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Multiplexed fibre-optic sensors using ring interferometers

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Abstract. We describe a number of multiplexing techniques which may be used for fibre-optic sensors in which the sensing element is a ring resonator. The basic optical unit comprises a sensing ring resonator connected to a second interrogating ring resonator of nearly equal length, and illuminated by a source of short coherence length. A number of such units may be combined to form an array, using coherence multiplexing techniques. Using such an arrangement, the high finesse of the ring resonator may be exploited to yield a sensor of increased sensitivity over that obtainable with a two-beam interferometer. When a high-coherence source is used, then multiplexing may be accomplished using frequency-division techniques. However, in this case, the resonators must have low finesse, in order to avoid cross-talk. We also show that when a source of moderate coherence length is used, it is possible to use both frequency division and coherence multiplexing simultaneously, with the potential of realizing a large sensor array.

1. Introduction

A large variety of fibre-optic interferometric sensors has been developed using different optical configurations and signal processing techniques [1]. Recently there has been increasing interest in multiplexing such sensors in one system to avoid unnecessary duplication of light sources, photodetectors, and fibre transmission lines. Several techniques have been previously reported including time-division multiplexing [2, 3], coherence multiplexing [4, 5], wavelength-division multiplexing [6], and frequency-division multiplexing [7]. The combination of time addressing with coherence multiplexing [8] and time addressing with frequency division [9] has also been reported with the advantage of increasing the number of sensors in a sensing system.

Most multiplexed sensor systems described to date have used the Mach–Zehnder interferometer as the primary sensing element although multiplexed systems based upon miniature low-finesse fibre Fabry–Perot interferometer sensing elements have recently been reported [10, 11]. The ring resonator is a multiple-beam interferometer, and has high finesse provided that a beam-splitting element (directional coupler), which satisfies the resonance conditions, is used [12, 13]; an advantage in sensitivity over that of a two-beam interferometer may therefore be achieved. The ring resonator has no separate fibre reference arm, and is therefore capable of absolute measurement; the Mach–Zehnder effectively makes differential measurements since only the optical path difference between the signal and reference arms can be determined.

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An objective of the present work was to realize a multiplexed sensor array, making use of high-finesse monomode optical-fibre ring resonators. A disadvantage of the ring resonator is that it is an inherently unbalanced interferometer, so that it requires a source with a long coherence length, and the phase of the interference is a strong function of the source wavelength. This effect limits the phase resolution of the ring resonator, especially when a diode laser source is used, since the wavelength of these lasers shows substantial drift and noise. Accordingly, in the present work we have adopted a tandem arrangement of ring resonators, in which each ring is of nearly equal length. A source is used whose coherence length is short in comparison with the length of either ring, but is longer than the difference between the optical paths of the two rings. Hence interference is only observed between nearly equal optical paths, so that the arrangement is balanced, and is hence relatively insensitive to wavelength variations.

It is convenient to use one of the rings in the tandem arrangement as the sensor, and the second ring as the receiver. The sensor ring may be deployed remotely from the receiver, whereas the receiver may incorporate any active components required for signal processing. The tandem arrangement is also well suited to coherence multiplexing. An array of sensing rings, all of different lengths, may be deployed. These sensors may then be connected by a single fibre cable to a corresponding set of receiving rings. Each receiving ring will show visible interference only in conjunction with the sensor ring of matching path length. We describe below an experimental demonstration of a multiplexed array based on these principles. In situations where a raised noise floor due to source frequency variation may be tolerated, then unbalanced optical arrangements may be used. Under these circumstances, frequency-division multiplexing is feasible. In this method, a frequency-chirped laser source is used to generate a pseudo-heterodyne carrier at the output of the unbalanced ring pair. The frequency of the carrier is dependent on the path imbalance, so that demultiplexing may be achieved by band-pass filtering. However, it is shown below that when high-finesse ring resonators are used in this way, then considerable cross-talk is introduced between the outputs. Nevertheless, it is demonstrated that frequency-division multiplexing may be satisfactorily employed when low-finesse rings are used. It is further shown that, with such low-finesse rings, it is possible to combine the techniques of coherence multiplexing and frequency-division multiplexing, and so to realize large arrays.

