must travel a longer distance to their newly assigned schools of Myers Park ($j = 3$) and Olympic ($j = 9$). As a cascading effect, some of the students originally assigned to Olympic High School ($j = 9$) are bussed to Ardrey Kell ($j = 11$) in the Southern edge of the county, incurring longer travel distances. Small numbers of students are sent to their second or third closest schools, which is also typical of capacitated facility location problems. In time period m_2, only three schools operate under physical capacity, and at time m_3 only the capacity at Butler High School ($j = 7$) is in check, not requiring modular units.

To respond to increasing demand, the CMS school network must lease an expanding number of modular units.

Scenario II: $\gamma_1 = 0.5$, $d_{max} = 5$km, $\beta = 0.01$, $\bar{E} = 30$ years.

The parameters used in scenario II are similar to the ones in scenario I, but we impose a tighter restriction on school age closure. As a result, the CMS system operates 16 schools through the planning horizon (total=48), without closing Waddell High School. Rising demand from m_1 to m_3 increases pressure on school capacity and forces the addition of modular capacity. At time m_1 several modular units are added at the schools located near the city center. As the time horizon progresses, demand for high school rises in the periphery, leading to an increase in modular units at schools located outside of the city center. At time m_3, Butler High School is the only facility operating without modular units. The most striking differences between the scenarios I and II come down to requirement of the age-specific closing constraint $\bar{E} = 30$ - scenario II - forcing Waddell High School ($j = 14$) to remain open throughout the optimization. Ultimately, enforcing a smaller value of $\bar{E}$ gives the flexibility to close older schools and those deemed less utilized in the existing network. Alternatively, a tighter age constraint will keep older schools open for a longer time, thus potentially making the construction of new schools unnecessary.

Scenario III: $\gamma_1 = 0.999$, $d_{max} = 5$km, $\beta = 0.01$, $\bar{E} = 30$ years.

Scenario III emphasizes the importance given to student-to-school travel and disregards school expenditures in effect. The system operated 17 schools at each time period, which leads to a smaller number of modular units per site than in the two previous scenarios. The increasing demand in the periphery coupled with greater importance given to travel costs ($\gamma_1 = 0.999$) forces the opening of Stumpton High School ($j = 19$), located in the Northern part of the county[5]. Scenario III contrasts from scenario II as follows: (i) the opening of

[5]Since a percentage of the local demand can easily be met with this new addition, the roles of Hopewell High School ($j = 1$) and North Meck High School ($j = 2$) fade: several of the students who were previously assigned

Stumpton High School reduces student travel and non-closest assignments, (*ii*) with a network of schools expanded to 17 units in this scenario, a larger number of students are sent to their closest school and CMS is expected to rely less on modular units than in scenario II, (*iii*) the number of schools operating above full capacity is smaller than in scenario II.

Scenario IV: $\gamma_1 = 0.999$, $d_{max} = 5$km, $\beta = 1$, $\bar{E} = 30$ years.

This scenario uses similar parameters as scenario III with the exception that $\beta = 1$, increasing the penalty incurred by non-closest assignments. The solution departs sharply from scenario III: all candidate schools are open throughout the planning horizon, reducing the average student travel to 5.22km and only 6.28% of non-closest assignments. Increasing demand in the periphery coupled with greater importance given to travel costs and penalty of non-closest assignments forces the opening of Hucks Road High School ($j = 17$), Stumpton High School ($j = 19$) and Bailey High School ($j = 20$), all located in the Northern part of the county. The addition of these three schools creates a tighter distribution of student to schools travels when compared to scenario III. Similarly, Palisades ($j = 18$) in the southwestern edge and Rocky River High School ($j = 21$) on the eastern side open (in 2011 Rocky River High School was added to the network), thereby reducing travel costs for these students located at the periphery of the county. Since the network is expanded to 21 schools at times m_1, m_2 and m_3, the total number of modular units (781) is significantly lower than in all other scenarios presented.

INSERT FIGURE 5 ABOUT HERE

to those schools are now bussed to Stumpton High School.

5 Conclusions

In this paper, we have proposed a new location modeling framework to expand (or contract) an existing network of schools over a finite planning horizon. The model has the flexibility to handle dynamic situations including significant changes in demand through the temporary adjustment of capacity with modular units as well as facility closing requirements, which allows schools to close only when they reach a certain age and guarantees that schools, once built, will remain open throughout the planning horizon. The ViFCLP should prove useful for capital investment planning of school systems due to its dynamic nature and its ability to account for a number of real-world decision-making considerations.

