Parameters of spinning FM reticles

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The literature describes tracking devices that allow a single detector coupled to a spinning FM reticle to determine target location. The spinning FM reticles presented were limited to single parameter reticles of frequency vs angle, frequency vs radius, or phase. This study presents these parameters with their capabilities and limitations and shows that multiple parameters can be integrated into a single reticle. Also, a general equation is developed that describes any FM reticle of the spinning type. *Key words:* Reticles, FM reticles, tracking devices.

I. Introduction

The term reticle has been defined[1] as "a pattern located in the focal plane of an instrument to measure or locate a point in an image." Reticles have been used and are still used in a multitude of operations from commercial applications or surveying to military applications of boresighting surveillance and fire control systems. The general case that most people are familiar with is the simple sight on a rifle or gun. There are as many types of reticle as there are uses for them. However, one type of reticle, commonly referred to as a spinning frequency modulated (FM) reticle, can be used to provide range and bearing information. We present here three parameters that completely describe any spinning FM reticle and derive a single equation that can be used to specify a spinning FM reticle transmission function to suit a particular application or requirement.

II. Background

FM reticles can be completely described by these three parameters, (1) frequency vs angle $f(\theta)$, (2) frequency vs radius $m(r)$, and (3) phase or spoke function $\rho(r)$.

In the literature spinning FM reticles are described by only one of these parameters [$f(\theta)$, $m(r)$, or $\rho(r)$]. A survey of this literature shows that the parameters above are not integrated into combinations of two or of all three. However, all the spinning FM reticles presented in the literature can be completely described by using one of these three parameters.

Sections III–V describe reticles that are modeled by a single parameter with the other two parameters held constant. This introduces the description and design of spinning FM reticles that involve only one of the three parameters before considering reticles of multiple parameters. Section VI presents the integration of two or all three of the parameters and shows examples of reticle designs where multiple parameters are used. Examples are introduced using a reticle transmission equation that is developed using the three FM reticle parameters. In fact, any FM reticle transmission function can be generated using the equation if the desired parameters are known.

The three parameters yield a simple model for reticles that can be used to design FM reticles or analyze existing ones. It is seen that the reticle transmission function does not have to be separable in polar coordinates for the parameters themselves to be separable. In fact, it will be seen that the transmission function is separable only when $m(r)$ and $\rho(r)$ are constant.

The frequency vs angle function $f(\theta)$ encodes azimuth, or bearing, target information. In the literature,[2] a reticle with a sinusoidal frequency vs angle function with $m(r)$ and $\rho(r)$ held constant is referred to as a spinning FM reticle. It is the classical case of the FM reticle. However, the function $f(\theta)$ need not be sinusoidal and the parameters $m(r)$ and $\rho(r)$ do not have to be constant. Also, the functional form of $f(\theta)$ can be discontinuous at some angular position. Older FM tracking reticles contained a continuous $f(\theta)$ to impress only azimuth target information for seeker

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guidance with $m(r)$ and $\rho(r)$ held constant. These reticles were inefficient for tracking since the rate of seeker turning was not weighted with radial target distance. This caused stability problems that resulted in loss of track or target overshoot. Section III describes the frequency vs angle parameter in more detail.

The frequency vs radius parameter $m(r)$ involves radial target location information. The classical case[3] of the frequency vs radius parameter is a linear increase in frequency as the target location increases radially from the center of the reticle. Since a reticle of this parameter encodes only radial location information, this type of reticle is usually coupled to a phasing sector, or a wedge of 0.5 transmission, to provide angular target information along with radial target information. However, the phasing sector imposes undesirable harmonic distortion on the radial target location signal. There are current studies[4] on techniques for minimizing the phasing sector distortion such as variations in sector geometries. Also, while not shown in available literature, a combination of the design parameters $f(\theta)$ and $m(r)$ in an FM reticle can eliminate the requirement for a phasing sector and along with it, its peculiar type of harmonic distortion. Section IV describes, in detail, the idea of the frequency vs radius parameter in reticle design.

The phase or "spoke" function[5,6] $\rho(r)$ of a reticle is a parameter that can be adjusted to maximize or minimize the reticle response to target shapes and sizes. Targets with lines or edges that are collinear to a radial line from the center of a reticle correlate well with reticles of constant spoke functions. These large correlations drown out signals of smaller targets and targets of large spatial variations. Spoke functions in the shapes of curves and jagged edges provide reasonable compensation for large, straight, and smooth backgrounds. The spoke function is described in greater detail in Sec. V.

The full capability of this technique for modeling spinning FM reticles is realized in the design and analysis of reticles that combine two or three of the parameters. It is possible to design single variable FM reticles with insight and intuition, but combining variables is more easily accomplished with the model presented in this paper. The design and analysis of reticles that combine two or all three of the parameters are illustrated in Sec. VI. This section demonstrates that an arbitrary FM signal can be obtained from an appropriately designed FM reticle. Also, given an arbitrary FM reticle, the FM signal, i.e., its spinning impulse response, can be found.

