

## Normalized detectivity as a function of diffusion length for SPRITE detectors

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### ABSTRACT

This paper suggests a method for normalization of  $D^*$  for SPRITE detectors with respect to MTF-limiting parameters, primarily the diffusion spread. The purpose of the normalization is to obtain a single performance parameter for the SPRITE detector to make it more objectively comparable with conventional detectors with discrete elements.

The recalculation ratio is the rms noise calculated with a filter that compensates the impulse response back to a square pulse divided by the rms noise of the SPRITE element with no compensating filters. In this paper two filters are used: one that fully compensates back to a square impulse response, and one that compensates the MTF at two selected frequencies.

The recalculation ratio is calculated as a function of diffusion length with the ratio element length/(carrier life time\*scan speed) ( $L/vt$ ) as a parameter. The results show very little variation with  $L/vt$ , so the results should be valid for most applications for the SPRITE detector.

### 1. INTRODUCTION

SPRITE is an acronym for Signal PROcessing In The Element.<sup>1,2</sup> The detector was introduced some ten years ago and is at present one of the most widely used detector in high performance thermal imagers operating in the 8 to 12  $\mu\text{m}$  waveband. The main advantages with the SPRITE detector is that it has a time-delayed integration (TDI) implemented in the element. This feature saves the external electronics that would otherwise have performed the TDI. Furthermore, the sensitive area of the element array is compressed, which enables efficient cold shielding.

The key figure of merit for most infrared detectors is the detectivity, or  $D^*$ . For a SPRITE detector,  $D^*$  alone is not a very good descriptor of the detector's performance, because the detector is reducing the system's MTF not only by the subtense of the read-out length, but also by diffusion spread along the element. So, if  $D^*$  is to be used as the sole figure of merit for a SPRITE detector, the performance-degrading effects of the diffusion must be considered when specifying  $D^*$ .

To make the  $D^*$  of a SPRITE detector an objective performance parameter that could be used when comparing the detector against conventional detectors with discrete elements, the measured value of  $D^*$  must be recalculated into a square-pulse impulse response. In the literature, expressions for the spatial resolution and number of equivalent background-limited elements along a SPRITE bar have been derived<sup>1,3</sup>.

This paper proposes that the noise of a SPRITE detector is recalculated into what it would have been if the impulse response had been a square pulse, which is the case for a conventional detector with discrete elements. The  $D^*$  is then recalculated accordingly. In the text below, two methods for doing this noise normalization are presented. They both include a compensating filter in the rms noise calculation. In the first case, the filter fully compensates the SPRITE-detectors impulse response back to a square pulse. Such a filter cannot be practically implemented as is shown in this paper. So, the second method concerns a filter that compensates the SPRITE-detector MTF at two spatial frequencies. The methods are called "the square impulse compensation" and the "two-point boost compensation" respectively.

## 2. COMPENSATION FILTERS FOR THE D\* normalization

### 2.1 The SPRITE detector MTF

The SPRITE detector has two properties which reduce its spatial resolution, namely the readout length and the diffusion spread. The readout geometry for SPRITE detectors is not entirely rectangular. Instead the readout zone has a slight tapering.<sup>5</sup> The MTF for this geometry has previously been derived<sup>6</sup> and is given by

$$MTF_r(k) = \frac{\left| \int_{-\infty}^{\infty} \text{rect}\left(\frac{x-X/2}{X}\right) e^{-\gamma x} e^{-2\pi i k x} dx \right|}{\left| \int_{-\infty}^{\infty} \text{rect}\left(\frac{x-X/2}{X}\right) e^{-\gamma x} dx \right|} = \frac{\left| \int_0^X e^{-\gamma x} e^{-2\pi i k x} dx \right|}{\left| \int_0^X e^{-\gamma x} dx \right|} = \left| \frac{\gamma}{1 - e^{-\gamma a}} \frac{1 - \cos(2\pi k a) e^{-\gamma a} + j \sin(2\pi k a) e^{-\gamma a}}{\gamma + j 2\pi k} \right| \quad (1)$$

where  $MTF_r$  is the readout transfer function,  $k$  is a spatial frequency (cy/mm),  $a$  is the readout length (mm), and  $\gamma$  is the tapering factor of the readout zone (1/mm). The other MTF-limiting factor of the SPRITE detector is the diffusion spread during the charge transfer along the filament. The MTF for diffusion spread can also be found in the literature<sup>3,7</sup> and it is given by

$$MTF_q(k) = \frac{1 - \exp\left[-(1 + (2\pi k Q)^2) \frac{L}{v\tau}\right]}{[1 + (2\pi k Q)^2] \left[1 - \exp\left(-\frac{L}{v\tau}\right)\right]} \quad (2)$$

where  $MTF_q$  is the MTF contribution from diffusion spread,  $Q$  is the diffusion length (mm),  $L$  is the total length of the element (mm),  $v$  is the scan speed along the element (mm/s), and  $\tau$  is the carrier lifetime (s). The factor  $L/v\tau$  is an important parameter. It is essentially the filament length over the carrier drift length and it contains such application-specific parameters as the scan speed and the anamorphic ratio,<sup>4,5</sup> which is the focal length along line scan divided by the focal length across scan. The scan speed is in itself a combination of a variety of sensor parameters; see Eq (3).

$$v = \frac{f_R}{N_{//}} \frac{4}{3} \frac{N_l N_{pl}}{\eta_x} A c \quad (3)$$

where  $f_R$  is the frame rate (Hz),  $N_{//}$  is the number of elements in parallel scan,  $4/3$  is the aspect ratio,  $N_l$  is the total number of lines per frame,  $N_{pl}$  is the number of presented lines per frame,  $\eta_x$  is the line scan efficiency,  $A$  is the anamorphic ratio<sup>8,9</sup> (-, dimensionless), and  $c$  is the line distance in the focal plane (mm).