2. Tandem ring interferometers system

2.1. *Theory*

A tandem fibre ring system is shown in figure 1. The system is illuminated by a relatively short coherence-length source such that no interference occurs at each individual ring interferometer but the interference can be observed when the interferometers' path imbalances are matched within the coherence length of the source. Following the notation used in references [12, 13], the general form of a ring-interferometer transfer function—assuming polarization insensitive components—is given by:

$$\left|\frac{E_4}{E_1}\right|^2 = (1-\gamma_0)\left[1 - \frac{(1-k)(1-k_r)}{[1+(kk_r)^{1/2}]^2 - 4(kk_r)^{1/2}\sin^2\left(\frac{\beta L}{2} - \frac{\pi}{4}\right)}\right], \qquad (1)$$

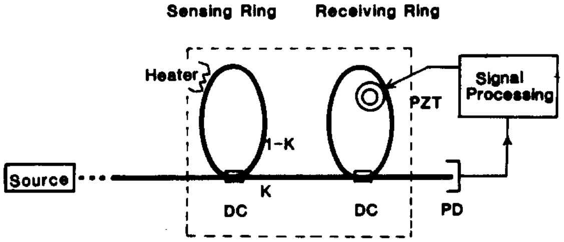

Figure 1. A tandem ring configuration: DC = directional coupler; PZT = piezoelectric cylinder; PD = photodiode.

where E_4 and E_1 are the electric-field vectors at the output and input of the ring respectively, k and γ_0 are the coupler's coupling ratio and loss factor respectively, β is the propagation constant (see below), and

$$k_r = (1-\gamma_0)\exp(-2\alpha_0 L), \tag{2}$$

is the resonant coupling ratio, where α_0 is the fibre attenuation coefficient and L is the length of the fibre. For a system of two tandem rings, then assuming no loss in the fibre-lead between the two rings, $E_4 = E_1'$ (where the prime denotes the second ring) and hence

$$\left|\frac{E_4'}{E_1}\right|^2 = (1-\gamma_0)^2 \left[1 - \frac{(1-k)(1-k_r)}{[1+(kk_r)^{1/2}]^2 - 4(kk_r)^{1/2}\sin^2\left(\frac{\beta L}{2} - \frac{\pi}{4}\right)}\right] \times \left[1 - \frac{(1-k)(1-k_r)}{[1+(kk_r)^{1/2}]^2 - 4(kk_r)^{1/2}\sin^2\left(\frac{\beta(L+\Delta L)}{2} - \frac{\pi}{4}\right)}\right], \tag{3}$$

with

$$\beta = \frac{2\pi n v}{c},$$

where v is the optical frequency, c is the velocity of light *in vacuo* and n is the effective refractive index of the fibre, and the two rings are assumed to be identical, except for a difference in length ΔL. For a relatively short coherence-length source with a Lorentzian spectrum [14], the output intensity of the system is

$$I \propto \frac{\Delta v}{2\pi} \int \frac{1}{(v-v_0)^2 + (\frac{1}{2}\Delta v)^2} \left|\frac{E_4'}{E_1}\right|^2 \mathrm{d}v, \tag{4}$$

where Δv is the spectral bandwidth of the source. By substituting for β in equation (3) using equation (4), we have evaluated this integral numerically for different values of k and k_r, and these results are shown in figure 2. Figures 2 (*a*)–(*c*) show the output of the two-ring system when the cavity lengths of the two rings are very well matched, that is ΔL is very small in comparison with the source coherence length. As can be seen, the sensitivity is a function of the coupling ratios, therefore high or low