III. Frequency vs Angle

The frequency vs angle parameter provides azimuth target information. Consider the simple rising sun reticle shown in Fig. 1(a). Let θ be the angular spatial position on the reticle with the $\theta = 0$ location at the right reticle edge and the $\theta = \pi, -\pi$ location at the left reticle edge traversing in the counterclockwise direc-

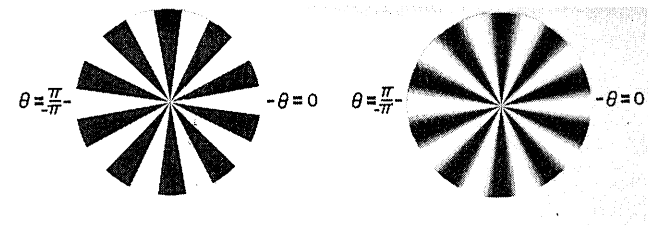


Fig. 1. Rising sun reticle.

tion. Also, let r be the radial position on the reticle. The spatial transmission function of the reticle is

$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}\,\mathrm{sgn}\,(\cos m\theta), \tag{1}$$

where m is the total number of spokes, or angular bars, in the reticle. In this example, m is 10. The signum function is equal to -1 when its argument is <0 and is equal to 1 when its argument is ≥ 0.

The signum function provides a binary on or off of the transmission characteristic of the reticle. For IR systems it is often difficult to construct materials of graded transmission accurately so the binary transmission is often used. This creates somewhat of a problem in tracking systems since the abrupt changes in transmission cause higher frequency components on the detector output of a spinning reticle system. It is more general to analyze the behavior of FM reticles without this constraint on the transmission. The reticles discussed here, therefore, have continuous tone transmission functions such as the reticle shown in Fig. 1(b) where the signum function has been removed.

If a point source were imaged onto the spinning rising sun reticle, the signal of the total transmitted light would be of sinusoidal form. If the point source is relatively stationary with respect to the reticle spin rate, the total light irradiance output from the spinning reticle can be found by

$$I(t) = \int_0^R \int_{-\pi}^{\pi} T(r,\theta)\delta[r - r_0, \theta - (\omega t - \theta_0)]\, r\, d\theta\, dr \tag{2}$$

$$= \int_0^R \int_{-\pi}^{\pi} [\tfrac{1}{2} + \tfrac{1}{2}\cos(m\theta)]\delta[r - r_0, \theta - (\omega t - \theta_0)]\, r\, d\theta\, dr, \tag{3}$$

where R is the radial spatial extent of the reticle, r_0 and θ_0 are the polar coordinates of the point source, and ω is the rate at which the reticle spins in radians per second. Note that Eqs. (2) and (3) show the point source spinning instead of the reticle. The reticle spinning is an equivalent to the point source spinning if the point source nutates around $r = 0$ (the point source moves only in angle). The temporal response shown in Eq. (2) is referred to as the spinning impulse response of a reticle.

It is important to note that Eqs. (2) and (3) are 1-D correlations of the reticle transmission function and the delta function in the θ direction. The correlation of any function with a shifted impulse, or delta function, is the function located at the delta function coordinate. Therefore,

$$I(t) = A[\tfrac{1}{2} + \tfrac{1}{2}\cos m(\omega t - \theta_0)], \tag{4}$$

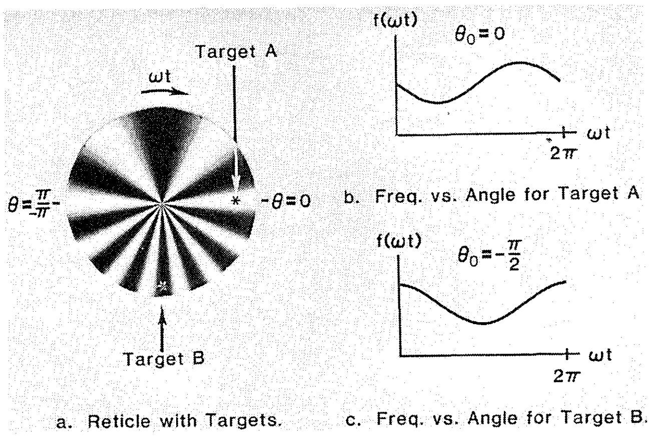


Fig. 2. Angular cosine modulated reticle.

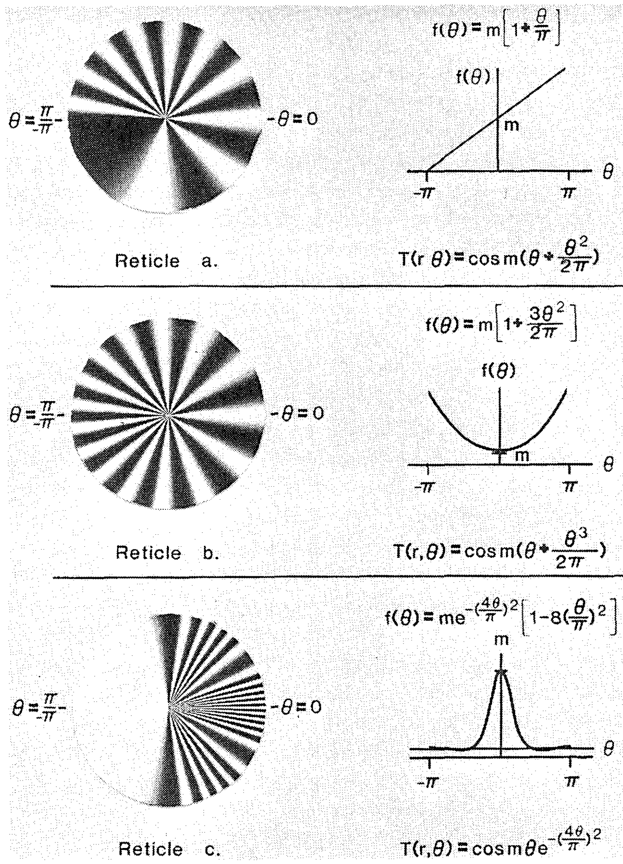


Fig. 3. Frequency vs angle functions with reticles.