### 2.2 Filter transfer function for the square-impulse compensation

In the previous section it has been shown that the SPRITE-detector MTF does not correspond to a square impulse response. For the SPRITE-detector  $D^*$  to be objectively compatible with detectors with discrete elements, it should have a square impulse response. The width of the square response should be  $62,5 \mu\text{m}$ , which is generally the detector width used when determining  $D^*$ .

The transfer function of this "perfect" compensation filter is given by the transfer function of a square with the width  $62,5 \mu\text{m}$  divided by the actual MTFs for the readout zone and the diffusion spread. The latter are given by Eqs (1) and (2) above. Consequently, the transfer function of the square pulse-compensation filter becomes

$$G(k) = \frac{C \text{sinc}(\pi k a_0)}{MTF_r(k) MTF_q(k)} = \frac{C \text{sinc}(\pi k a_0)}{\frac{1 - \exp\left[-(1 + (2\pi k Q)^2) \frac{L}{v\tau}\right]}{[1 + (2\pi k Q)^2] \left[1 - \exp\left(-\frac{L}{v\tau}\right)\right]} \left( \frac{\gamma}{1 - e^{-\gamma a}} \frac{1 - \cos(2\pi k a) e^{-\gamma a} + j \sin(2\pi k a) e^{-\gamma a}}{\gamma + j 2\pi k} \right)} \quad (4)$$

where  $G(k)$  is the transfer function of the compensation filter,  $a_0$  is the detector width used when normalizing  $D^*$  (generally  $62,5 \mu\text{m}$ ),  $L$  is the total length of the element (mm), and  $C$  is a constant to normalize the average output signal measured over one readout length.

The transfer function of a filter that compensates for the response of a SPRITE detector back to a square pulse will have a rather complicated expression [see Eq (4)]. In Fig 1, the transfer function is plotted against the product spatial frequency in the focal plane with the diffusion length as a parameter. The application factor  $L/vt$  is set to 3, the tapering factor is set to  $28.5 \text{ 1/mm}^6$ , and both the reference ( $a_0$ ) and the actual readout length are set to  $62,5 \mu\text{m}$ . The transfer curves below have been calculated with parameters that are fairly typical to a long-bar application in the 8 to  $12\text{-}\mu\text{m}$  region. However, the technique is equally applicable in the 3 to  $5\text{-}\mu\text{m}$  waveband, where the diffusion spread is larger and the need for compensation is greater.

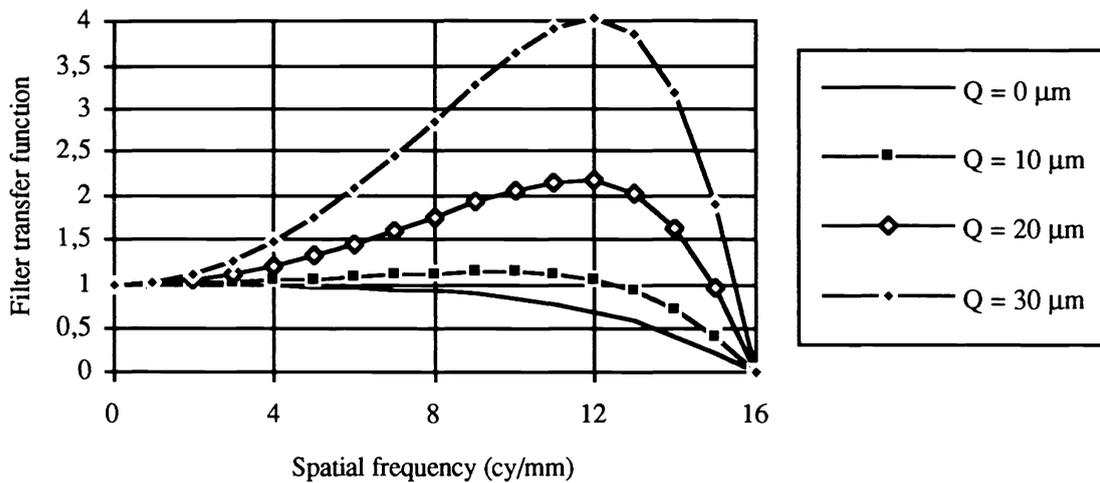


Figure 1. Filter transfer function for the square impulse compensation

In Fig 1, the filter transfer function has been plotted only up to  $16 \text{ cy/mm}$ , which corresponds to a spatial frequency of  $1 \text{ cy/readout width}$ . Regrettably, the transfer function of the compensation filter for the square impulse technique will have infinite response at higher spatial frequencies, and, as will be shown later, the noise spectral density after compensation filtering does not roll off to zero. Consequently the rms noise cannot be calculated by integrating the noise spectrum over all frequencies.