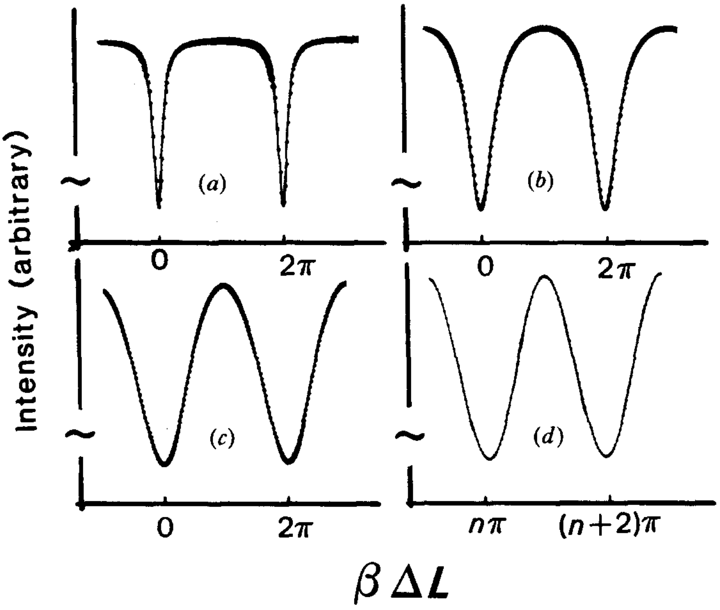

Figure 2. Output intensity of a tandem ring arrangement against $\beta \Delta L$ for: (*a*) $k=k_r=0{\cdot}87$. (*b*) $k=0{\cdot}5$, $k_r=0{\cdot}9$. (*c*) $k=0{\cdot}13$, $k_r=0{\cdot}9$. (*d*) $n\Delta L=L_c$, $k=0{\cdot}5$, $k_r=0{\cdot}9$. The intensity units are arbitrary, and the zero is suppressed in each case.

sensitivity (figures 2 (*a*) and (*c*)) systems can be designed by selecting appropriate coupling coefficients. For $n\Delta L$ of the order of the source coherence length, the output intensity (equation (4)) has a transfer function similar to those of two-beam interferometers (figure 2 (*d*)). This is because light which has circulated the sensor ring once can interfere with light which has circulated the receiver ring once with reasonable visibility. However, for higher-order circulations, the path imbalance increases, so that the recombining beams become mutually incoherent, regardless of coupling ratio.

2.2. *Experiment*

A system similar to the one shown in figure 1 was constructed to investigate these concepts experimentally. The two ring resonators were made using couplers with k factor $\approx 0{\cdot}87$ and loss factor $\gamma_0 \sim 0{\cdot}1$, and spliced to each other in series. The cavity lengths (physical length) of both interferometers were 561 cm and they were carefully chosen to be equal (∓ 2 mm). The attenuation coefficient of the fibre was $\sim -8\,\mathrm{dB\,km^{-1}}$, so that $\alpha_0=0{\cdot}921\times 10^{-3}$; the resonant coupling ratio, k_r, is hence 0·899. A source with a coherence length of 5 mm was used to illuminate the system and the output was monitored using a p–i–n photodiode. A piezo-electric transducer was incorporated in the second ring resonator to strain the fibre and hence to modulate the phase over one period of its transfer function. The visibility was determined by applying a slowly increasing d.c. voltage to the same PZT to achieve the highest visibility (in this case maximum finesse), thus ensuring that the path difference between the two rings was minimized. In other words, ΔL was very small in comparison with the coherence length of the source (L_c), and maximum circulation was achieved.

2.3. *Results and discussion*

The coupling ratios were close to the resonant values, and hence a transfer function similar to that theoretically predicted and shown in figure 2(*a*) was expected. Figure 3 shows the photodiode output when the two rings were carefully balanced and the PZT was driven by a ramp generator. As expected, a high finesse was observed, thus indicating that a greater sensitivity (expressed as a rate of change of intensity with phase) is available in comparison with two-beam interferometers. This enhanced sensitivity is valuable in sensor applications, and we show below that the optical arrangement is practical for use in this way.