where A is the peak integrated irradiance of the point source through the reticle transmission function. Since the transmission function was not a function of r, the output irradiance waveform is not a function of r. Note the output is a pure cosine, or shifted sine, of frequency $m\omega/2\pi$ with a phase shift of $m\theta_0$. The phase shift is not useful in finding the angular point source location since the waveform reproduces itself m times for one reticle rotation and results in an ambiguity. That is, there are m possible angular target positions for the same phase shift. However, it should be noted that, if m is equal to 1, the angular location of the point source can be found. For $m = 1$, the reticle looks like a circle composed of a semicircle permitting full transmission and a semicircle blocking all transmission. This type of reticle is referred to as a phase sector reticle. Phase sector reticles are powerful tools when coupled to the frequency vs radius reticle parameter. While not addressed in this paper, they are a topic of further investigation.[4]

It has been shown that the reticle in Fig. 1 has a constant output frequency of $m\omega/2\pi$ regardless of the point source location in r or θ. For a FM reticle to give angular target location information, an angular variation in frequency must be imposed on the reticle transmission function. The case shown most often in the literature for this variation of frequency vs angle is shown in Fig. 2(a). The spatial transmission function of the reticle is of the form

$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}\cos m(\theta + \alpha\cos\theta). \tag{5}$$

The spinning impulse response of this reticle is

$$I(t) = \tfrac{1}{2} + \tfrac{1}{2}\cos m[(\omega t - \theta_0) + \alpha\cos(\omega t - \theta_0)]. \tag{6}$$

The angular frequency content of the reticle can be derived from the basic definition of frequency. For a given transmission function

$$T(\theta) = \tfrac{1}{2} + \tfrac{1}{2}\cos g(\theta), \tag{7}$$

where $g(\theta)$ is the transmission function's cosine argument, the frequency content of $T(\theta)$ is the derivative of g with respect to θ. Hence, $f(\theta)$, the angular spatial frequency content for the reticle described by Eq. (5), is

$$f(\theta) = \frac{dg(\theta)}{d\theta} = \frac{d}{d\theta}[m(\theta + \alpha\cos\theta)] = m(1 - \alpha\sin\theta). \tag{8}$$

The frequency content of the temporal spinning impulse response is found by simply taking the derivative of its cosine argument with respect to time. That is,

$$f(\omega t) = \frac{d}{dt}\{m[\omega t - \theta_0) + \alpha\cos(\omega t - \theta_0)]\} \tag{9}$$

$$= m\omega[1 - \alpha\sin(\omega t - \theta_0)] \tag{10}$$

in radians per second. The frequency content of the spinning impulse response is the frequency content of the detector output signal. Note that the temporal frequency $f(\omega t)$ and the spatial frequency $f(\theta)$ are of the same functional form. The temporal frequency of the spinning impulse response varies from $(1 - \alpha)m\omega/2\pi$ when the point source travels through the $\theta = \pi/2$ location of the reticle to a frequency of $(1 + \alpha)m\omega/2\pi$ when the reticle travels through the $\theta = -\pi/2$ location of the reticle. That is, the irradiance output is of FM form where the frequency of the output signal follows a sinusoidal variation of one period per reticle rotation. The value of α for the reticle shown is 0.4. Two target point sources are shown in Fig. 2(a). Figure 2(b) shows the output frequency plotted with respect to ωt for target A and Fig. 2(c) shows the output frequency for target B. The phase from the $\omega t = 0$ axis to the center of the downward cosine slope is equivalent to the negative value of the angular target location in radians.

The difference between the cosine peak for target A and the cosine peak for target B is equivalent to the angular offset between the targets. Note that these waveforms are periodic with a period equal to the reticle rotation time.

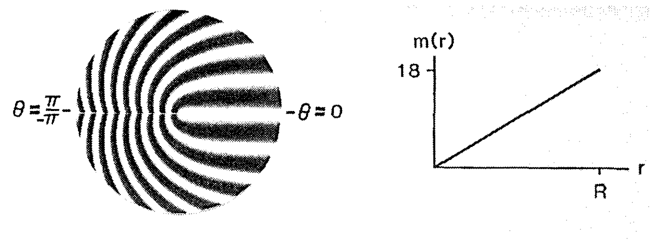
The simple angular FM reticle shown in Fig. 2 is of the most basic periodic form. However, the variation in frequency vs angle can be any function imaginable. Some examples of reticles with other frequency variations in angle are shown in Fig. 3. The angular frequency content $f(\theta)$ is shown along with the corresponding transmission function for each reticle. Note that in each case the argument of the reticle transmission cosine function $g(\theta)$ is related to the reticle angular frequency $f(\theta)$ by its derivative

$$f(\theta) = \frac{dg(\theta)}{d\theta}. \tag{11}$$

Consider a base angular spatial frequency of m cycles per 2π rad and let $f(\theta)$ either add to or subtract from this base frequency. The reticle in Fig. 3(a) both adds to and subtracts from the base frequency in a linear manner from $-\pi$ to π. The reticle in Fig. 3(b) only adds to the base frequency as a square function with a minimum frequency of m at $\theta = 0$. The reticle in Fig. 3(c) only subtracts from the base frequency with a peak frequency of m at $\theta = 0$. The frequency content of the reticle in Fig. 2 oscillates around its base frequency in a sinusoidal manner. With a little work, it becomes apparent that the frequency vs angle parameter $f(\theta)$ is the derivative of the cosine argument of the reticle transmission function.

