Facet model for photon-flux transmission through rough dielectric interfaces

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When a photon flux is incident upon a rough interface that separates media with different refractive indices, the interface roughness influences the angular distribution of the transmitted flux. For the case of very rough surfaces with slopes of the order of unity, we find that a simple facet model is sufficient to describe the main features of the flux-transmission behavior. We demonstrate the effect observed by Nieto-Vesperinas *et al.* [Opt. Lett. **15,** 1261 (1990)] for plane-wave incidence, that the interface roughness tends to suppress the refractive-index contrast. In addition, for cases in which the incident flux is distributed in angle, we find that the direction of maximum transmitted flux can be predicted from the surface roughness. © 1996 Optical Society of America

Applications ranging from light scattering in biological tissues or powdered media to remote sensing of sea ice require models for random volume-scattering media (VSM's) bounded by rough dielectric interfaces[1] or placed behind random phase screens.[2]

In this Letter we treat the problem of photon-flux transfer through very rough dielectric interfaces when ray optics is a sufficient approximation (the rms roughness σ and the surface correlation length L are both larger than the wavelength λ) and the interface has a steep average slope (the ratio L/σ is of the order of unity). This situation is encountered, for example, in reflectance measurements of skin tissues[3] and in highly packed granular composites.[4]

Experimental[5] observations and numerical[6] studies of a new aspect of light scattering from highly sloped random surfaces were recently presented. When a plane wave is incident upon a rough dielectric interface the propagation direction of the transmitted flux spreads, and the mean direction of refraction for the transmitted beam shifts from that predicted by a flat-interface Snell law. For $L > \lambda$ and $\sigma > \lambda$ and L/σ of the order of unity the roughness tends to suppress the refractive-index contrast, and the angular distribution of the transmitted light peaks around the straight-through direction.

The simplest model that can account for surface effects in light scattering from a slab of a VSM with rough surfaces uses a geometric-optics approach. In this model the photon flux at the interface experiences Fresnel transmission or reflection by planar surface facets. In this Letter we present a rough-surface facet model that explains the angular distribution of the photon flux transmitted through a rough dielectric interface, demonstrating the transmission effect described in Refs. 5 and 6 for plane-wave incidence. Moreover, when an angularly distributed photon flux impinges upon the interface, our treatment can be used to predict the corresponding modification of the angular distribution of the transmitted flux.

When one is describing a surface there are two independent aspects of the roughness: the distribution of heights as measured from a reference plane and the variation of heights along the surface. Usually these variables follow independent statistics. We assume that the distribution of heights $h(x)$ is Gaussian with zero mean and that σ is the rms roughness. For surfaces with a single correlation length a simple model consists of randomly oriented facets with horizontal projections equal to L (see Fig. 1). The statistical character of the surface is described by the probability density $P_s(s)$ of the distribution of local slopes:

$$s = \tan^{-1}\left[\frac{h(x) - h(x + L)}{L}\right]. \qquad (1)$$

In this model the heights of points separated by L, $h_1(x)$ and $h_2(x + L)$, are statistically independent random variables that have the same probability density. Hence the joint probability density $P(h_1, h_2)$ factors as $P(h_1)P(h_2)$. When we define the height difference along a correlation length $\Delta(x) = h(x) - h(x + L)$, it follows from Eq. (1) that $P_\Delta(\Delta)$ also has a Gaussian distribution. The probability density of the local slope $s = \tan^{-1}(\Delta/L)$ satisfies $P_s(s)\mathrm{d}s = P_\Delta(\Delta)\mathrm{d}\Delta$ and can be written as

$$P_s(s) = \frac{L}{2\sqrt{\pi}\cos^2(s)} \exp\left[-\frac{(L \tan s)^2}{4\sigma^2}\right]. \qquad (2)$$

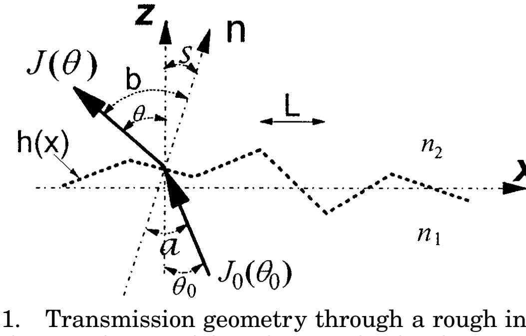

Fig. 1. Transmission geometry through a rough interface $h(x)$ having facets of length L. The local normal **n** is rotated with an angle s with respect to the average normal **z**.