A number of techniques are available to process the signal from an interferometer to give an output which varies linearly with the phase. The active homodyne technique [16, 17] may be applied here straightforwardly, in which feedback is applied to the phase modulating PZT in the receiving interferometer to compensate for a phase change in the sensing element. The output from the pair of rings is hence maintained at a constant value. For maximum sensitivity, the output is maintained at the position of maximum slope in its transfer function.

As an example of the use of this technique, the pair of rings was demonstrated as a temperature sensor. The sensing ring was placed in an insulated enclosure with provision for electrically heating an 8 cm length of the fibre, a thermocoupler was used to provide an independent measurement of temperature. The receiving interferometer was maintained in an insulated enclosure at constant temperature. Figure 4 shows the variation of the feedback voltage, which is proportional to the phase change, as a function of the temperature of the sensing element.

The temperature sensor described above is suitable for applications where very high-resolution is required, provided that a relatively large contact area is available. However, the arrangement is also well suited to measurements of periodic quantities (such as acoustic pressure). In such cases, where the amplitude of the measurand is small, it would be appropriate to use a low gain–bandwidth product servo to compensate for slow ambient temperature changes, and to determine the measurand directly from the photodiode output [17].

A well known disadvantage of homodyne signal processing is that the servo-locking range, and hence the measurement range, is inevitably limited. The obvious alternative is to use heterodyne processing, of which the most easily implemented technique in this case is to generate a pseudo-heterodyne carrier by ramping the

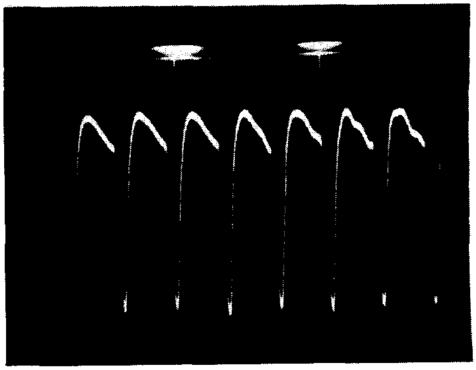

Figure 3. The photodiode output for the arrangement of figure 1 when the PZT is modulated with a sawtooth waveform.

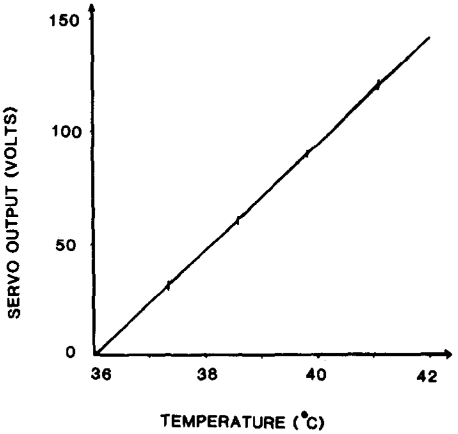

Figure 4. The tracking voltage applied by the electronic servo to the PZT against the temperature change of the sensing fibre for the arrangement of figure 1.

phase in the receiving interferometer. Figure 3 shows the photodiode output obtained when the phase modulator in the receiving interferometer was driven with a sawtooth, whose amplitude was sufficient to drive the tandem interferometer over one period of its transfer function. By comparing the phase of the photodiode output with that of the driving sawtooth, the optical phase change induced by the measurand may be determined [15].

3. Multiplexed sensing systems

3.1. *Coherence multiplexing*

Coherence multiplexing of interferometers is achieved by using a source of short coherence length. The interferometers are deployed in pairs: one element of the pair is the sensing interferometer, the other is a receiving interferometer. Each interferometer has a path length imbalance which is large in comparison with the source coherence length. However, for each pair the difference in path length imbalance between the interferometers is less than the source coherence length. Therefore, when the interferometers are connected together, interference is observed, where the phase is dependent on the relative path imbalance (as discussed before). Interference fringes of high finesse are observed as before; however, the visibility is somewhat reduced by incoherent light from the other ring combinations.