The spinning impulse response $I(t)$ of the reticles shown in this section can be found by simply replacing θ in the transmission function with $(\omega t - \theta_0)$ and scaling its amplitude to some peak integrated irradiance A. The spinning impulse response is of the same functional form as the reticle transmission function. The frequencies of the reticles (spatial and temporal) can be found as shown previously for the reticle in Fig. 2. All the frequency variations of the reticles in Figs. 2 and 3 have a period of $\omega t = 2\pi$. That is, the recurrence in the frequency function is periodic in a time $\omega t = 2\pi$, or one rotation of the reticle. This periodicity allows the phase of the waveform to be used to locate the point source. Now a restriction can be placed on the frequency vs angle function $f(\theta)$ for use in azimuth target location determination. The function $f(\theta)$ cannot reproduce itself in a period $<2\pi$. Also, since the reticle transmission will now allow $f(\theta)$ to have a period $>2\pi$, all the useful frequency vs angle functions will have a period equal to 2π.

One last note on frequency vs angle reticles regarding the separability of the reticle transmission function. If the reticle is purely of the frequency vs angle type (i.e., constant radial frequency and constant phase), the reticle transmission function is separable. That is, the reticle transmission function can be written $T(r,\theta) = T(r)T(\theta)$. The reticles considered in this section are of this form where $T(r) = 1$.



a. Lovell Reticle. b. m(r) Versus r.

Fig. 4. Lovell reticle with radial dependence.

IV. Frequency vs Radius

Unlike the frequency vs angle parameter, the frequency vs radius parameter $m(r)$ is nonperiodic. Since the reticle is moving in the θ direction only, the spinning impulse response repeats in angle but not in radius. For the reticles presented so far, the number of spokes or bars subtended onto a circular contour centered at the reticle origin for any radius is constant. Therefore, radial target information was not contained in these reticles.

In frequency vs radius FM reticles, the radial information is obtained by changing the number of subtended bars for circle contours of different radii. That is, $m(r)$ must change vs radius. The classic case of this type of reticle is the Lovell[7] reticle shown in Fig. 4(a). For any circle drawn around the reticle origin, a cosine of constant frequency lies under the circular arc. This is a deceiving phenomenon since the reticle bars on the left are smaller and closer together than the bars on the right. The manner in which the arc subtends through the smaller bars corrects the bar widths to be equivalent. At any rate, the arc overlays a larger number of bars as the radius increases. The reticle transmission function for the reticle is

$$T(r,\theta) = \frac{1}{2} + \frac{1}{2}\cos[\theta m(r)] = \frac{1}{2} + \frac{1}{2}\cos\left[\theta\left(18\,\frac{r}{R}\right)\right]. \tag{12}$$

Note that the angular spatial frequency and the temporal frequency of the spinning impulse response is still found by taking the derivative of the cosine function with respect to theta or time, respectively. Also, $m(r)$ is not a function of theta and is considered a constant in the derivative calculation. However, $m(r)$ can change the angular frequency since the frequency of Eq. (12) is simply $m(r)$ which increases linearly with respect to radius.

The reticle shown in Fig. 4 has a linear one-to-one mapping of r to $m(r)$. A requirement for a useful tracking $m(r)$ function is that the mapping be single valued. The $m(r)$ function for the reticle in Fig. 4 satisfies the single value mapping constraint for tracking in the r direction. Consider the reticle shown in Fig. 5(a) along with its $m(r)$ vs r plot in Fig. 5(b). The angular frequency vs radius parameter is

$$m(r) = 18\exp\left\{-2\left[\frac{\left(r - \frac{R}{2}\right)}{R}\right]^2\right\}. \tag{13}$$

The reticle has a two-to-one mapping from r to $m(r)$ for

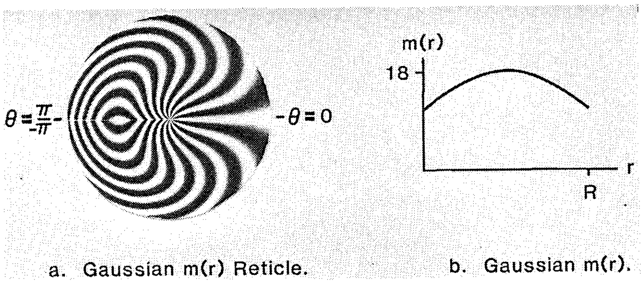


Fig. 5. Reticle with a Gaussian $m(r)$.

all frequencies other than $m(R/2)$ and there exists two radial positions corresponding to a single frequency. Therefore, the temporal frequency of the spinning impulse response allows two radial target locations to correspond to one temporal frequency. Hence, the reticle would not be useful to locate a target in radial position. This reticle shows the importance of the single value mapping constraint.

One similarity to the $f(\theta)$ parameter is that the $m(r)$ parameter can be any function imaginable with the one-to-one mapping restriction. Some examples of frequency vs radius reticles are shown in Fig. 6 along with their corresponding $m(r)$ and $T(r,\theta)$ functions.

