Parameters of spinning AM reticles

Ronald G. Driggers, Carl E. Halford, and Glenn D. Boreman

A new method of obtaining amplitude modulation (AM) for determining target location with spinning reticles is presented. The method is based on the use of graded transmission capabilities. The AM spinning reticles previously presented were functions of three parameters: amplitude vs angle, amplitude vs radius, and phase. This paper presents these parameters along with their capabilities and limitations and shows that multiple parameters can be integrated into a single reticle. It is also shown that AM parameters can be combined with FM parameters in a single reticle. Also, a general equation is developed that relates the AM parameters to a reticle transmission equation. *Key words:* Reticles, tracking devices, AM modulation.

I. Introduction

Amplitude modulation (AM) in reticles has been studied[1,2] as a means of providing target position information to tracking systems. The production of an AM signal in reticles has been different from production of AM signals in other systems. This difference results in part from a lack of variable transmission characteristics in infrared reticle materials. This paper presents a simple and direct approach of obtaining an AM signal with reticles using parameters similar to the parameters involved in FM modulation.[3]

Consider the historical[2] case of amplitude modulation in spinning reticles as shown in Fig. 1(a). A target is imaged onto the spinning reticle and the target image is transmitted through the transparent white sectors and blocked by the opaque dark sectors. Usually, the spatial integral of the light is collected by a single detector and a temporal signal from the detector is analyzed for target location information. If a circular target were located at the center of the spinning reticle (location A), the amount of image light transmitting through the reticle does not change with the spinning motion of the reticle. Hence, the spatial integral of the light transmitting through the reticle would be a dc value and the modulation (ac response) would be zero. A dc signal would correspond to a target located at the reticle center. As the target is moved from the center of the reticle, the reticle begins to chop the target image and the modulation grows as the target is moved towards the reticle periphery. It can be seen that once the target passes beyond the radius at which it overlaps one sector, the modulation is at a maximum and remains constant to the reticle periphery. It is not obvious, however, that the modulation goes through peaks and troughs as the target is moved from the center of the reticle to the location of the modulation peak. The peak occur when the target image overlaps an odd number of sectors and the troughs occur when the target overlaps an even number of sectors. Consider target location B where the target image overlaps an even number of sectors. As the reticle spins, a white sector is leaving the target area and a dark sector is entering the target area resulting in a large change in transmitted image light. Now consider the target location C where the image size corresponds to an even sector overlap. A white sector is leaving the target area and a white bar is entering the target area, minimizing the change in transmitted light (modulation). If the modulation were plotted as a function of radial target location, a curve similar to the one shown in Fig. 1(b) would be generated.

It can be seen that the modulation curve depends upon target size. Also, for small targets, or point sources, the AM configuration is useless in generating an error signal for target location since the modulation peaks at a very small distance from the reticle center. A method for obtaining an AM signal for any size target is presented in this paper. The ability to grade the transmission characteristics of the reticle materials is assumed, but the grading of transmission is feasible even in infrared materials using halftone techniques.[4] With this assumption, spinning AM reticles can be described using three amplitude parameters

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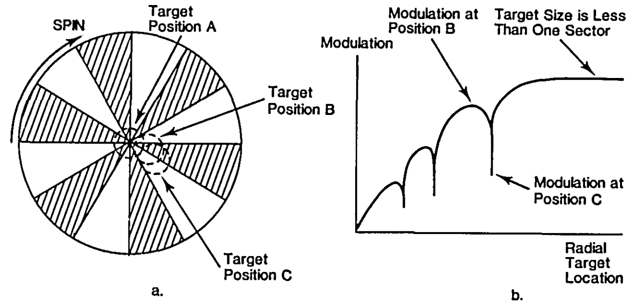


Fig. 1. Classical amplitude modulation in reticles.

(similar to FM parameters[5]): amplitude vs angle, $f(\theta)$, amplitude vs radius, $g(r)$, and phase, $\rho(r)$.

The following sections describe these parameters in detail. After the parameters are presented, a combination of the parameters are shown to provide an error signal useful in tracking targets. The last section shows how AM can be combined with FM in reticles to provide improved error signals without the constraints of pure AM signals.