Our problem is formulated as a single multi-period model, minimizing student travel costs. The model is unique in its flexibility to (*i*) modify the maximum effective capacity of each school using the addition of modular units, (*ii*) incorporate minimum facility age closure, (*iii*) model travel costs and (*iv*) reflect the uncertainty of demand projections over the planning horizon. We demonstrated the applicability of the ViFCLP to the Charlotte-Mecklenburg Schools (CMS) system. We came to realize important and interesting features of the ViFCLP, when exploring the relationship between travel costs (f_1) and school expenditures (f_2). Specifically, we note that a similar increase in school expenditures may impact travel costs differently according to the number of schools in operation. We presented a *bi-objective* alternative to approximate the Pareto curve of the ViFCLP model in Eqs. (1)-(12). Results from the real-world case study indicate that shorter student-to-school travel distances can be obtained by increasing γ_1, which controls the importance of the travel cost sub-objective. Excessively small d_{max}-values can force the school system to add new schools to its network, instead of augmenting school capacities.

The ViFCLP model proposed in this paper can assess the impact of different scenarios of future growth of the student population on the optimal location of schools. A status quo in the center and a larger number of schools in the periphery is a likely outcome of an increase in population in the outskirts of Charlotte. The latter may amplify population sprawl when families decide to relocate to the outskirts of town to be closer to school. If the school-age population moves closer to the city center and to employment centers (reverse sprawl movement), due to increasing gas price, centrally located schools will experience rapid enrollment growth and a pressing need for capacity expansion, while schools located in the periphery will tend to have a lower-than-one utilization rate.

We see the following aspects for future research on the ViFCLP. First, the development of heuristic techniques is needed to find faster solutions for very large problems (Bigotte and Antunes 2007). Second, in some circumstances, a student may travel a longer distance to attend a school that offers specific programs. As such, a more realistic behavioral model of school choice may be more appropriate in some cases (Muller 2008, Araya et al. 2012). Third, our multiperiod school location formulation can be generalized by replacing the exogenous school age constraint by a scheme driven by investment depreciation rates and return on investment considerations. In effect, the location model would become an investment model where school age is endogenized and school renovation or expansion are part of the portfolio of decision options. Finally, the ViFCLP model can be enhanced by accounting for (social) costs associated with the reassignment of students to a different school over successive periods within the planning horizon.

Acknowledgements We would like to thank the Center for Operations Research and Econometrics, Université catholique de Louvain, Louvain-la-Neuve, Belgium which made it possible for the first author to reside at CORE during the Summer 2011. In addition, we appreciate the feedback we received from Scott McCully who is the Executive Director of

Planning and Student Placement for Charlotte-Mecklenburg Schools, as well as two colleagues, who helped us improve a first version of our paper. The first and last author acknowledge support from the Renaissance Computing Institute of North Carolina.

References

Albareda-Sambola M., E. Fernandez, Y. Hinojosa and J. Puerto (2009). The multi-period incremental service facility location problem. Computers and Operations Research, vol. 36(5): 1356-1375.

Allison J. F. (1988). Relocatable classrooms: An alternative to permanent construction. School Business Affairs, vol. 54(1): 24-25.

Antunes A. and D. Peeters (2000). A dynamic optimization model for school network planning. Socio-Economic Planning Sciences, vol. 34(2): 101-120.

Antunes A. and D. Peeters (2001). On solving complex multi-period location models using simulated annealing. European Journal of Operational Research, vol. 130(1): 190-201.

Antunes A., O. Berman, J. Bigotte and D. Krass (2009). A location model for urban hierarchy planning with population dynamics. Environment and Planning A, vol. 41(4): 996-1016.

Araya F., R. Dell, P. Donoso, V. Marianov, F. Martinez and A. Weintraub (2012). Optimizing location and size of rural schools in Chile. International Transactions in Operational Research, vol. 19(5): 695-710.

Balinski M. (1965). Integer programming: Methods, uses, computations. Management Science, vol. 12(3): 253-313.

Balou R. (1968). Dynamic warehouse location analysis. Journal of Marketing Research, vol. 5(3): 271-276.

Bigotte J.F. and A.P. Antunes (2007). Social infrastructure planning: A location model and solution methods. Computer-Aided Civil and Infrastructure Engineering, vol. 22(8): 570-583.

Buchanan B. (2003). Moving to modular. American School Board Journal, vol. 190(6): 36-38.

Caro T., T. Shirabe, M. Guignard and A. Weintraub (2004). School redistricting: Embedding GIS tools with integer programming. Journal of the Operational Research Society, vol. 55(8): 836-849.

Church R. and O.B. Schoepfle (1993). The choice alternative to school assignment. Environment and Planning B, vol. 20(4): 447-457.

Church R. and A.T. Murray (1993). Modeling school utilization and consolidation. Journal of Urban Planning and Development ASCE, vol. 119(1): 23-38.

Church R. and A.T. Murray (2009). Business Site Selection, Location Analysis and GIS. John Wiley, Hoboken, NJ.

Clarke S. and J. Surkis (1998). An operations research approach to racial desegregation of school systems. Socio-Economic Planning Sciences, vol. 1(3): 259-271.

Daskin M. (1995). Network and Discrete Location. Wiley, New York.

Dost J. (1968). Reassessment of the school location problem: A multifunctional role for the school in the urban environment. Annals of Regional Science, vol. 2(1): 136-141.