The reticle shown in Fig. 6(a) has an $m(r)$ parameter that begins with some nonzero frequency at a small radius. This type of reticle may be useful for a tracking system that has an ac coupled detector that does not respond well to signals at lower frequencies. In fact, the slope of the $m(r)$ curve adjusts the reticle bandwidth so that the reticle frequencies can match the temporal frequencies of the system electronics. One last note about the reticle in Fig. 6(a) is that, if a target were located at the center of the reticle, a dc signal would be seen on the output of the detector. When the target is moved to a small neighborhood of the reticle origin, a jump in frequency occurs. Hence, the $m(r)$ function is discontinuous at the reticle origin.

The reticle shown in Fig. 6(b) has an $m(r)$ function of the squared form. This type of reticle may be used when a larger frequency change is required to give a tracking system more target location resolution. For the reticle shown, the larger frequency change occurs at the outer edges of the reticle. So the reticle may be useful for tracking targets at a higher resolution around the periphery of the reticle and at a lower resolution near the reticle center.

The reticle shown in Fig. 6(c) is the converse of the reticle in Fig. 6(b). The reticle has a larger frequency change near the center of the reticle and a lower frequency change near the periphery. This allows target locations to be resolved better toward the center of the reticle, much like human visual resolution. Also note that the frequency change is negative as the radius increases in contrast with the other frequency vs radius reticles presented.

Now that various frequency vs radius reticles have been presented and some insight to the $m(r)$ parameter has been gained, it is easy to verify that a frequency vs radius reticle is designed by imposing a particular vari-

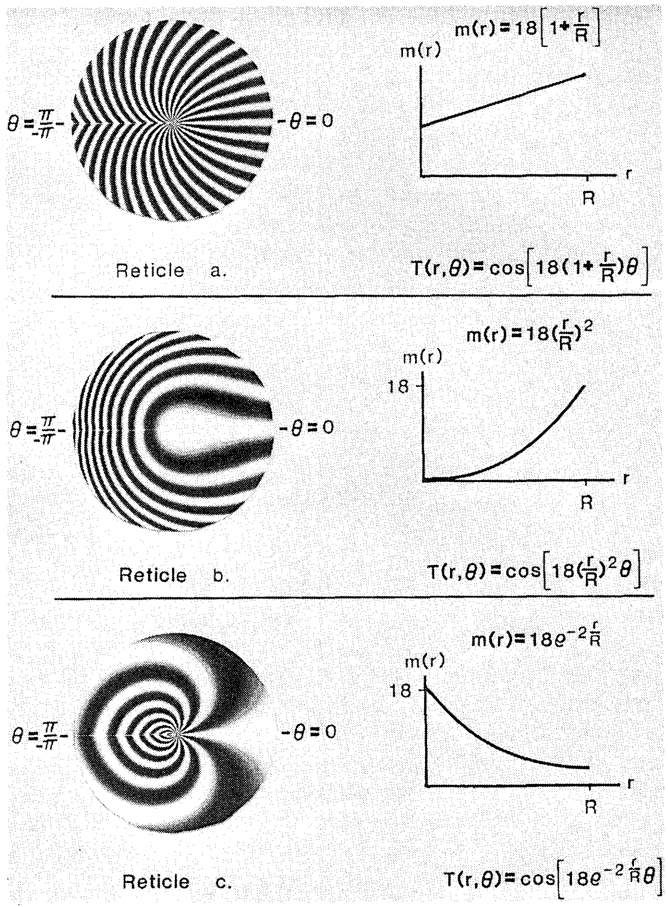


Fig. 6. Frequency vs radius reticles.

ation in m as a function of radius. Also, $m(r)$ is the same base frequency presented in Sec. II. The base frequency $m(r)$ has a radial dependence and the angular frequency $f(\theta)$ modulates on this base frequency. The angular spatial frequency becomes a function of r and θ,

$$\mathrm{freq}(r,\theta) = m(r)f(\theta), \tag{14}$$

where

$$f(\theta) = \frac{dg(\theta)}{d\theta},$$

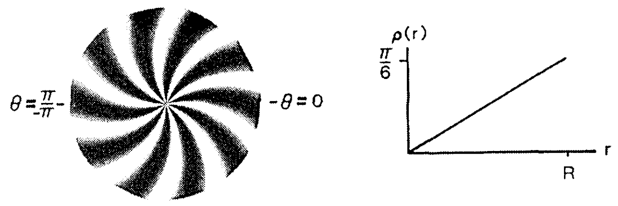
and $g(\theta)$ is contained in the reticle transmission function

$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}\cos[m(r)g(\theta)]. \tag{15}$$

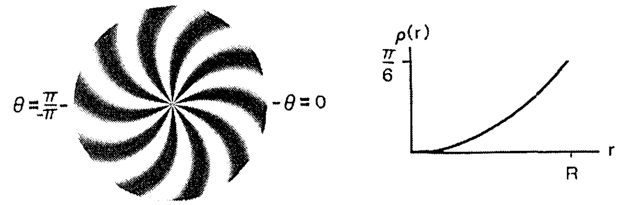
The spinning impulse responses of the reticles are found as before with the exception that r is replaced with r_o. The temporal frequency content is also found as before by taking the derivative of the spinning impulse cosine function with respect to time. The temporal frequency functional form is the same as the angular spatial frequency functional form with $r = r_0$ and $\theta = \omega t - \theta_0$.