The spatial-frequency range that the square impulse compensation filter covers must be limited. A suitable cutoff frequency for it is  $1 \text{ cycle/readout length}$ . This corresponds to an ideal anti-aliasing filter for a system that samples twice per readout length. The limited spatial frequency range will result in ringing in the impulse response. The impulse response after attempted square pulse compensation is calculated with an inverse Fourier transform in accordance with

$$h(x) = C a_0 \int_{-1/a_0}^{1/a_0} \text{MTF}_r(k) \text{MTF}_q(k) \frac{\text{sinc}(\pi k a_0)}{\text{MTF}_r(k) \text{MTF}_q(k)} e^{j2\pi k x} dk C a_0 \int_{-1/a_0}^{1/a_0} \text{sinc}(\pi k a_0) e^{j2\pi k x} dk \quad (5)$$

where  $h(x)$  is the impulse response,  $x$  is a coordinate along the element (mm),  $a_0$  is the desired square width of the impulse response, and  $C$  is to normalize the average output signal over one readout length. In this study the desired average is unity.

For a conventional element, the detector output does not vary over the element. We intend to recalculate the SPRITE detectivity into what it would have been for a square pulse, instead of the diffusion-degraded point response that it actually has. To perform normalization as fairly as possible, the average of the compensated signal taken over one readout length has been normalized, rather than the maximum amplitude after attempted restoration.

Eq (5) shows that the impulse response after attempted square pulse restoration will depend only on the desired width of the impulse response. Figure 2 shows the result. The coordinate along the SPRITE element is given in IFOVs, or detector widths. The target is a function that is unity between  $\pm 0,5$  detector widths and zero elsewhere.

In that diagram, the restored square impulse response is also calculated with larger cutoff frequencies for the compensation filter. However, increasing the cutoff frequency further does not improve the impulse tremendously. The curves have all been normalized to set the average for the compensated signal measured over one readout length to unity. The correction factor C was 1.052, 1.027, and 1.018 for the cut-off frequencies 1, 2, and 3 cy/readout length respectively.

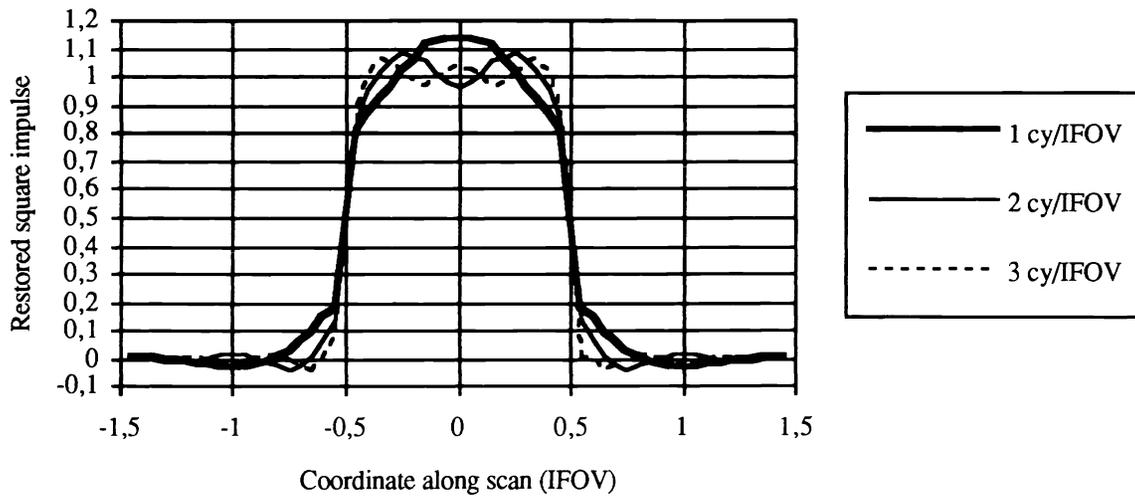


Figure 2. Impulse response after attempted square impulse restoration

### 2.3 Filter parameters for the two-point boost compensation

The compensation filter in the previous section that gives the near-square impulse response for the SPRITE detector cannot easily be physically implemented. So, in practical cases, where there is an interest to measure a recalculated  $D^*$ , a number of discrete frequencies for compensation must be selected. In this analysis two such frequencies will be chosen. The amplification of the compensating filter, which in effect is similar to a high-frequency boost filter, are calculated with Eq (4) above.

In most SPRITE-based sensors, a high frequency boost filter in the electronics enhances the spatial resolution and overcomes the performance degradation that results from diffusion spread. The cost for the high-frequency boost is ringing in the output signal and a loss in sensitivity caused by noise amplification. An attempt to optimise the boost-filter parameters together with the anamorphic ratio has been previously performed.<sup>10</sup>

In a typical application of a military thermal imager, the spatial frequencies for target detection and recognition are somewhere around 0.5 cycles/vertical IFOV. Since the standard SPRITE detector width is 62.5  $\mu\text{m}$ , the critical spatial frequency becomes 8 cy/mm. To make it analytically possible to derive the two boost-filter parameters, resonance frequency and relative damping, the two selected spatial frequencies must be fairly close to each other. In this paper, the two critical spatial frequencies where the boost filter is located to compensate back to the transfer function of a square pulse are 7.5 and 8.5 cy/mm.