Drezner Z. (1995a). Facility Location. Springer, New York.

Drezner Z. (1995b). Dynamic facility location: The progressive p-median problem. Location Science, vol. 3(1): 1-7.

Fong C. and V. Srinivasan (1981). The multi-region dynamic capacity expansion problem, Part II. Operations Research, vol. 29(4): 801-816.

Fleming J.A. (1997). The saga of relocatable classroom. The School Administrator, vol. 54(6): 18-21.

Gerrard R. and R. Church (1996). Closest assignment constraints and location models: Properties and structure. Location Science, vol. 4(4): 251-270.

Greenleaf N. and T. Harrison (1987). A mathematical programming approach to elementary school facility decisions. Socio-Economic Planning Sciences, vol. 21(6): 395-401.

Jacobsen S. (1990). Multiperiod capacitated location models. In: Mirchandani, P., Francis, R. (Eds.), Discrete Location Theory, Wiley, New York, pp. 173-208.

Harris D. (2006). Class size and school size: Taking the trade-offs seriously. Brookings Papers on Education Policy, vol. 1: 137-161.

Lyons J.B. (2001). Do school facilities really impact a childs education? Council of Educational Facility Planner International (CEFPI) Brief, Issue 1:6.

Manne A.S. (1967). Investments for Capacity Expansion, Size, Location, and Time-phasing. The MIT Press, Cambridge, MA.

Maxfield D. (1972). Spatial planning of school districts. Annals of the Association of American Geographers, vol. 62(4): 582-590.

Marianov V. and D. Serra (2002). Location problems in the public sector. In: Drezner, Z. and H. Hamacher (Eds.), Facility Location: Applications and Theory, Springer-Verlag, Berlin, Germany, pp. 119-150.

Mirchandani P.B. and R.L. Francis (1990). Discrete Location Theory. New York, Wiley.

Muller S. (2008). Dynamic school network planning in urban areas: A multi-period, cost-minimizing location planning approach with respect to flexible substitution patterns of facilities. LIT Verlag Munster.

Murray A.T. (2010). Advances in location modeling: GIS linkages and contributions. Journal of Geographical Systems, vol. 12(3): 335-354.

Murray A.T. and R. Gerrard (1997). Capacitated service and regional constraints in location-allocation modeling. Location Science, vol. 5(2): 103-118.

National Center for Education Statistics (2011). Digest of education statistics: 2010, chapter 2, Washington, D.C.

ReVelle C., D. Bigman, D. Schilling, J. Cohon and R. Church (1977). Facility location: A review of context-free and EMS models. Health Services Research, vol. 12(2): 129-146.

ReVelle C. and G. Laporte (1996). The plant location problem: New models and research prospects. Operations Research, vol. 44(6): 864-874.

Ribeiro A. and A. Antunes (2002). A GIS-based decision support tool for public facility planning. Environment and Planning B, vol. 29(4): 553-569.

Roodman G.M. and L.B. Schwartz (1975). Optimal and heuristic phase-out strategies. AIIE Transactions, vol. 7(2): 177-184.

Sridharan R. (1995). The capacitated plant location problem. European Journal of Operational Research, vol. 87(2): 203-213.

Taylor R., M. Vasu and J. Causby (1999). Integrated planning for school and community: The case of Johnston County, North Carolina. Interfaces, vol. 29: 67-89.

Teixeira, J., A. Antunes and D. Peeters (2007). An optimization-based study on the redeployment of a secondary school network. Environment and Planning B, vol. 34(2): 296-315.

Teixeira, J. and A. Antunes (2008). A hierarchical location model for public facility planning. European Journal of Operational Research, vol. 185(1): 92-104.

Thill J.-C., E.C. Delmelle, J.S. Son, K. Venkitasubramanian, Y. Zhou, Z. Gong and A. Ali (2011). Charlotte Land Use and Economic Simulator. Presented at the Annual North American Meetings of the Regional Science Association International, Miami, FL.

Toregas C., R. Swain, C. ReVelle and L. Bergman (1971). The location of emergency service facilities. Operations Research, vol. 19(6): 1363-1373.

U.S. Department of Education (1999). A back to school report on the baby boom echo: No end in sight. Washington, D.C.

Van Roy T. and D. Erlenkotter (1982). A dual-based procedure for dynamic facility location. Management Science, vol. 28(10): 1091-1105.

Yeates F. (1963). Hinterland delimitation: A distance minimizing approach. The Professional Geographer, vol. 15(6): 7-10.

Viegas J. (1987). Short and mid-term planning of an elementary school network in a suburb of Lisbon. Sistemi Urbani, vol. 1: 57-77.

Wang Q., R. Batta, J. Bhadury and C.M. Rump (2003). Budget constrained location problem with opening and closing of facilities. Computers and Operations Research, vol. 30(13): 2047-2069.