V. Phase or Spoke Function

The last parameter of FM reticles is the phase or spoke function $\rho(r)$. Until this point, all our transmis-

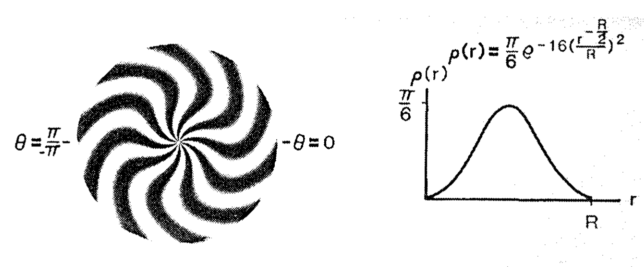


a. Linear Phase Reticle. b. Linear $\rho(r)$ Versus r.

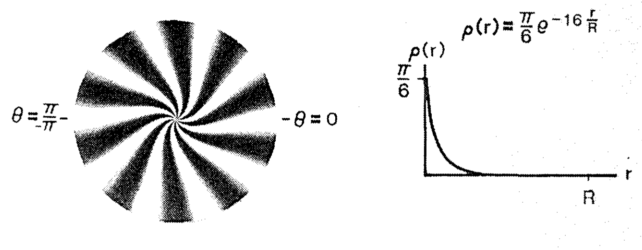


c. Squared Phase Reticle. d. Squared $\rho(r)$ Versus r.

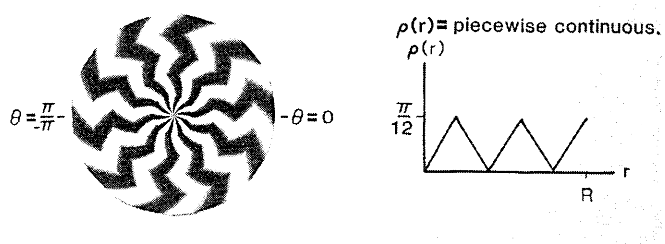
Fig. 7. Linear and squared phase reticles.



Reticle a.



Reticle b.



Reticle c.

Fig. 8. Various phase reticles.

sion functions have been written with respect to some phase reference. This reference has been the $\theta = 0$ line on the reticle. This line can be thought of as a phase line. The transmission function can be thought of as a set of angular functions that occur on discrete radii. The phase is the line where each discrete angular function begins. That is, the arguments of the angular functions are zero and $T(r,\theta) = 1$ on this line.

In the early days of FM reticle design, it was found that reticles that contained spokes with straight lined edges correlated well with large objects with smooth, straight edges such as clouds, horizons, and buildings. These large correlations drowned out signals of small targets which were usually the targets of interest. To dampen the larger spatial signals, either a curvature or a high frequency variation was imposed on the edges of the spokes to decorrelate the reticle with large objects. In fact, the type of spoke function used in a reticle depended directly on the size and shape of the desired target and the sizes and shapes of the expected background clutter.

Consider the rising sun reticle in Fig. 1. The reticle has a constant $f(\theta)$ and $m(r)$ parameters of 10 cycles per rotation as do all the reticles presented in this section. The reticle also has a constant phase of $\rho(r) = 0$ at any radius. That is, the transmission function cosine begins, or is equal to one, on the $\theta = 0$ line for any radius chosen. Now consider the reticle in Fig. 7(a). The $T(r,\theta) = 1$ line, or the phase line, curves around clockwise as the radius increases. This curving is linear in angle as the radius increases as shown in Fig. 7(b). Note that the reticle looks different from the original rising sun reticle but its $f(\theta)$ and $m(r)$ parameters are equivalent. Hence, the phase or spoke function changes the geometric shape of the spokes but does not change the fundamental frequency variations of the reticle.

The transmission function that describes the phase reticles in this section is

$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}\cos\{10[\theta + \rho(r)]\}, \tag{16}$$

where

$$\rho(r) = \frac{\pi}{12}\frac{r}{R} \tag{17}$$

for the reticle shown in Fig. 7(a). The phase of the reticle in Fig. 7(c) is

$$\rho(r) = \frac{\pi}{12}\left(\frac{r}{R}\right)^2. \tag{18}$$

Note that a linear increase in phase angle vs radius is not a straight line in spoke geometry. Only a constant phase can give a straight spoke line. The reticle shown in Fig. 7(c) is similar to the linear phase reticle with the exception of a squared dependence of phase angle on radius. The reticle curves more toward its periphery than the linear phase reticle. Figure 7(d) shows the $\rho(r)$ graph vs r for the reticle.

As in the last two sections, the variations in $\rho(r)$ are limitless. A few of these variations are shown in Fig. 8. Again, the reticles all have constant $f(\theta)$ and $m(r)$ parameters of 10 cycles per 2π rad. Also, their corresponding $\rho(r)$ functions are presented. All their transmission functions are described by Eq. (16).

The first reticle, Fig. 8(a), has a Gaussian phase function. A one-to-one mapping is often not of concern in phase functions since enhancement of the correlation of the spoke edge with target objects and suppression of the correlation of the spoke edge with background objects are the usual design constraints. The reticle shown in Fig. 8(b) has an inverse exponential phase function. This type of phase function may be useful in giving good correlation with straight line targets near the reticle periphery while discriminating against straight line targets near the reticle center by providing a poor correlation. The reticle shown in Fig. 8(c) is a common reticle in applications of cloud filtering. The higher frequency variation on the edges of the reticle spokes appears to filter out clouds, horizons, and other large objects well, while maintaining a good correlation with smaller targets of interest.