The transfer function of a boost filter that compensates at two frequencies is given by the expression in Eq (6) below. The amount of boost required to give the desired spatial-transfer function at the selected frequencies depends on the parameters in the used detector and on the ratio  $L/vt$  [see Eq (4) above].

$$H_c(k) = \frac{C}{1 + j2\xi \frac{k}{k_0} - (\frac{k}{k_0})^2} \tag{6}$$

where  $k_o$  is the resonance frequency (cy/mm),  $\xi$  is the relative damping for the boost filter and  $C$  is a constant to normalize the average compensated signal measured over one readout length.

The designing criteria for the compensating boost filter used in the recalculation of a SPRITE detector's  $D^*$  is that at two spatial frequencies it compensates back to the transfer function of a square with the width  $62,5 \mu\text{m}$ . In terms of equations these criteria can be expressed as:

$$|H_c(k_1)| = C G(k_1) , k_1 = 7.5 \text{ cy/mm} \quad (7)$$

$$|H_c(k_2)| = C G(k_2) , k_2 = 8.5 \text{ cy/mm} \quad (8)$$

where  $H_c(k)$  is the boost-filter transfer function [Eq (5)], and  $G(k)$  is the transfer function of a filter that truly restores the impulse response of the SPRITE detector into a square pulse [Eq (4) above]. The resonance frequency  $k_o$  in the boost-transfer function in Eq (6) is determined by using the conditions in Eqs (7) and (8). After some algebraic manipulations, the following expression for the boost resonance frequency emerges.

$$k_o^4 = \frac{k_1^4 k_2^2 - k_1^2 k_2^4}{\frac{k_2^2}{G(k_1)^2} - \frac{k_1^2}{G(k_2)^2} - k_2^2 + k_1^2} \quad (9)$$

where  $k_1$  is a selected frequency (here 7.5 cy/mm),  $k_2$  is another selected frequency (here 8.5 cy/mm), and  $G$  is the required boost amplification at the selected frequencies. The relative damping in the boost filter is subsequently derived from Eq (5) and given by

$$\xi = \frac{k_o}{2k_1} \sqrt{\frac{1}{G(k_1)^2} - \left[1 - \left(\frac{k_1}{k_o}\right)^2\right]^2} \quad (10)$$

where  $\xi$  is the relative damping in the boost filter,  $k_o$  is the resonance frequency of the reference boost filter,  $k_1$  is a selected frequency (here 7.5 cy/mm), and  $G(k_1)$  is the required boost amplification at this selected frequency.

The two boost-filter parameters, resonance frequency and damping, can now be determined by using Eqs (9) and (10) together with the filter design criteria in Eqs (7) and (8). The desired transfer function at the two selected spatial frequencies for a restoration of a square impulse response is taken from Eq (4). The normalization factor  $C$  in Eq (5) is determined by calculating the average of the compensated impulse response measured over one readout length. The target value is unity.

In the diagram below, the detector transfer function after a two-point boost compensation is plotted against spatial frequency with the diffusion length as a parameter. The transfer function of a true square pulse with the width  $62.5 \mu\text{m}$  is shown with the dashed sinc curve, which is the target for the compensated transfer. The same assumptions regarding detector and application parameters as in Fig 1 have been used. The amplitude normalization factor increases with diffusion length.

Note that the normalization factor  $C$  sets the average signal to unity. The average is taken over one readout length of the SPRITE detector. The more diffusion spread there is, the larger the value of  $C$ . This is shown at the origin of the curves in Fig 3 below, where the curves start at an amplitude equal to  $C$ , which increases with diffusion length. Because of the variation of  $C$ , the curves will not match at the selected frequencies 7.5 cy/mm and 8.5 cy/mm. And remember that  $C$  is out when the resonance frequency and the relative damping are calculated.