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This form for $P_s(s)$ can also be obtained on the basis of a normal distribution of heights and a Gaussian form of the autocorrelation function.[7] The interesting feature of the slope distribution of Eq. (2) is the presence of the two maxima at approximately $s_M = \pm\cos^{-1}(L/2\sigma)$. However, the maxima appear only in the case of very rough surfaces having $L \leq 2\sigma$.

We treat the problem of light incident upon the interface from the dense-medium (n_1) side, as shown in Fig. 1. The random inclination of the surface normal $\mathbf{n}$ with respect to the average normal $\mathbf{z}$ is described by the local slope s. Light impinges upon a facet at an angle of incidence θ_0 as measured from the direction of the average normal $\mathbf{z}$, and a and b are the local angle of incidence and the corresponding transmission angle, respectively. We want to find the distribution of the transmitted light as a function of the scattering angle θ. For the sake of brevity we consider the incident light to be unpolarized. For an angle of incidence $a = s - \theta_0$ the Fresnel transmission coefficient is $T(a) = 1/2[T_\perp(a) + T_{||}(a)]$, where $T_\perp(a)$ and $T_{||}(a)$ are the perpendicular- and parallel-polarization Fresnel coefficients, respectively.[8] If $J_0(\theta_0)$ denotes the angle-dependent photon flux incident upon a facet of slope s, the angle-dependent photon flux transmitted through the facet is $J(\theta) = J_0(\theta_0)T(s - \theta_0)$. For rough surfaces we are interested in the transmitted photon flux averaged over the whole range of surface slopes:

$$\langle J(\theta)\rangle = \int_{-\pi/2}^{\pi/2} J_0(\theta_0)T(s - \theta_0)P(s)\mathrm{d}s\,, \tag{3}$$

where the angles are related through Snell's law, $n_2 \sin(s + \theta) = n_1 \sin(s - \theta_0)$.

For the case of a plane wave incident at Θ_0 upon the dielectric interface, Eq. (3) is evaluated for an incident flux $J_0(\theta_0) = \delta(\theta_0 - \Theta_0)$. Figure 2(a) presents the transmitted flux corresponding to a unit flux incident at Θ_0 equal to 0°, 20°, 40°, and 60° for rough surface with slopes described by Eq. (2) having $L/\sigma = 1.5$ and a refractive-index contrast $n_1/n_2 = 1.5$. Also indicated in Fig. 2(a) are the expected directions of refraction $b(\Theta_0)$ if the surface had been flat. For all angles of incidence the transmitted flux peaks at smaller angles than expected, given $n_1/n_2 = 1.5$. The transmitted flux tends to center around the straight-through direction, behaving as if no refractive contrast were present at the interface. This effect was also observed by Nieto-Vesperinas *et al.*[5] The large surface slope means that small local angles of incidence are less probable for large Θ_0. Therefore there are fewer contributions to the forward flux for large Θ_0. Note that for a smooth interface and for the largest angle of incidence, $\Theta_0 = 60°$, Snell's law would predict no transmission at all. However, in the case of a rough interface the slope distribution permits local angles of incidence smaller than the angle of total internal reflection. A direct comparison with experimental data of Ref. 5 is not straightforward because our analysis is concerned with a dielectric-to-vacuum geometry. However, the main conclusion of Ref. 5 is that surface interface roughness acts to suppress the refractive-index contrast between the media and is successfully explained by our model.