Wesolowsky G. (1973). Dynamic facility location. Management Science, vol. 19(11): 1241-1247.

Witten K., T. McCreanor, R. Kearns and L. Ramasubramanian (2001). The impacts of a school closure on neighborhood social cohesion: Narratives from InverCargill, New Zealand. Health and Place, vol. 7(4): 307-317.

Ye X., K. Konduri, R. Pendyala, B. Sana and P. Waddell (2009). A methodology to match distributions of both housefold and person attributes in the generation of synthetic populations. Paper presented at the annual meeting of the Transportation Research Board, Washington, D.C.

id(j)	Name	z^+_{jm}	$U_{j,0}$	e^E_{jm}	$Y_{j,m=0,\ldots,3}$
1	Hopewell	2237	33	2001	-
2	NorthMeck	2441	37	1951	-
3	*MyersPark*	2823	21	1951	1
4	*SouthMeck*	1832	20	1959	1
5	*EastMeck*	2058	30	1950	1
6	*Providence*	2247	9	1989	1
7	Butler	2136	20	1997	-
8	Independence	2378	39	1967	-
9	Olympic	1877	21	1966	-
10	Vance	1856	34	1997	-
11	ArdreyKell	2314	-	2001	-
12	MallardCreek	2107	-	2007	-
13	WestMeck	2041	11	1951	-
14	Waddell	1164	-	2001	-
15	Garinger	1833	27	1959	-
16	WestCharlotte	2029	16	1938	-
17	HUCKSROAD	2250	-	-	-
18	PALISADES	2250	-	-	-
19	STUMPTON	2250	-	-	-
20	BAILEY	2250	-	-	-
21	ROCKYRIVER	2250	-	-	-

Table 1: School names, capacities, number of modular units $\sum_{j \in J^N} U_{j,m=0} = 318$, date of construction and non-closing requirement. School names in italics indicate facilities that must remain open through the planning horizon, while school names in capital letters are candidate school locations.

Budget($) $[in 10^6]$	β	d_{max} (km)	f_1($) $[in 10^6]$	f_2($) $[in 10^8]$	Open Schools	Modular Units	Travel avg.(km)	% non-closest	Time (sec)
$^{(A)}$6645	0.01	5	1.0901	6.6449	54	654	5.43	13.47	550.85 (int., 0.01% gap)
6650	0.01	5	1.0835	6.6499	54	691	5.40	12.26	194.34
$^{(B)}$6680	0.01	5	1.0700	6.6795	54	949	5.34	7.16	32.67
6700	0.01	5	1.0588	6.6999	57	696	5.30	9.56	25.47
$^{(C)}$6733	0.01	5	1.0557	6.7329	60	511	5.31	11.63	129.76
6750	0.01	5	1.0497	6.7499	60	672	5.25	8.56	28.32
$^{(D)}$6773	0.01	5	1.0480	6.7716	60	858	5.24	6.39	76.61
6800	0.01	5	1.0420	6.7990	63	673	5.23	7.65	15.66
6900	0.01	5	1.0410	6.8140	63	800	5.21	5.85	14.76
7000	0.01	5	1.0410	6.8140	63	800	5.21	5.85	13.84

Table 2: ViFCLP instances with varying budget parameters. Schools and modular units are the total number of schools and modular units in use through the planning horizon. We use parameter $\bar{E}$=10 years. Instances (A), (B), (C) and (D) are also illustrated in Fig. 3.