Figure 5 shows the experimental arrangement, which incorporated two sensors, and two corresponding receivers. The arrangement was illuminated using the short coherence-length source described above ($L_c \cong 5\,\mathrm{mm}$). The ring resonators were made using couplers with a k factor of 0·5 and the cavity lengths of the rings were chosen to be

$$R_b = R'_b = 195\,\mathrm{cm},$$

$$R_a = R'_a = 214\,\mathrm{cm},$$

such that $|R_i - R'_i| \ll L_c$. As would be expected, experiments showed that interference could not be obtained, except for the appropriate sensor and receiver pair. Figures 6 (*a*) and (*b*) show the outputs from the respective photodetector where a ramp

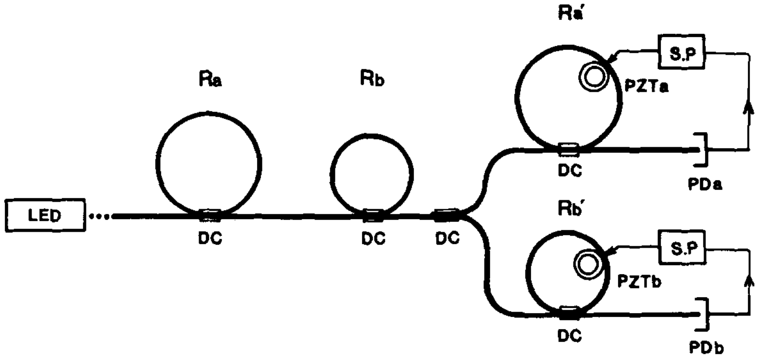

Figure 5. Experimental arrangement for the coherence multiplexing of two pairs of ring resonators. SP = signal processing.

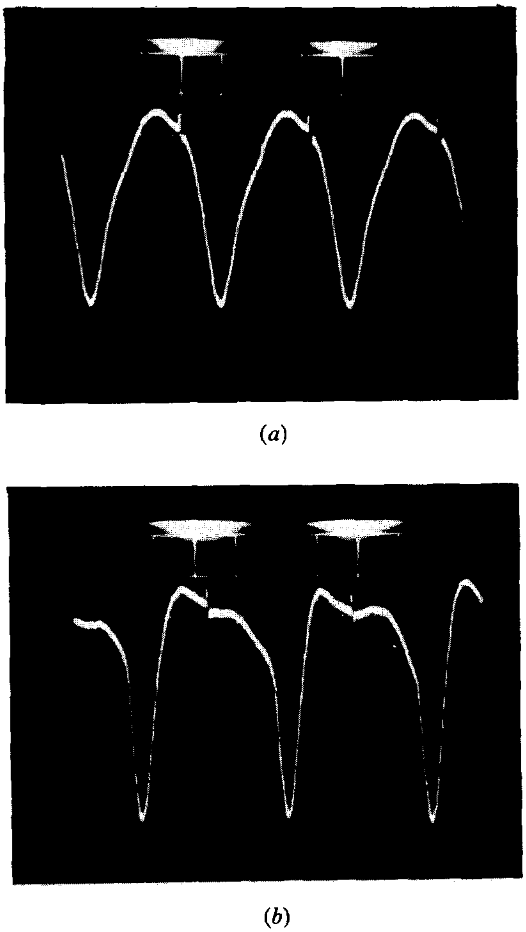

Figure 6. The interference patterns obtained experimentally for the arrangement of figure 5 corresponding to (*a*) the R_a, R'_a pair and (*b*) the R_b, R'_b pair.

voltage was applied to the receiving interferometer's phase modulators. In these experiments, the coupling coefficients of the available couplers deviated from the ideal resonant value, so that the finesse observed was rather lower than that demonstrated in the system of section 2.2. Nevertheless, the transfer function is clearly sharper than that of a two-beam interferometer. As before, signal processing may be carried out using a phase modulator in the relevant receiving interferometer. Alternatively, a passive technique—for example, using two sources of different wavelengths [18]— may also be employed.