In Fig. 2(b) we show the transmitted flux corresponding to a plane wave incident at $\Theta_0 = 20°$ upon surfaces with various L/σ ratios between 1 and 5.5 as indicated. For very rough surfaces the transmitted flux has a broad angular distribution. For $L/\sigma < 1$, shadowing effects and contributions of multiple total internal reflections are not negligible, and the simple single-scattering model fails to provide an accurate description at large θ. When the surface roughness decreases ($1 < L/\sigma < 5$), the maximum transmission shifts toward larger angles, and a narrow peak develops. Within this range of surface roughness the transmission effect pointed out in Ref. 5 can be observed. For smoother surfaces with $L/\sigma > 10$ the transmitted flux tends to peak near the specular direction given by Snell's law, $b(\Theta_0) = \sin^{-1}[n_1/n_2 \sin(\Theta_0)]$.

We now generalize our treatment for the case of angularly distributed (rather than plane-wave) illumination. First we consider an angularly uniform incident flux $J_0(\theta_0) = 1$ for $-\pi/2 < \theta_0 < \pi/2$. The results of evaluating Eq. (3) are shown in Fig. 3(a) for rough surfaces with L/σ ratios between 1 and 5 and a refractive-index contrast $n_1/n_2 = 1.5$. The overall effect of a dielectric interface is to attenuate the flux transmitted at large angles. Moreover, for very rough interfaces a depletion zone appears around the normal to the interface, and the transmitted flux does

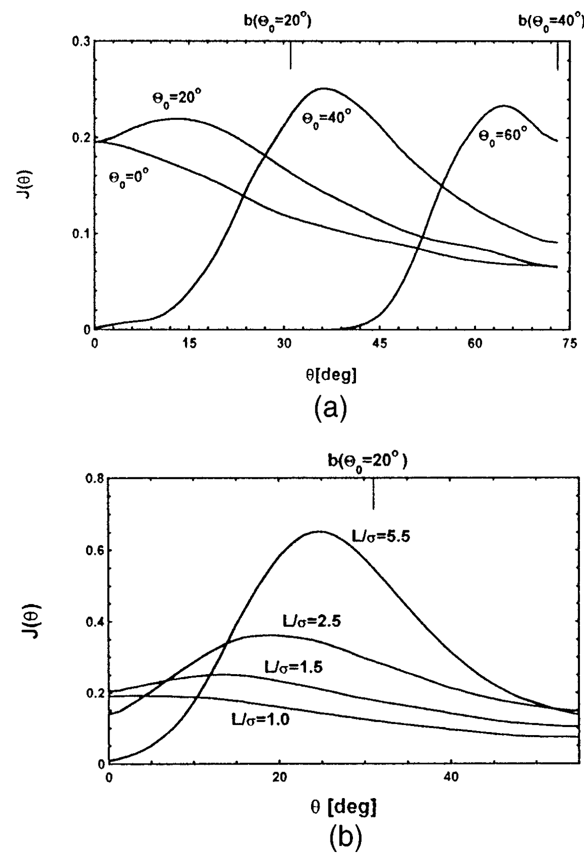

Fig. 2. (a) Angular dependence of the transmitted photon flux corresponding to plane waves with different angles of incidence for a rough interface with $L/\sigma = 1.5$ and $n_1/n_2 = 1.5$. (b) Angular dependence of the transmitted photon flux corresponding to a plane wave incident at 20° upon interfaces with $n_1/n_2 = 1.5$ and roughness parameters as indicated.