All the reticles presented in this section have been reticles of variation in phase only. Given this insight in phase functions, it is possible to combine the phase function with the other reticle parameters in the transmission function. Now, for any given three reticle parameters, $f(\theta)$, $m(r)$, and $\rho(r)$, the reticle transmission function can be constructed

$$T(r,\theta) = \frac{1}{2} + \frac{1}{2}\cos\left[m(r)\int_0^{\theta+\rho(r)} f(\alpha)d\alpha\right], \tag{19}$$

where the spinning impulse response is found by replacing θ and r with $\theta_0 - \omega t$ and r_o, respectively.

VI. Combinations of Reticle Parameters

Equation (19) is an important relationship in that it relates the FM signals (spatial and temporal) to a reticle transmission function. Given any desired FM signal, a reticle can be constructed to provide that signal given a point object in the FOV. Or conversely, given any FM reticle transmission function, the FM signal or a spinning impulse response can be found.

A convenient means to analyze or synthesize a FM reticle is to make use of its parameter graphs. For example, the simple rising sun reticle can be completely described with the three parameter graphs shown in Fig. 9(a). It has a constant $f(\theta)$, a constant $m(r)$, and a constant zero phase. Figure 9(b) shows the parameter graphs for the Lovell reticle in Fig. 4(a). The Lovell reticle has a constant $f(\theta)$, a linear increase in $m(r)$ as r increases, and a constant zero phase. Figure 9(c) shows the parameter graphs for the reticle in Fig. 8(a). The reticle has constant $f(\theta)$ and $m(r)$ parameters along with a Gaussian phase function. The point is that any FM reticle can be described with the three frequency parameters. And once the parameters are known, a straightforward evaluation of Eq. (19) allows the generation of the reticle transmission function.

Combinations of the three various types of FM reticle have not previously been shown in the literature. The usual treatments were that when one parameter was varied, such as the spoke parameter, the other parameters were held constant. To use a reticle for finding a target in both the radial and azimuth directions, at the very least, nonconstant $f(\theta)$ and $m(r)$ parameters must be imposed on the reticle. Then, to correlate the reticle to a desired target and decorrelate the reticle to background clutter, a phase function must be imposed on the reticle.

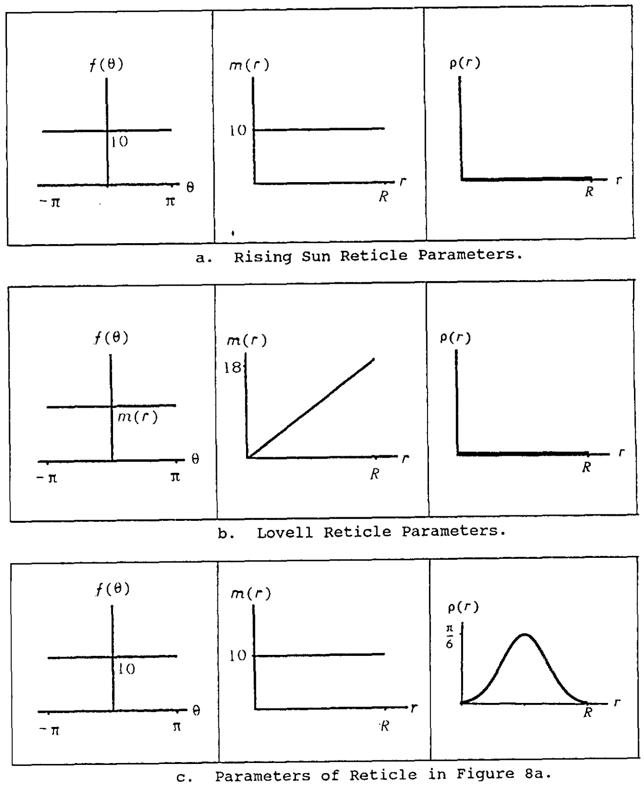


Fig. 9. Reticle descriptions using FM parameters.

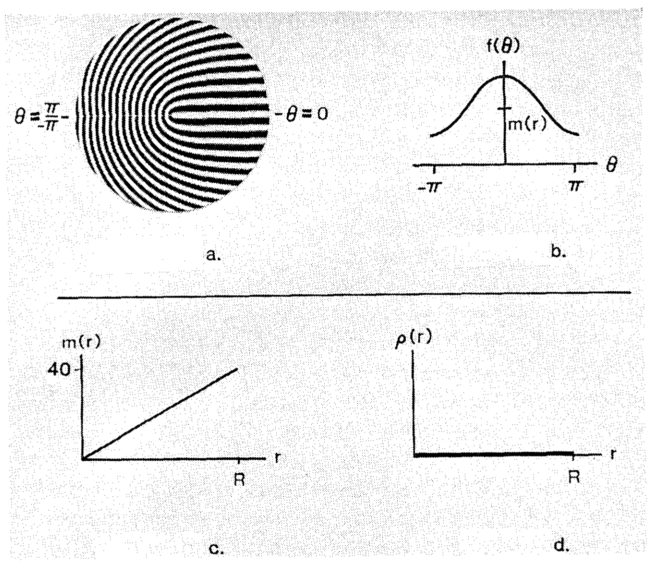


Fig. 10. Marriage with two nonconstant parameters.

