Modulation Transfer Function Measurement Using Three- and Four-bar Targets

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Abstract

Modulation-transfer-function (MTF) measurement often involves the use of three- and four-bar resolution targets. In the conversion of three- and four-bar image data to MTF, biased results can occur when we use series-expansion techniques appropriate for square-wave targets of infinite extent. For systems where the image data are digitally recorded, a convenient and accurate conversion of bar-target data to MTF can be performed using a Fourier-domain method.

Method

Modulation transfer function (MTF) is the ratio of the output to input modulation depth as a function of spatial frequency ξ for sinusoidal input targets. Modulation depth M is defined for a general target as

$$M = \left[\frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right], \tag{1}$$

where I_{max} and I_{min} are the maximum and minimum values of W/cm^2 describing either the input-object exitance or output-image irradiance. Assuming sinusoidal input targets, MTF is written as

$$\mathrm{MTF}(\xi) = \left[\frac{M_{output}(\xi)}{M_{input\text{-}sine\text{-}wave}(\xi)}\right]. \tag{2}$$

Although MTF is defined for sine-wave targets, practice often dictates that resolution-target sets other than sine waves be used. A target set consists of a number of different-sized targets, which can be specified by a spatial period X equal to one cycle of the pattern. Binary target sets with equal-sized lines and spaces are commonly used. Visible-wavelength systems are often characterized using a three-bar (Air Force) target set.[1] Infrared systems are typically characterized with a four-bar target set.[2]

Targets that have an infinite number of square-wave cycles are simpler to analyze mathematically. For such targets, we can define a contrast transfer function (CTF) as a function of fundamental spatial frequency $\xi_f = 1/X$ as

$$\mathrm{CTF}(\xi_f) = \left[\frac{M_{output}(\xi_f)}{M_{input\text{-}square\text{-}wave}(\xi_f)}\right]. \tag{3}$$

Like the MTF for sine waves, the CTF also is a transfer function because it describes the image modulation depth over a basis set of square-wave components. The modulation depth of the input square wave is usually 1 for all targets in the set.

The infinite-square-wave CTF is generally higher than the MTF at the same spatial frequency because the odd harmonics of the infinite-square-wave test pattern (which are absent from sine-wave targets) will also contribute to the image modulation depth. The modulation depth, and hence CTF, at any frequency can be expressed as a summation of harmonic components. These components are weighted by two multiplicative factors in the summation: their relative strength in the input waveform and the MTF of the system under test at each harmonic frequency. This process yields an expression[3] for CTF in terms of MTF

$$\mathrm{CTF}(\xi_f) = \frac{4}{\pi}\left\{\mathrm{MTF}(\xi = \xi_f) - \frac{\mathrm{MTF}(\xi = 3\xi_f)}{3} + \frac{\mathrm{MTF}(\xi = 5\xi_f)}{5} - \frac{\mathrm{MTF}(\xi = 7\xi_f)}{7} + \frac{\mathrm{MTF}(\xi = 9\xi_f)}{9} \cdots\right\}. \tag{4}$$

Inversion of the series[3] yields an expression for MTF in terms of CTF

$$\mathrm{MTF}(\xi) = \frac{\pi}{4}\left\{\mathrm{CTF}(\xi_f = \xi) + \frac{\mathrm{CTF}(\xi_f = 3\xi)}{3} - \frac{\mathrm{CTF}(\xi_f = 5\xi)}{5} + \frac{\mathrm{CTF}(\xi_f = 7\xi)}{7} + \frac{\mathrm{CTF}(\xi_f = 11\xi)}{11} \cdots\right\}. \tag{5}$$

The series representations of Eqs. (4) and (5) are analytically valid for infinite-square-wave targets because these targets have discrete harmonic components. However, the spectra of finite-length square-wave targets such as the three- and four-bar targets have broader features and do not have discrete harmonic components.[4] A series representation is not as accurate for bar-target data because contributions to the image modulation depth occur at spatial frequencies other than those in the series. Bar-target data maybe interpreted in terms of image modulation depth as a function of fundamental spatial frequency, $\mathrm{IMD}(\xi_f)$, according to Eq. (6)

$$\mathrm{IMD}(\xi_f) = \left[\frac{M_{output}(\xi_f)}{M_{input\text{-}bar\text{-}target}(\xi_f)}\right], \tag{6}$$

where again the input modulation is usually 1.

The modulation depth of the output image is calculated in terms of maximum and minimum values, according to Eq. (l). For an infinite-square-wave target, the maxima are all equal and the minima are all equal. For the case of three- and four-bar targets, edge effects produce maxima and minima that are not equal for each bar in the output image. To be consistent with usual laboratory practice, we calculated the modulation depth using the highest maximum and the lowest interbar minimum that occur in any particular image.

When computed in this fashion, the resulting three- and four-bar IMD curves differ from each other, and are different from the CTF defined for infinite-square-wave targets. The closer the IMD curves are to the CTF curve for the infinite-square-wave target, the more accurate will be the series in Eq. (5) (substituting IMD for CTF) for three- and four-bar IMD-to-MTF conversion. If substantial differences exist among the curves, a direct application of the series conversion will yield biased results. In that case, techniques discussed below will convert bar-target data directly to MTF.