γ_1	β	d_{max} (km)	f_1($) $[in 10^6]$	f_2($) $[in 10^8]$	Open Schools	Modular Units	Travel avg.(km)	% non-closest	Time (sec)	Notes
0.999	0.01	5	1.1026	6.6285	51	926	5.49	8.75	12.58	
0.999	0.01	5	1.1026	6.6285	51	926	5.48	8.85	16.04	(III,*)
0.99	0.01	5	1.2110	6.5932	46	1049	5.95	10.97	64.60	
0.975	0.01	5	1.2277	6.5952	45	1095	5.97	10.90	34.56	
0.9	0.01	5	1.2274	6.5952	45	1095	5.97	10.90	60.74	
0.75	0.01	5	1.2276	6.5952	45	1095	5.97	10.90	41.30	
0.5	0.01	5	1.2278	6.5952	45	1095	5.97	10.90	39.93	(I)
0.5	0.01	5	1.2016	6.5891	48	956	5.85	9.92	50.91	(II,*)
0.999	0.01	2	1.1223	6.6284	51	925	5.49	8.86	13.85	
0.99	0.01	2	1.2291	6.5932	46	1049	5.95	10.86	54.34	
0.975	0.01	2	1.2452	6.5952	45	1095	5.98	10.93	37.40	
0.9	0.01	2	1.2451	6.5952	45	1095	5.98	10.94	52.67	
0.75	0.01	2	1.2449	6.5952	45	1095	5.97	11.00	51.24	
0.5	0.01	2	1.2451	6.5952	45	1095	5.98	11.19	56.65	
0.999	0.5	2	2.4825	6.7095	57	778	5.30	8.01	13.95	
0.99	0.5	2	3.0088	6.5973	48	1024	5.84	8.12	33.42	
0.975	0.5	2	3.0691	6.5939	47	1025	5.92	9.84	67.11	
0.9	0.5	2	3.2274	6.5952	45	1095	5.99	10.42	49.75	
0.75	0.5	2	3.2276	6.5952	45	1095	5.99	10.52	47.91	
0.5	0.5	2	3.2277	6.5952	45	1095	5.99	10.47	58.77	
0.999	1	1	542.6739	6.8152	63	804	5.22	5.81	11.32	
0.99	1	1	542.6894	6.8141	63	795	5.22	5.93	10.84	
0.975	1	1	542.6818	6.8134	63	789	5.22	6.09	10.34	
0.9	1	1	552.0302	6.7123	57	799	5.29	8.07	15.99	
0.75	1	1	561.8898	6.6669	54	833	5.35	9.11	18.31	
0.5	1	1	635.1541	6.5978	48	1028	5.83	8.35	60.33	
0.999	1	2	419.4911	6.8152	63	804	5.22	5.81	11.40	
0.99	1	2	419.4897	6.8141	63	795	5.22	5.92	15.45	
0.975	1	2	419.4999	6.8131	63	786	5.22	6.03	12.19	
0.9	1	2	428.6430	6.7122	57	799	5.29	8.05	20.16	
0.75	1	2	438.3672	6.6671	54	835	5.35	9.07	19.19	
0.5	1	2	511.0590	6.5978	48	1028	5.83	8.17	72.82	
0.999	1	5	129.9321	6.8127	63	782	5.23	6.26	11.04	
0.999	1	5	129.9374	6.8125	63	781	5.22	6.28	7.78	(IV,*)
0.99	1	5	129.9375	6.8119	63	775	5.23	6.27	14.75	
0.975	1	5	129.9513	6.8110	63	767	5.23	6.45	8.55	
0.9	1	5	136.2120	6.7106	57	783	5.29	8.40	23.31	
0.75	1	5	145.1026	6.6641	54	809	5.37	9.63	23.31	
0.5	1	5	210.7947	6.5972	48	1023	5.84	8.41	29.76	
0.999	1.5	5	13653.5280	6.8136	63	790	5.24	6.40	12.2	
0.99	1.5	5	13654.1517	6.8134	63	788	5.24	6.42	12.65	
0.975	1.5	5	13653.8940	6.8132	63	787	5.24	6.45	11.41	
0.9	1.5	5	13653.8888	6.8133	63	787	5.24	6.42	12.07	
0.75	1.5	5	13653.1717	6.8134	63	787	5.24	6.39	17.49	
0.5	1.5	5	13654.0963	6.8121	63	777	5.24	6.56	11.15	

Table 3: *Bi-objective* ViFCLP instances with varying γ_1, d_{max}, β parameters. Schools and modular units are the total required number of schools and modular units necessary throughout the planning horizon. We use $\bar{E}$=10 years as default. *(I)*, *(II)*, *(III)*, *(IV)* = scenario I, II, III, IV. $^{(*)}$ = $\bar{E}$ of 30 years

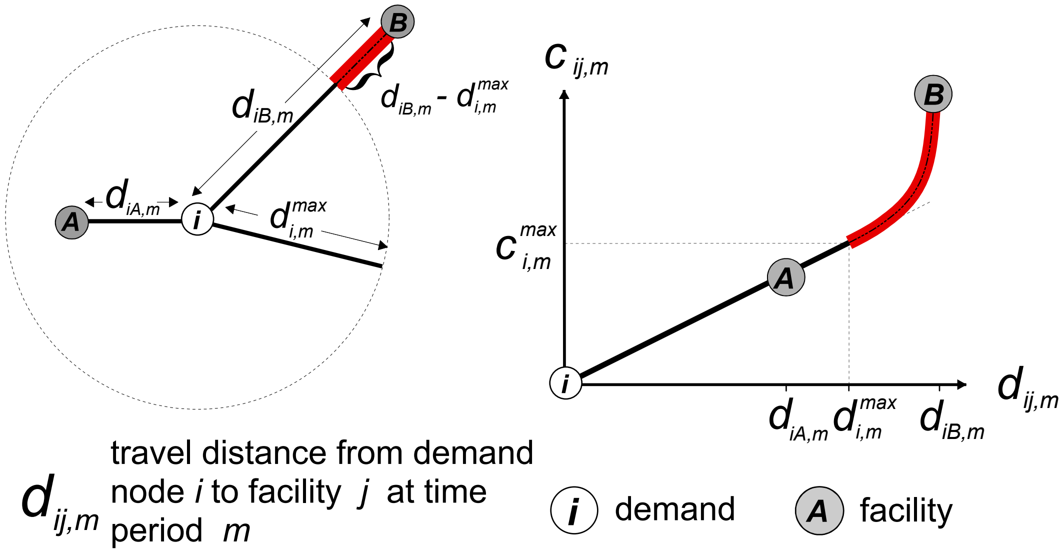


Fig. 1: Travel impedance as a function of distance from demand node to facility. If a node is assigned to a facility beyond a travel distance d_{im}^{max}, the marginal impedance increases non-linearly.