II. Amplitude vs Angle

The amplitude vs angle parameter encodes azimuth target location. Consider the fan blade reticle shown in Fig. 2. There are thirty angular cycles in the transmission function as θ traverses from $-\pi$ to π. If a point source were imaged onto the reticle and the reticle were spinning in the direction shown, the reticle would modulate the point source light at a rate of thirty cycles per reticle rotation time. This frequency can be considered the carrier frequency k for all the reticles presented in this paper. Amplitude modulation can be described by the equation

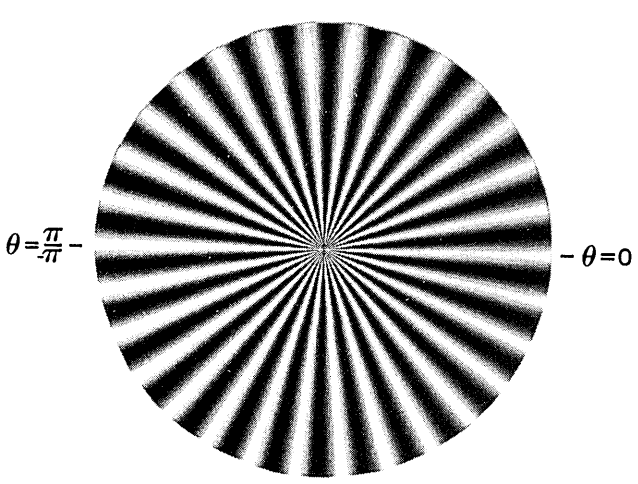
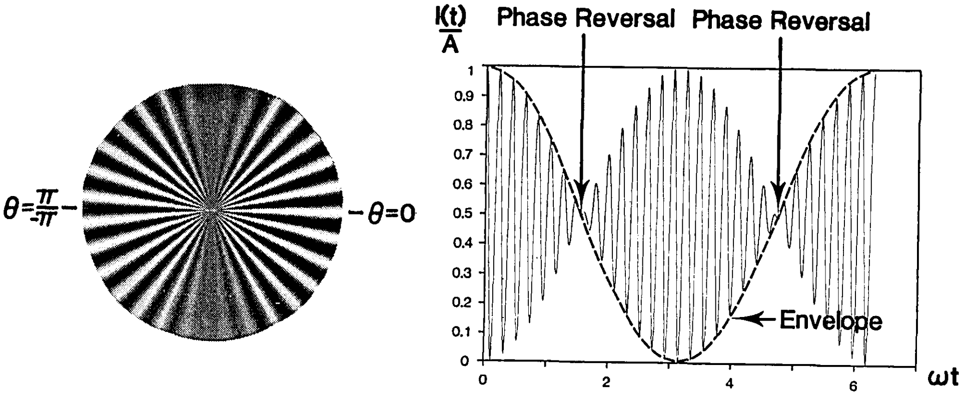
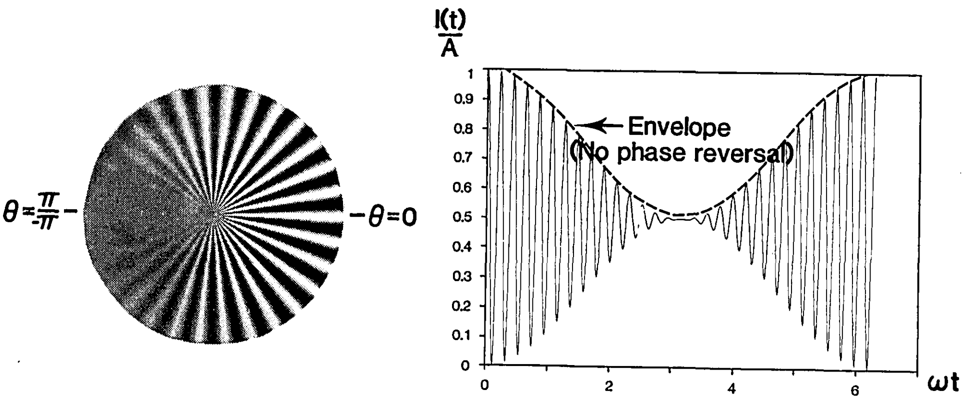


Fig. 2. Fan blade reticle.



a. Reticle one.