To illustrate the integration of two nonconstant parameters, consider the reticle in Fig. 10(a). Figures 10(b), (c), and (d) show the $f(\theta)$, $m(r)$ and $\rho(r)$ parameters, respectively. The $f(\theta)$ parameter is of the sinusoidal form and angular target location information is provided through the phase offset of the angular frequency variation. The base frequency of the angular frequency variation is contained in the $m(r)$ parame-

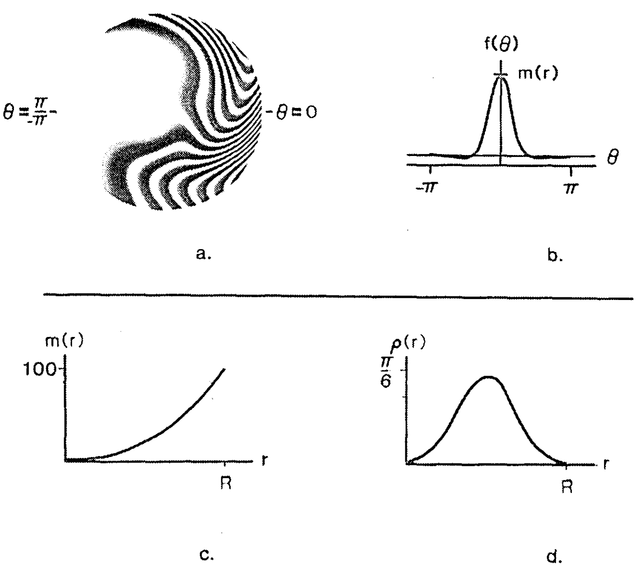


Fig. 11. Marriage with three nonconstant parameters.

ter. The $m(r)$ parameter increases linearly as the radial target location increases. The phase function for the reticle was held constant and equal to zero. The transmission function of the reticle was evaluated to be

$$T(r,\theta) = \frac{1}{2} + \frac{1}{2}\cos\left\{40\,\frac{r}{R}\,[\theta + 0.4\,\sin(\theta)]\right\}. \qquad (20)$$

This reticle appears to be a powerful reticle in that it provides a FM detector signal that is unique for every point source target location in the FOV. In fact, the reticle can be altered to match the electronic specifications of a tracking system. The $m(r)$ slope and offset along with the reticle spin rate can be fixed to correspond with the tracking system electronics bandwidth. Also, the modulation index of the angular frequency variation can be set to provide a phase signal compatible with the electronics.

There are an infinite number of parameter combinations. Many combinations can be found that are more elaborate than the reticle shown in Fig. 10. The reticle in Fig. 11 has a combination of parameters where none of the parameters was held constant. The $f(\theta)$ parameter is of Gaussian form that is offset by a Gaussian phase over the radial extent of the reticle. The Gaussian phase function can be clearly seen by taking the white bar at $\theta = 0$ point at the reticle periphery and traversing it to the center of the reticle. The $m(r)$ function is of squared form where the frequency vs radius increases toward the periphery. The bar pair size at the $\theta = 0$ point on the reticle periphery is such that 100 cycles would fit around the reticle parameter. However, the bar pair size increases at a one over Gaussian rate from the $\theta = 0$ point.

It easy to get carried away and combine parameters that may not provide a useful tracking FM reticle. Each parameter must be evaluated closely to determine whether it can provide a useful signal for a particular tracking system and target/background scenario. For example, the spoke size of a reticle may have to be large for a specific application of a system to track large targets. The large spoke limitation places an upper limit on $m(r)$. However, once the three parameters are found to provide a useful signal, the reticle can be generated using Eq. (19).

VII. Conclusion

Three parameters have been presented that completely describe any spinning type FM reticle. The parameters were frequency vs angle, frequency vs radius, and phase or spoke function of the reticle. A simple relationship was shown relating the parameters to a reticle transmission function. It was also shown that a spinning impulse response of the reticle could be found by replacing the spatial variables of the transmission function, θ and r, with $\omega t + \theta_0$ and r_o, respectively. ω was the angular spin rate of the reticle and (θ_0, r_0) was the spatial location of the impulse or point source. The temporal frequency of the output of the single detector placed behind the reticle could be found by taking the derivative of the spinning impulse response with respect to t.

Since there is no one universal reticle that can satisfy all applications, each parameter must be individually specified for a particular application. Each parameter must be analyzed to give an optimum signal on the output of the detector due to a desirable target in undesirable background clutter. Typical questions to be answered in the specification of reticle parameters are as follows: What type of spoke edges correlate well with the desired target while filtering backgrounds? What is the optimum size of the spokes for the desired target size? What bandwidth does the detector electronics operate at? Is a higher resolution desired at particular regions of the system FOV? Once the parameters are picked, it is a simple task to generate the reticle.

Two areas that may be interesting for future study using the three FM reticle parameters are as follows: The first is the application of reticle theory to spatial light modulators (SLMs). It appears that with insight into the three reticle parameters, algorithms for modulation schemes of tracking systems using SLMs are similar to the requirements and limitations of FM reticle design. The second area is the marriage of focal plane geometries to reticle geometries. This includes the marriage of SLMs to focal plane geometries. The insight of reticle parameters may provide a means to decide the optimum focal plane geometries for marriage to certain types of reticle. This can even include adaptive filtering since the capabilities of SLMs allow an effective reticle pattern to change over time to optimize the tracking response of regions that contain candidate targets.

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