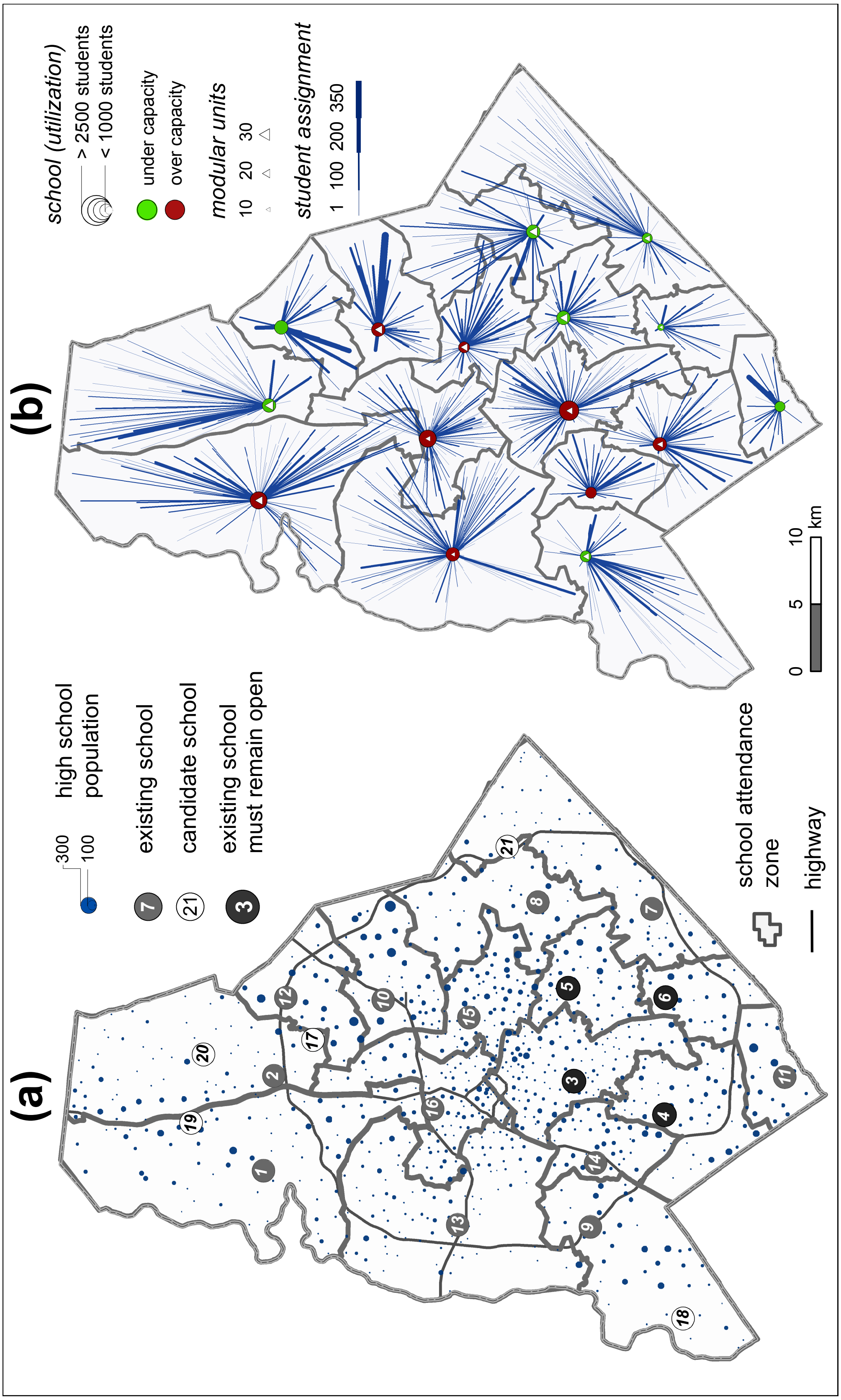
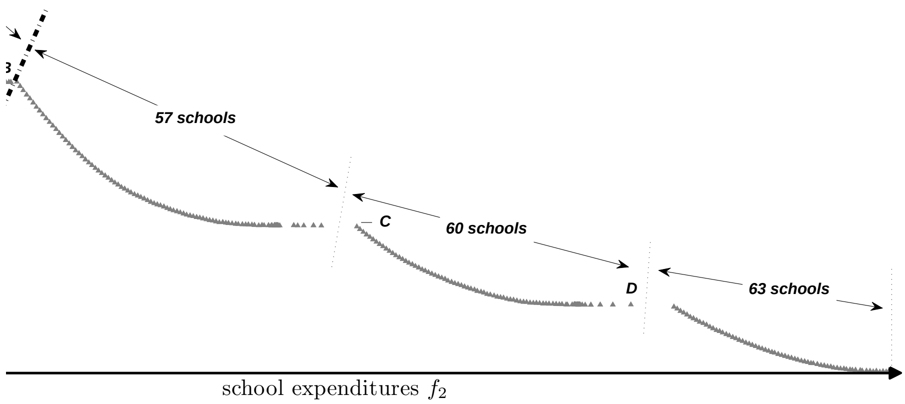