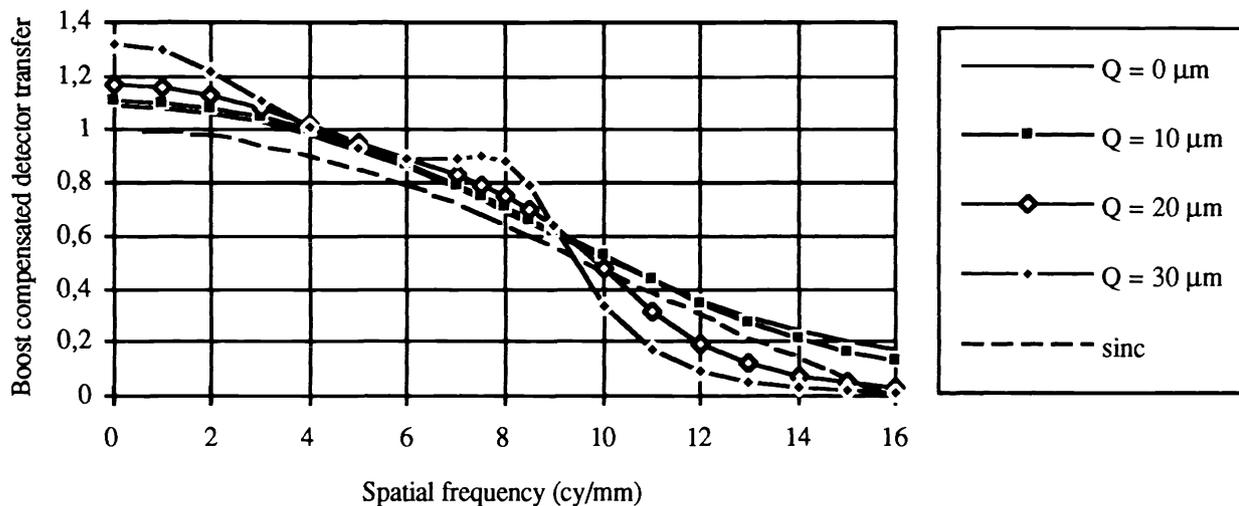


Figure 3. Detector transfer function after a two-point boost compensation

### 3. RESULTS OF D\*-NORMALIZATIONS

#### 3.1 Conditions for the normalization

The relationship between  $D^*$  and MTF for SPRITE detectors has been investigated previously and an empirical relation has been derived from measured results on a number of detectors.<sup>4</sup> In this paper a somewhat different approach has been applied. It is suggested here that the measured detectivity for a certain detector is recalculated to a compensated square impulse response with the following expression

$$D^* = D^*_m \sqrt{\frac{\Delta f_o}{\Delta f_c}} \quad (11)$$

where  $D^*$  is the recalculated value,  $D^*_m$  is the measured value,  $\Delta f_o$  is the noise equivalent bandwidth of the SPRITE detector without any filter,  $\Delta f_c$  is the noise-equivalent bandwidth with a filter that compensates the impulse response back to a square pulse either at a number of selected frequencies, or continuously up to a cut-off frequency. The worse MTF the detector has, the more compensation is needed to obtain the square impulse, and consequently the recalculated value for  $D^*$  will drop.

The noise-equivalent bandwidth of the SPRITE detector can be calculated analytically by using residual calculus and the expression for the noise spectral density that is given in the integrand of Eq (12) below. The noise integral is performed over spatial frequencies, so the noise bandwidth is expressed as a spatial frequency. The noise bandwidth of the SPRITE detector itself without compensatory filters is given by

$$\Delta f_o = \int_0^\infty S_n(k) dk = \int_0^\infty \frac{\text{sinc}(\pi k \tilde{a})^2}{1 + (2\pi k \tilde{Q})^2} dk = \frac{1}{2a} \left[ 1 - \frac{\tilde{Q}}{a} \left( 1 - e^{-\frac{a}{\tilde{Q}}} \right) \right] \quad (12)$$

where  $S_n$  is the normalized noise power spectrum (1/cy/mm) and the values with the  $\sim$  symbol on top denote that it is not the actual values for the readout and diffusion lengths that are to be put in, but ones that have been adjusted. Steven Braim has suggested that the adjusted values should be 5  $\mu\text{m}$  and 12  $\mu\text{m}$  for the readout and diffusion lengths respectively.<sup>11</sup> With these values inserted in Eq (12), the noise-equivalent bandwidth becomes 18.2 cy/mm.

The noise-equivalent bandwidth after compensation is given by a similar expression. However, the noise power spectrum is now multiplied by the square of the transfer function for the compensating filter, which is either the "square pulse-compensation" filter in Eq (4) and Fig 1, or the "two-point boost compensation" filter in Eq (6) and Fig 3.

$$\Delta f_c = \int_0^{\infty} \frac{\text{sinc}(\pi k \tilde{a})^2}{1 + (2\pi k \tilde{Q})^2} |H_c(k)|^2 dk \quad (13)$$

where  $H_c(k)$  is the transfer function of the compensation filter.

The noise-equivalent bandwidths in this paper are expressed in spatial frequencies. Normally, it is given in temporal frequencies. The conversion factor is simply the scan speed along the element. The scan speed depends on the application, [Eq (3)]. In the analysis presented here it is more convenient to express the noise bandwidth in spatial frequencies that relate to the dimensions of the SPRITE detector. All application-specific parameters are lumped together in the ratio  $L/vt$  (filament length / scan speed \* carrier lifetime). In the results shown below,  $L/vt$  will in most cases be used as a parameter.

### 3.2 D\*-normalizations for the square impulse compensation

The compensation filter that truly restores the impulse response for a SPRITE detector back into a square pulse cannot be physically implemented, because it has infinite response at higher spatial frequencies [see Eq (4) above]. However, if the compensation has a limited frequency range and a cut-off frequency, the noise bandwidth can be calculated with numerical methods. In a previous section, a suitable cut-off frequency has been found to be 1 cycle / readout length. The integral for determining the noise-equivalent bandwidth is given in Eq (14) below:

$$\Delta f_c = \int_0^{1/a_0} \frac{\text{sinc}(\pi k \tilde{a})^2}{1 + (2\pi k \tilde{Q})^2} \left| \frac{C \text{sinc}(\pi k a_0)}{\frac{1 - \exp\left[-(1 + (2\pi k \tilde{Q})^2) \frac{L}{v\tau}\right]}{[1 + (2\pi k \tilde{Q})^2] \left[1 - \exp\left(\frac{-L}{v\tau}\right)\right]} \left( \frac{\gamma}{1 - e^{-\gamma a}} \frac{1 - \cos(2\pi k a) e^{-\gamma a} + j \sin(2\pi k a) e^{-\gamma a}}{\gamma + j 2\pi k} \right)} \right|^2 dk \quad (14)$$

where  $a_0$  is the detector width used when normalizing  $D^*$  (generally 0,0625 mm) and  $C$  is an amplitude normalization factor. When the cut-off frequency is 1 cy/detector width, the factor  $C$  becomes approximately 1.05.