3.2. *Frequency-division multiplexing*

Figure 7 shows an arrangement for frequency-division multiplexing, which has a series topology and comprises rings of limitingly low finesse, such that each one may be assumed to behave as a two-beam interferometer. In practice, this may be achieved by using directional couplers having coupling coefficients far from the resonance value. Suppose that the *i*th ring has an optical path length of m_iL_0 where m_i is an integer. The optical frequency of the laser is then modulated by a sawtooth ramp of amplitude $\Delta\nu$. The phase difference produced in the *i*th interferometer is thus

$$\Delta\phi_i = 2\pi m_i L_0\,\Delta\nu/c, \tag{5}$$

and with suitable choice of $\Delta\nu$, we may arrange that

$$\Delta\phi_i = 2\pi m_i. \tag{6}$$

Therefore, considering the *i*th ring alone, the output shows an approximately sinusoidal form, with power concentrated at a frequency $m_i f$, where f is the ramp frequency.

Consider now the two rings connected in series. Outputs will be observed at frequencies $m_1 f$ and $m_2 f$ correspondingly to the first and second rings respectively, which may hence be distinguished by band-pass filtering. However, outputs at $(m_1 - m_2)f$ and $(m_1 + m_2)f$ will also be observed, so that to avoid cross-talk, suitable values of m_1 and m_2 must be chosen—for example 1 and 3. This result may be generalized to any number of rings, where the progression of m_i may be determined from the difference relation [7]:

$$m_i = m_{i-1} + N\sum_{j=1}^{i-1} m_j + R, \tag{7}$$

where N and R are integers.

It is now clear why the condition of low finesse was imposed. If higher order circulation were permitted, then in the two-ring system (for example) outputs will be observed at frequencies $(am_1 + bm_2)f$, where a and b are integers corresponding to the higher order circulations. It may thus be seen that values of m_i determined from equation (7) would give rise to cross-talk between the outputs. Other values of m_1

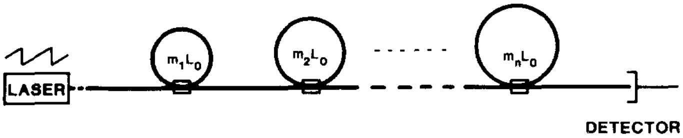

Figure 7. Series topology for the frequency-division multiplexing array of ring resonators.

and m_2 may be found for given a and b to produce outputs free of cross-talk, but these values would require a large difference between the cavity length of the two rings, which is undesirable, and the number of multiplexed sensors would be severely restricted.

In practice, this method of multiplexing suffers from a number of disadvantages. Practical considerations show that for systems containing only a few sensors, even in the case of low finesse, the path lengths become impractically large, and quickly exceed the source coherence length, L_c. An alternative arrangement is shown in figure 8. This comprises an array of sensing rings of optical lengths $L_1, L_2 \ldots L_n$, together with a receiving interferometer of optical length L_r. A fibre delay line is induced between each of the sensing interferometers, of length much greater than L_c. This effectively prevents coherent mixing between the outputs of any two sensing interferometers, thus relaxing the path imbalance requirements for the series of sensing interferometers. For satisfactory operation we require that

$$L_i,\ L_r > L_c,$$

but

$$|L_i - L_r| < L_c.$$

As before, we generate a series at the output of frequencies $m_i f$, where

$$m_i \propto |L_i - L_r|.$$

For a parallel topology (figure 8) the set of m_is is more compact than the series configuration (equation (7)) and is given by [7]