Fig. 2: Location of existing and candidate high schools in the CMS system in 2008 in (a) and student assignment based on 2008 school attendance zone boundaries in (b). Red dots in (a) - in dark gray on the hard copy - denote schools above permanent capacity (and thus with modular equipment), while a triangle is used to represent the number of mobile units added at each site). School names are listed in Table 1.

(a)



(b)

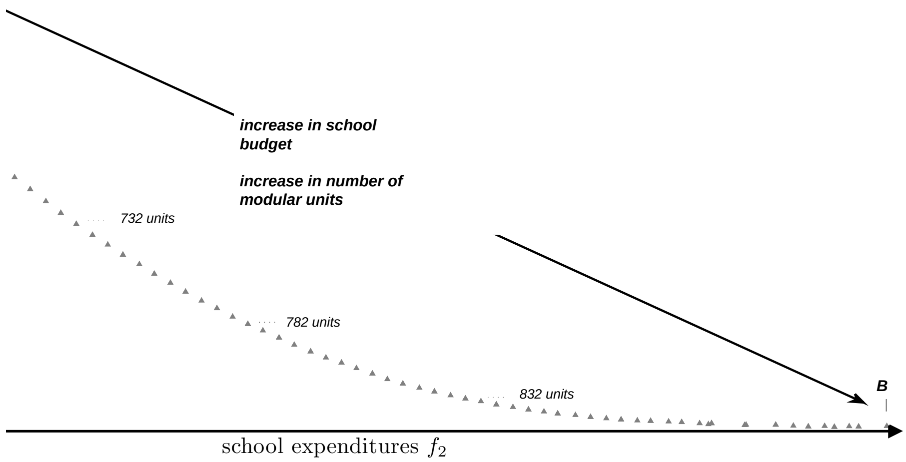


Fig. 3: Variation in travel costs and school expenditures for the ViFCLP model [Eqs. (1)-(12)], optimized under different budget regimes. Variation in the number of schools in (a) and in the number of modular units in (b).

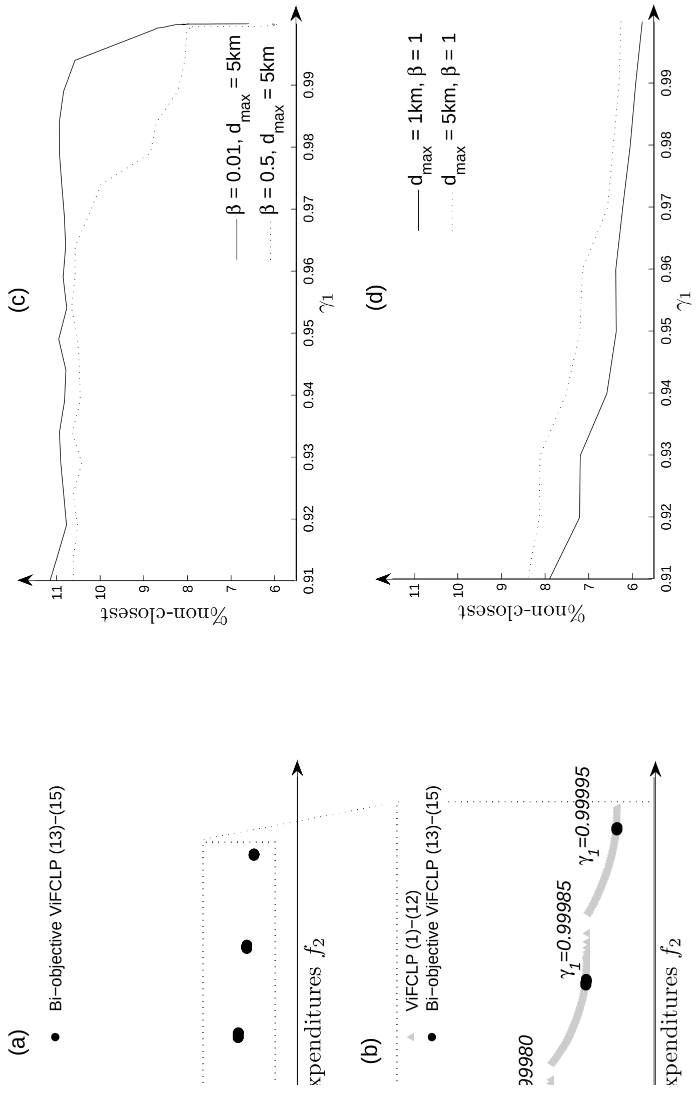


Fig. 4: Variation in travel costs and school expenditures for the *bi-objective* ViFCLP model (Eqs. 13-15), optimized for different values of γ_1 in (a) and (b). Variation in non-closest assignments under different β and d_{max} regimes in (c) and (d), respectively.