The integral above can not be solved analytically, so numerical methods have to be employed. In the diagram below, the noise-equivalent bandwidth has been calculated as a function of diffusion length with the application-dependent factor  $L/vt$  as a parameter. The following assumptions have been made: the tapering factor is set to 28.5 1/mm<sup>6</sup> and both the reference ( $a_0$ ) and the actual readout length are both set to 62.5  $\mu\text{m}$ .

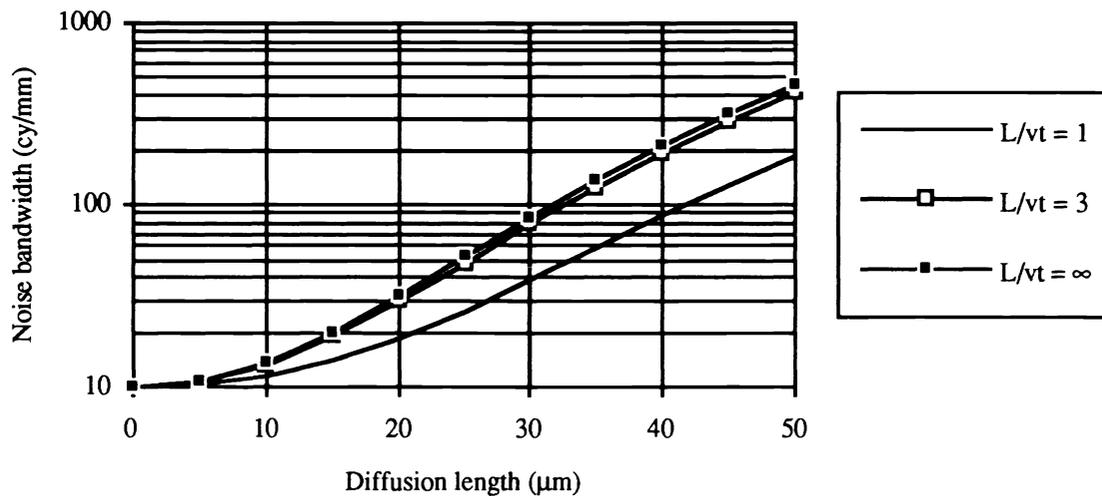


Figure 4. Noise-equivalent bandwidth after square impulse compensation

The  $D^*$ -recalculation factor can now be calculated by using the diagrams in Fig 4, the uncompensated noise bandwidth calculated below Eq (12), and the definition in Eq (11). In the diagram below, the  $D^*$ -recalculation factor after a square impulse compensation with the finite bandwidth filter is shown. The normalization factor is presented as a function of diffusion length with the application-dependent ratio element length / carrier life-length ( $L/vt$ ) as a parameter.

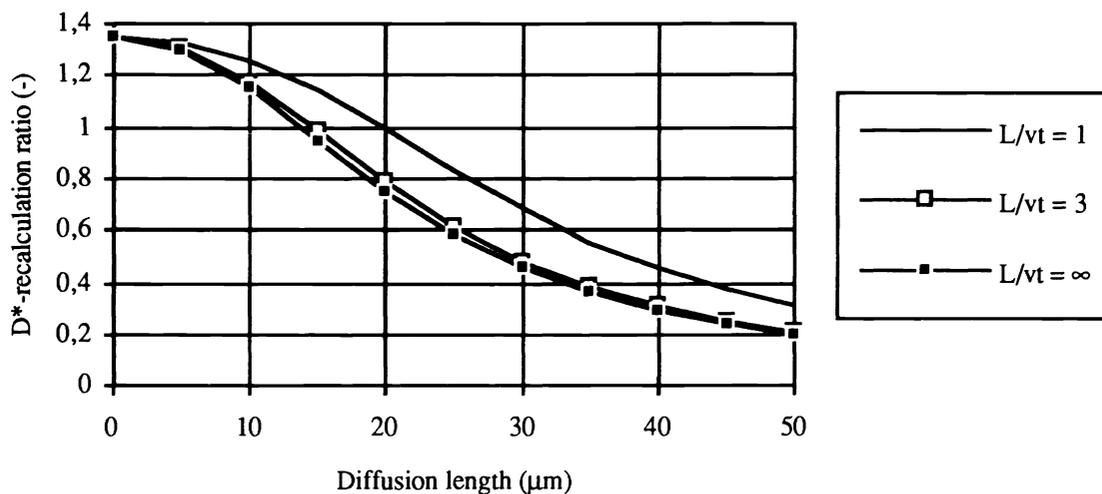


Figure 5.  $D^*$ -normalization factor as a function of diffusion length after square impulse compensation

The curves above show that for a diffusion length of 30  $\mu\text{m}$ , which is a fairly typical value for an 8 to 12- $\mu\text{m}$  SPRITE detector, the measured  $D^*$  should be multiplied by about 0.5 to make it a performance parameter that is more "objectively" compatible with the  $D^*$  values quoted for conventional detectors with discrete elements. The ratio element length/carrier lifelength ( $L/vt$ ) in Fig 6 corresponds to the detectivity measuring conditions, and should correspond not to the sensor where the detector will be used.