$$m_i = N(i-1) + R. \tag{8}$$

Ring interferometers used in such a system should have a very low finesse (sinusoidal transfer function) in order to minimize the cross-talk (as discussed before). In our experiments on such a system two sensing ring interferometers were used with cavity lengths of 1706 cm, 1866 cm and a receiving ring interferometer with a cavity length of 1746 cm. The ring resonators were made using couplers with 50–50 coupling ratios ($k \cong 0{\cdot}5$). A laser source with a moderate coherence length ($L_c \cong 5$ m) illuminated the system. The splice losses were such that higher-order circulations were not observed. The frequency of the laser was modulated with a sawtooth wave form at 330 Hz. The amplitude of the ramp current was such that the first pair (1706 cm, 1746 cm, $\Delta L = 40$ cm) was driven over 2π rad, and hence the

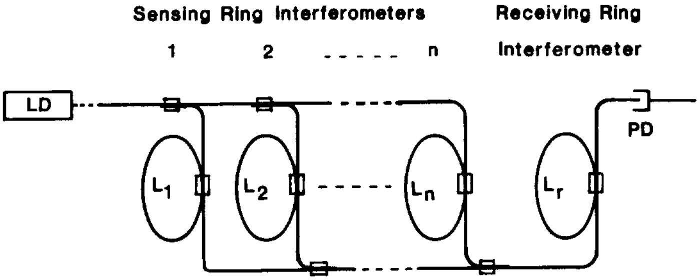

Figure 8. A parallel topology frequency-division multiplexed array of ring resonators interrogated by a coherence-tuned receiving interferometer.

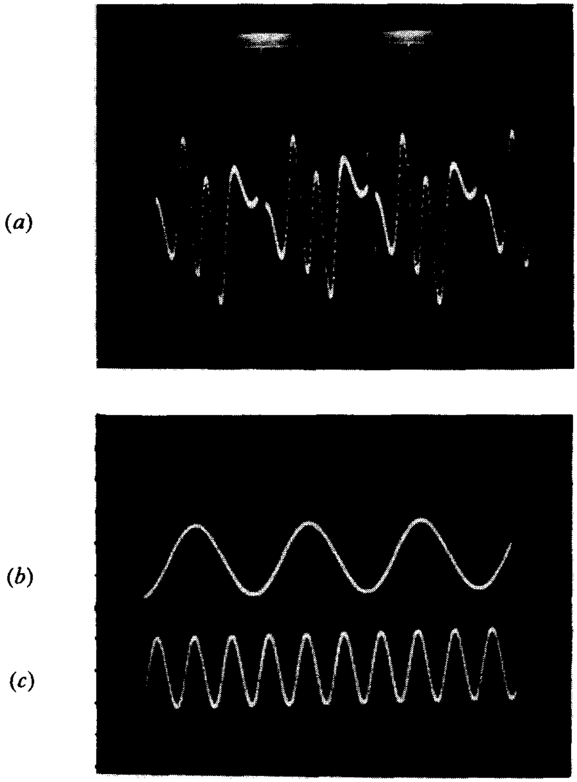

Figure 9. Experimental results obtained with the arrangement of figure 8 with two sensing interferometers, showing the multiplexed output of (1706 cm, 1746 cm) and (1866 cm, 1746 cm) pairs (*a*) and their corresponding band-pass filtered signals (*b*) and (*c*).

second pair (1866 cm, 1746 cm, $\Delta L = 120$ cm) was driven over three cycles of its transfer function. The photodiode output is shown in figure 9 with the corresponding filtered outputs at 330 and 990 Hz. Obviously, as $L_c < \Delta L_i$, ΔL_r, no interference due to the individual interferometers was observed.