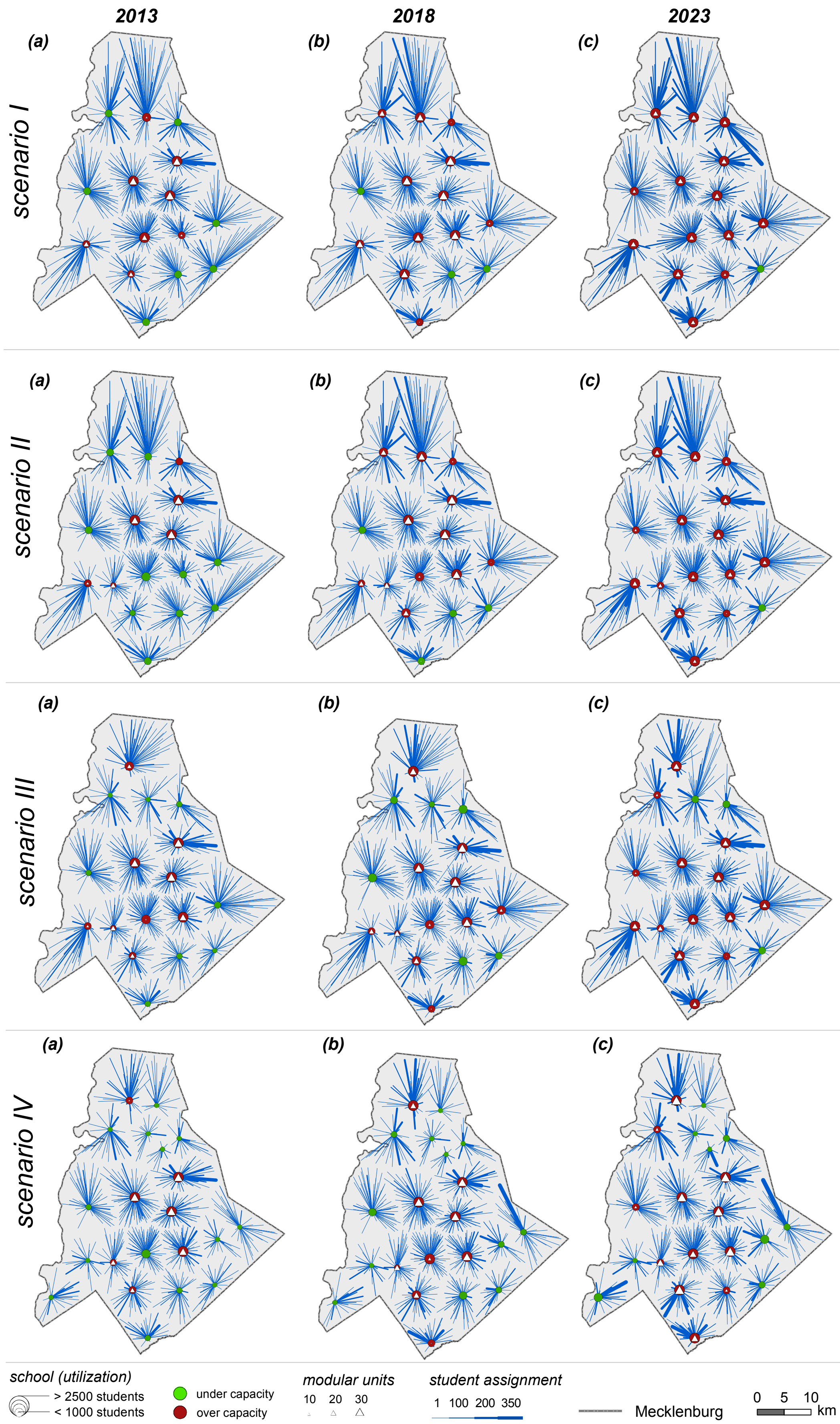


Fig. 5: School locations and student-to-school assignments for four different scenarios (I, II, III, IV). Scenario I: γ_1=0.5, d_{max} = 5km, β = 0.01, $\bar{E}$ = 10 years; Scenario II: γ_1 = 0.5, d_{max} = 5km, β = 0.01, $\bar{E}$ = 30 years, Scenario III: γ_1 = 0.999, d_{max} = 5km, β = 0.01, $\bar{E}$ = 30 years. Scenario IV: γ_1 = 0.999, d_{max} = 5km, β = 1, $\bar{E}$ = 30 years.