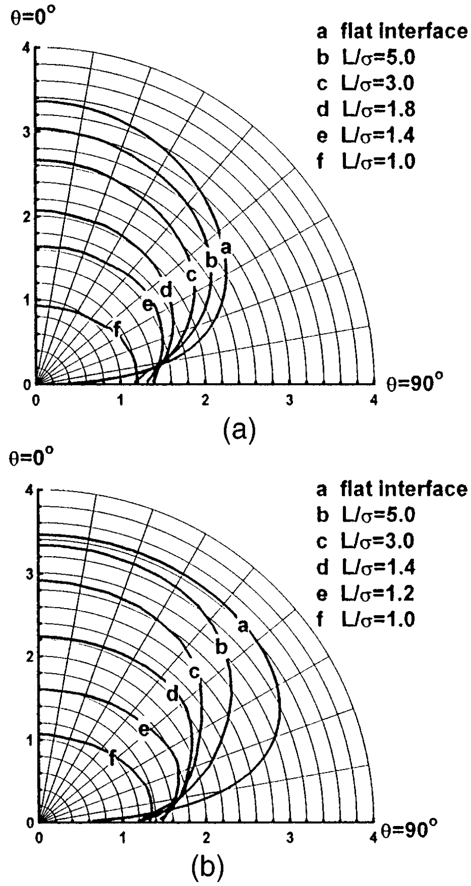

Fig. 3. (a) Polar plot of the transmitted photon flux corresponding to an angularly uniform incident flux, $n_1/n_2 = 1.5$, and roughness parameters as indicated. (b) Polar plot of the transmitted photon flux corresponding to a cosine distribution of the incident flux, $n_1/n_2 = 1.5$, and roughness parameters as indicated.

not peak at $\theta = 0$. For these surfaces the transmitted flux peaks along scattering angles that depend on the average slope. A similar effect is visible in the Monte Carlo calculations of Ref. 6 as well as in the experiments of Ref. 5, where the effect is attributed to misalignment. We interpret these maxima as being caused by the total internal reflection that occurs for small values of θ_0. In this case, because of the large surface slopes, local angles of incidence have large values and, consequently, there is a lack of transmitted flux in the forward ($\theta = 0$) direction. When the interface becomes smoother the values of the local angle of incidence span a large interval, and the transmitted flux loses the depletion zone. As can be seen from Fig. 3(a), for $\sigma/L > 1.5$ the flux starts to peak along a forward direction, and eventually it develops an angular dependence similar to the one corresponding to a flat interface.

Of particular importance is the case in which the incident photon density follows a $\cos(\theta_0)$ distribution. Within the framework of the diffusion approximation for photon propagation through a VSM with an interface that is flat in comparison with the transport mean free path, the angular dependence of the emerging flux obeys the Lambert law: $J_0(\theta_0) = \cos(\theta_0)$.[8] In Fig. 3(b) we plot the angular distribution of the transmitted flux for interfaces with L/σ ratios between 1 and 5. As in the case of an angularly uniform incident flux, the dominant effect of the roughness is to flatten the angular distribution. For very rough surfaces one again observes depletion zones around the surface normal. Because this geometry corresponds to the case of many natural light-scattering media, it is important to see how rough interfaces lead to significant deviations from the Lambert law of diffuse transmittance.

We have presented a simple ray-optics model that describes a recently observed effect in the flux transmission through rough dielectric interfaces. The facet model explains the angular broadening of the transmitted flux and the effective suppression of the refractive-index contrast. The present approach also predicts an attenuation of the forward flux resulting from total internal reflections caused by an increase in the local angle of incidence for surfaces with steep slopes.

More-rigorous treatments can be developed for $L < \lambda$ and $\sigma < \lambda$, but Ref. 6 shows that for smooth surfaces the transmission is predominantly specular in spite of a small L/σ ratio. For surfaces with $L > \lambda$ and $\sigma > \lambda$ and high slopes (L/σ near unity) this ray-optics model is sufficient to describe most of the flux transmission phenomena.

In describing combinations of VSM's and rough dielectric interfaces the practical significance of this study is twofold. Photon propagation through a VSM is relatively well understood in the frame of a diffusion approximation that predicts a Lambertian cos θ distribution of the emergent flux. However, in most applications the measurement is done outside the VSM, and therefore the measurable quantities are strongly influenced by the interface roughness. In other cases, when the surface roughness is of interest, we want to isolate its effect from that of the multiple scattering in the VSM. Provided that the propagation and scattering through the VSM is correctly described, roughness characterization can be based on measurements of the angular dependence of the flux transmitted through the interface.

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