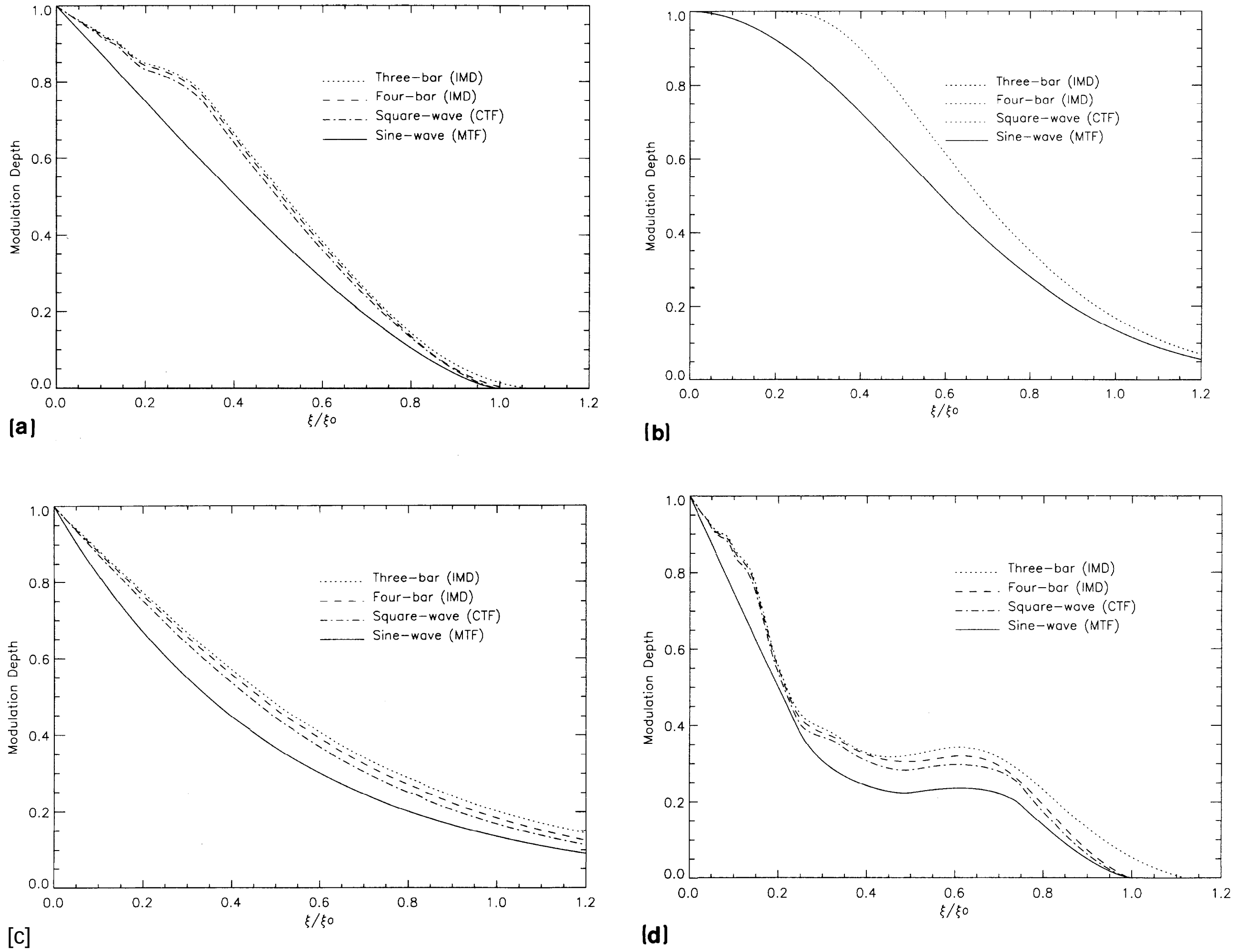

Figure 1. Comparison of IMDs for three- and four-bar targets, and CTFs for infinite-square-wave targets, given the following MTFs: la) a diffraction-limited circular-aperture MTF with cutoff frequency ξ_0; lb) a Gaussian MTF = $\exp\{-2(\xi/\xi_0)^2\}$ (both IMD curves are identical to the CTF); 1c) an exponential MTF = $\exp\{-2(\xi/\xi_0)\}$; and 1d) a diffraction-limited annular-aperture MTF (50% diameter obscuration) with cutoff frequency ξ_0.