3.3. *Combined coherence and frequency-division multiplexing*

It is possible to combine the techniques of frequency-division and coherence multiplexing, and a schematic arrangement is shown in figure 10. The sensors are arranged into p sub-arrays each containing n rings of optical path lengths L_{ij} for the jth member of the ith sub-array. Each element of the sub-array, and each sub-array, is separated by a delay line as in section 3.2. Each sub-array is interrogated by its own receiving interferometer of optical path length L_{ri}. As before, we require that

$$L_{ij},\ L_{ri} > L_c; \quad i = 1, \ldots p; \quad j = 1, \ldots, n.$$

In order to interrogate the ith sub-array with the ith receiver, we require

$$|L_{ij} - L_{ri}| < L_c$$

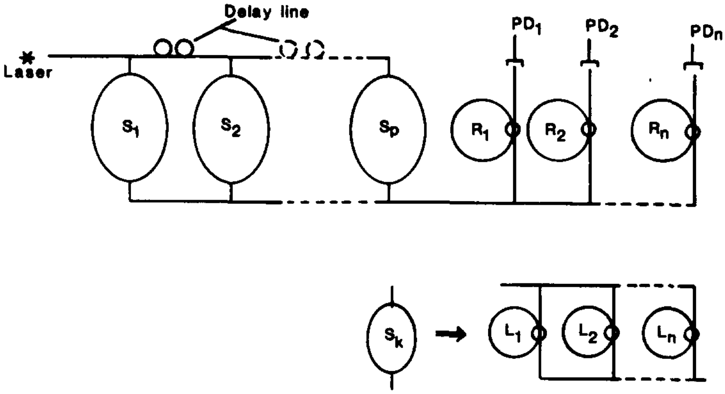

Figure 10. A sensor array combining frequency-division and coherence-division multiplexing techniques. $S_1 \ldots S_p$: sub-arrays of sensing interferometers comprising ring resonators $L_1 \ldots L_n$. $R_1 \ldots R_n$: receiving interferometers. $PD_1 \ldots PD_n$: photodetectors.

but to avoid cross-talk (that is interference between members of the *i*th sub-array with the *k*th receiver, $i \neq k$), then

$$|L_{ij} - L_{rk}| > L_c.$$

The various members of the *i*th sub-array are distinguished by frequency-division multiplexing, so that the path differences follow a sequence of the form

$$|L_{ij} - L_{ri}| \propto m_q,$$

where m_q are integers, following the sequence given in equation (8). For example for a source with a coherence length of ~100 cm, it is possible to make an array consisting of ring resonators with the following cavity lengths:

$$\begin{aligned} &L_{r1} = 110, \quad L_{11} = 120, \quad L_{21} = 140, \quad L_{31} = 190\,\text{cm}, \\ &L_{r2} = 300, \quad L_{12} = 290, \quad L_{22} = 270, \quad L_{32} = 220\,\text{cm}, \\ &L_{r3} = 400, \quad L_{13} = 410, \quad L_{23} = 430, \quad L_{33} = 480\,\text{cm}. \end{aligned}$$

4. Summary

We have described a passive remote-sensing system, using as its basic element, a pair of ring interferometers arranged in tandem. We have considered the effect of using a short coherence-length source and the relative path imbalance of the pair of interferometers on the final sensitivity of the system. We have shown that a system with higher sensitivity than for two-beam inteferometers can be constructed. We have also used a pair of such high-finesse ring resonators in a coherently multiplexed system and shown that high sensitivity can be obtained with appropriate signal processing. It was shown that cross-talk in a frequency division multiplexed system is increased when high-finesse ring resonators are used, but this problem can be resolved by using low-finesse ring interferometers. Such ring resonators are especially suitable for use in a multiplexed system having a series topology.

We have shown that frequency-division multiplexing (FDM) is also compatible with the use of moderate coherence-length sources, using a sensing ring matched with a receiving ring. Clearly, FDM allows several sensors to be used for each receiver, and we have demonstrated this with two sensors and one receiver. The technique may be extended to combine FDM with coherence multiplexing, so that a number of receiving rings are used, where each receiver interrogates several sensors. An alternative approach is to combine either coherence multiplexing or FDM, or both, with time addressing techniques. The number of multiplexed sensors may thus be increased substantially.

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