Fig 5 shows also that the measured value of  $D^*$  for a SPRITE detector with zero diffusion length should be increased before it is compared with  $D^*$ -values for conventional detectors. This is because the finite bandwidth of the compensation filter cuts out much of the high-frequency noise that is normally there. The result can appear surprising. However, diffusion lengths below 20  $\mu\text{m}$  are not very likely.

### 3.3 D\*-normalizations for the two-point boost compensation

By combining the expressions in Eqs (6) and (12) with the definition in Eq (13), the following is obtained for the relation between the noise-equivalent spatial bandwidth and the parameters describing the SPRITE detector and the boost compensation filter

$$\Delta f = \int_0^{\infty} \frac{\text{sinc}(\pi k \tilde{a})^2}{1 + (2\pi k \tilde{Q})^2} \left| \frac{C}{1 + j2\xi \frac{k}{k_0} - \left(\frac{k}{k_0}\right)^2} \right|^2 dk \quad (15)$$

The integral in Eq (15) can, after trigonometric manipulations in the nominator, be solved analytically by using residual calculus.<sup>12</sup> After a few substitutions to get a less complex integrand, the following expression for the noise-equivalent spatial bandwidth,  $\Delta f$ , emerges.

$$\Delta f = k_0 C^2 \int_0^{\infty} \frac{1 - \cos(2\alpha x)}{[1 + (\gamma x)^2] \{ [1 - x^2]^2 + (\beta x)^2 \} (\alpha x)^2} dx \quad (16)$$

with  $\alpha = \pi a k_0$ ,  $\beta = 2\xi$ ,  $\gamma = 2\pi Q k_0$ ,  $x = k/k_0$ . The noise equivalent spatial bandwidth can now be calculated by using residual calculus. The denominator of the integrand in Eq (16) has 8 poles. These are

$$x_{1,2} = 0 \quad (17a)$$

$$x_{3,4} = \cos\delta \pm j\sin\delta, \quad \delta = \frac{1}{2}\arccos(1-2\xi^2) \quad (17b)$$

$$x_{5,6} = -\cos\delta \pm j\sin\delta, \quad \delta = \frac{1}{2}\arccos(1-2\xi^2) \quad (17c)$$

$$x_{7,8} = \pm j\frac{1}{\gamma} \quad (17d)$$

The integral in Eq (15) is solved by summing the residues in the poles in the upper half plane and the result becomes

$$\Delta k = \frac{\pi k_0}{4\alpha^2} \left[ 2\alpha + \frac{\cos 3\delta + \gamma^2 \cos 5\delta - e^{-2\alpha \sin \delta} [\cos(3\delta - \epsilon) + \gamma^2 \cos(5\delta - \epsilon)]}{\sin 2\delta (1 + \gamma^4 + 2\gamma^2 \cos 2\delta)} - \frac{\gamma^5 (1 - e^{-2\alpha \gamma})}{(1 + \gamma^2)^2 - \gamma^2 \beta^2} \right] \quad (18)$$

with  $\epsilon = 2\alpha \cos \delta$  where  $\delta$  is defined in Eq (17b) and  $\alpha$ ,  $\beta$ ,  $\gamma$  are substitutions given below Eq (16).

The parameters for the two-point boost-compensation filter can be calculated by the methods described in section 2.2. The noise-equivalent bandwidth can then be calculated with Eq (18). Figure 6 presents below the result. The noise bandwidth is shown as a function of diffusion length with the application-dependent ratio  $L/v\tau$  as a parameter. The amplitude normalization factor is chosen so that the average of the output signal becomes unity when measured over one readout length.

Not surprisingly, the noise bandwidth increases with diffusion length, because the required boost amplification also increases with diffusion. Larger values on  $L/v\tau$ , (element length/carrier drift length) correspond to worse diffusion and consequently the noise bandwidth also increases with  $L/v\tau$ .