Figure 1 illustrates the behavior of the IMD curves for three- and four-bar targets and CTF curves for infinite-square-wave targets, given four typical system MTFs: a diffraction-limited circular-aperture system with cutoff frequency ξ_0 (Fig. la); a Gaussian MTF = $\exp\{-2(\xi/\xi_0)^2\}$ (Fig. lb); an exponential MTF = $\exp\{-2(\xi/\xi_0)\}$ (Fig. 1c); and a diffraction-limited annular-aperture system (50% diameter obscuration) with cutoff frequency ξ_0 (Fig. id). To produce the infinite-square-wave CTF curves, the series in Eq. (4) was used directly with each MTF curve. To produce the three- and four-bar IMD curves, spectra were calculated for 120 bar targets of various frequencies, which were then filtered by each MTF curve. The resulting filtered spectra were inverse transformed, and the IMDs were calculated from the image data according to Eqs. (1) and (6). The Gaussian MTF produced identical curves for the three-bar, four-bar, and infinite-square-wave cases. The other three-bar IMDs were higher than the four-bar IMDs, which were higher than the infinite-square-wave CTFs. For these examples, the difference between the three-bar IMD and infinite-square-wave CTF curves is within 5% in absolute modulation depth, but the relative difference between the curves is as high as 20%. Thus, in some cases three-or four-bar IMD data should not be converted to MTF using a series such as Eq. (5).

Digital Data Measurement

Where the image data are recorded digitally, a convenient and accurate IMD-to-MTF conversion can be performed by using a Fourier-domain calculation. Figure 2 shows a measured three-bar-target magnitude spectrum, and the corresponding calculated input spectrum. Both spectra are normalized to 1 at the origin, which yields MTF = 1 at dc. The measured output spectrum has been filtered by the MTF of the system under test. Because of the falloff of MTF with frequency, the peak of the output spectrum will occur at a frequency somewhat lower than the fundamental frequency of the input spectrum. We can determine the fundamental frequency of input spectrum ξ_f from the location of the first zero in the output spectrum. The location of the first zero is not shifted by the system MTF. For the three-bar target, the fundamental frequency is three times the frequency of the first zero. For the four-bar target, the fundamental frequency is four times the frequency of the first zero. These relations are revealed from the expressions for

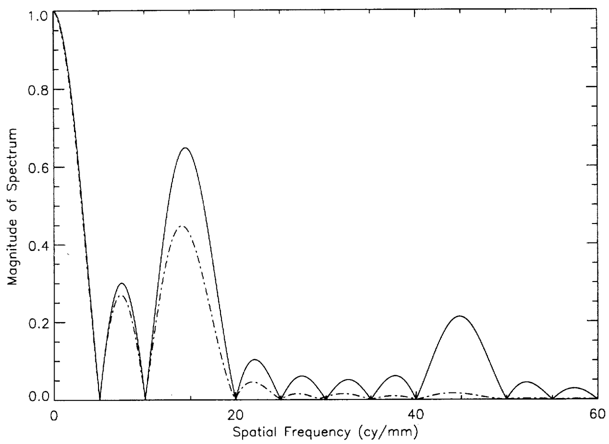

FIGURE 2. Measured output spectrum magnitude for a three-bar target (dashed line) and calculated input spectrum magnitude (solid line).

the magnitude spectra for each case. For the three-bar target

$$S_{\text{input, three-bar-target}}(\xi) = \left| X \operatorname{sinc}\left(\frac{X}{2}\xi\right)\left[\cos(2\pi X\xi) + \frac{1}{2}\right]\right|, \quad (7)$$

where the term in square brackets first goes to zero at

$$\xi_{\text{first zero,3-bar}} = \frac{1}{3}\frac{1}{X} = \frac{\xi_f}{3}. \quad (8)$$

For the four-bar target

$$S_{\text{input,four-bar-target}}(\xi) \quad (9)$$

$$= \left| X \operatorname{sinc}\left(\frac{X}{2}\xi\right)[\cos(3\pi X\xi) + \cos(\pi X\xi)]\right|,$$

where the term in square brackets first goes to zero at

$$\xi_{\text{first zero,4-bar}} = \frac{1}{4}\frac{1}{X} = \frac{\xi_f}{4}. \quad (10)$$

The fundamental frequency is the only free parameter in the calculation of the normalized input spectrum. Once the fundamental frequency has been found using Eq. (8) or Eq. (10), the input spectrum needed for the MTF calculation is determined.

The MTF at the fundamental frequency&of any particular target of the set can be calculated as

$$\text{MTF}(\xi = \xi_f) = \left[\frac{S_{\text{output}}(\xi = \xi_f)}{S_{\text{input-bar-target}}(\xi = \xi_f)}\right], \quad (11)$$

where $S_{\text{input-bar-target}}(\xi = \xi_f)$ is the magnitude (normalized to 1 at dc) of the input spectrum at its fundamental frequency, and $S_{\text{output}}(\xi = \xi_f)$ is the dc-normalized magnitude Of the output spectrum at the same frequency. Using Eq. (11) for various test targets allows MTF to be measured directly from bar-target data without need for a series conversion. To verify the procedure, we used this MTF measurement technique to test a commercial visible CCD camera-and-lens combination over a spatial frequency range up to one half of the spatial Nyquist frequency of the detector array. The results obtained agreed with a sine-wave MTF measurement within 2%. At spatial frequencies higher than approximately half Nyquist, both the sine-wave test and the bar-target test will suffer appreciably from sampling artifacts, and other measurement techniques such as an oversampled knife-edge response[5] or random-noise targets[6] must be used instead.

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