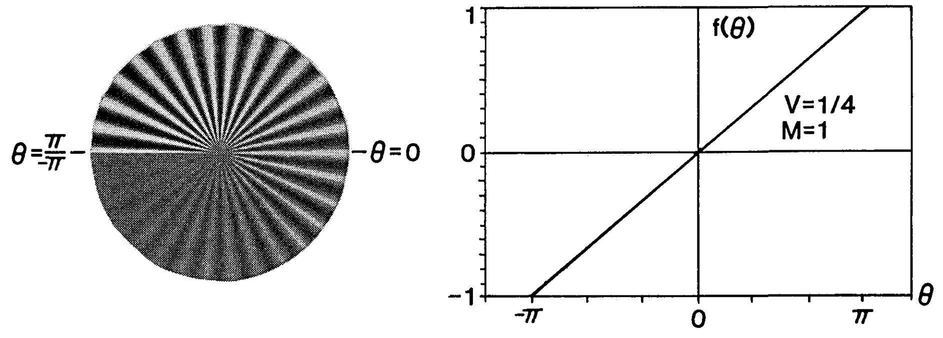
b. Spinning impulse response for reticle one.



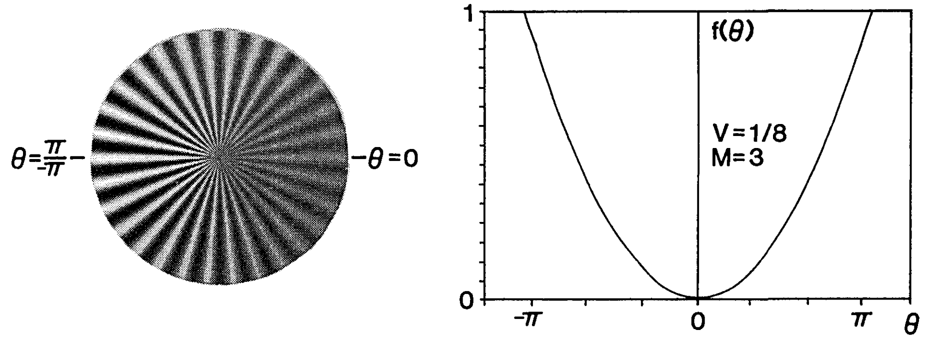
c. Reticle two.

d. Spinning impulse response for reticle two.

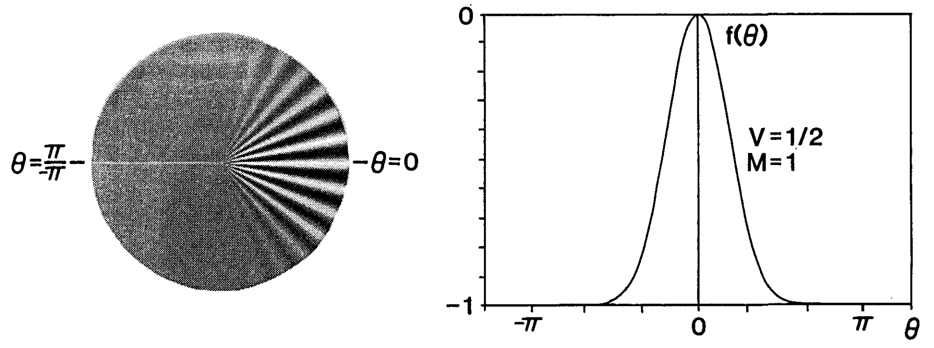
Fig. 3. Two angular AM reticles.



Reticle a.



Reticle b.



Reticle c.

Fig. 4. Various amplitude vs angle reticles.

$$S(\theta) = \tfrac{1}{2} + V[1 + mf(\theta)] \cos(k\theta), \tag{1}$$

where $s(\theta)$ is the modulated signal, V is a constant, m is the modulation index, and $f(\theta)$ is the low frequency modulation signal. For the reticle shown in Fig. 2, m is 1, $f(\theta)$ is a constant 1, and V is 1/4. The 1/2 dc term in Eq. (1) allows an average reticle transmission of 1/2 rather than zero (i.e., no light passing the reticle).