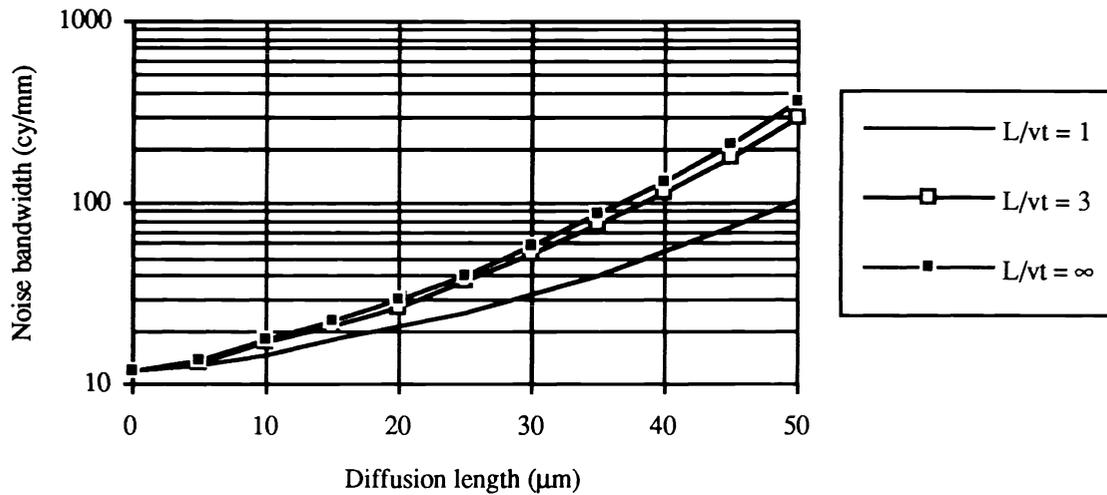


Figure 6. Noise-equivalent spatial bandwidth as a function of diffusion length and with  $L/vt$  as a parameter

The  $D^*$ -recalculation factor can now be calculated by using Figs 4 and 7 and the definition in Eq (11). In Fig 7 the  $D^*$ -recalculation factor after a square impulse compensation with the finite bandwidth filter is shown. The normalization factor is presented as a function of diffusion length with the application-dependent ratio element length / carrier drift length ( $L/vt$ ) as a parameter.

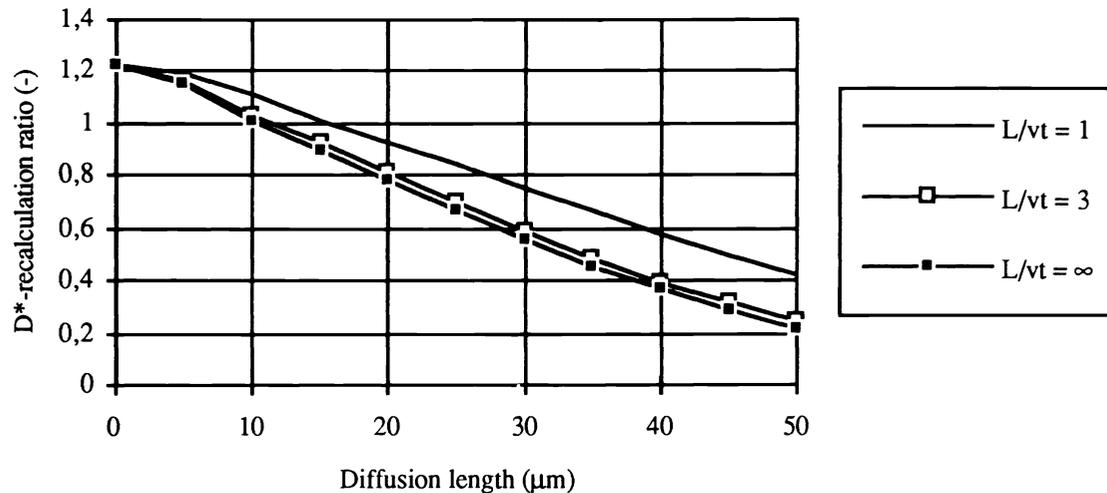


Figure 7.  $D^*$ -normalization factor as a function of diffusion length after two-point boost compensation

The curves above show that for a diffusion length of  $30 \mu\text{m}$ , which is a fairly typical value for an 8 to  $12\text{-}\mu\text{m}$  SPRITE, the measured  $D^*$  should be multiplied by about 0.6 to make it a performance parameter that is more "objectively" compatible with the  $D^*$ -values quoted for conventional detectors with discrete elements. The ratio element length / carrier lifelength in Fig 7 corresponds to the detectivity measuring conditions, and not to the sensor where the detector will be used.

Again, the normalized  $D^*$  should be somewhat increased in the case of zero diffusion. This is because the compensation filter blocks out some of the high-frequency contents of the detector noise. This effect is less promoted with the boost compensation than it was with the square-pulse technique. This is explained by the fact that the boost filter does not have a cut-off frequency.

#### 4. CONCLUSIONS

For a SPRITE detector, the detectivity ( $D^*$ ) alone is not a very descriptive performance parameter. To make the  $D^*$  of a SPRITE more objectively compatible with  $D^*$  for conventional detectors, it must be recalculated. A conventional detector has a square pulse impulse response. The SPRITE detector has not, primarily due to diffusion spread. Consequently, the rms noise of a square detector should be calculated after a compensatory filter that amplifies higher spatial frequencies to restore a "box-car" impulse response.

It is suggested in this paper that  $D^*$  is recalculated with the following expression:

$$D^* = D^*_m \sqrt{\frac{\Delta f_o}{\Delta f_c}} \quad (11)$$

where  $D^*$  is the recalculated value,  $D^*_m$  is the measured value,  $\Delta f_o$  is the noise-equivalent bandwidth of the SPRITE detector without a filter and  $\Delta f_c$  is the noise-equivalent bandwidth with a filter that compensates the impulse response back to a square pulse.

The normalization ratio has been calculated with two types of filters and has been presented as a function of diffusion length. For a diffusion length of 30  $\mu\text{m}$ , the measured  $D^*$  for a SPRITE detector should be reduced by some 40% - 50% to make it more objectively compatible with conventional detectors.

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