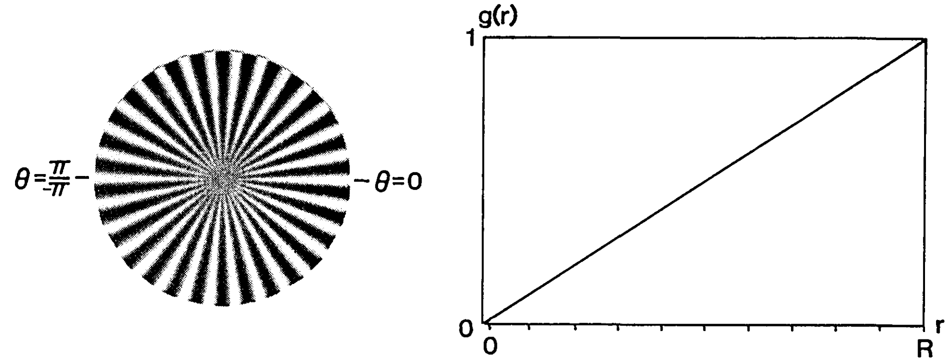
For symbol convention, let us define r and θ as the spatial variables of the reticle transmission function with ranges of 0 to R and $-\pi$ to π, respectively. Also, let the reticle spin rate be ω rad/s and let r_o, θ_0 be the spatial coordinates of a point source that is imaged onto the reticle. Usually, a single detector collects all of the modulated light of the target through the spinning reticle. This light collected is known as the spinning impulse response and can be found by

$$I(t) = \int_0^R \int_{-\pi}^{\pi} T(r,\theta) A\delta[r - r_0, \theta - (\omega t - \theta_0]r d\theta dr, \tag{2}$$

where the impulse function δ represents the spatial extent of the point source with a light brightness amplitude of A. The transmission function of the reticle shown in Fig. 2 is

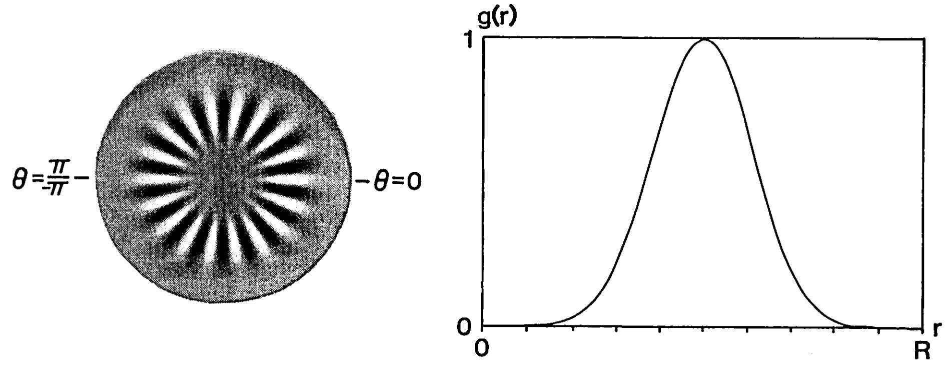
$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}\cos(30\theta). \tag{3}$$

Noting that Eq. (2) is a 1-D correlation function, it can be easily applied to Eq. (3). The spinning impulse response of the reticle shown in Fig. 2 is



a. Reticle one.

b. Amplitude versus radius for reticle one.



c. Reticle two.

d. Amplitude versus radius for reticle two.

Fig. 5. Two radial AM reticles.

$$I(t) = \frac{A}{2} + A\cos[30(\omega t - \theta_0)]. \tag{4}$$

Note the signal is not dependent on r and that there are thirty possible target locations in angle that provide identical spinning impulse response signals.

For an AM reticle to provide angular target location, a variation in amplitude must be imposed on the reticle as a function of angle. That is, an amplitude vs angle parameter is formed. One must be careful in imposing this variation since it is easy to mistake the reticle requirements for angular variations as being similar to FM reticles. Consider the reticle transmission function

$$T(r,\theta) = \tfrac{1}{2} + \tfrac{1}{2}f(\theta)\cos(30\theta), \tag{5}$$

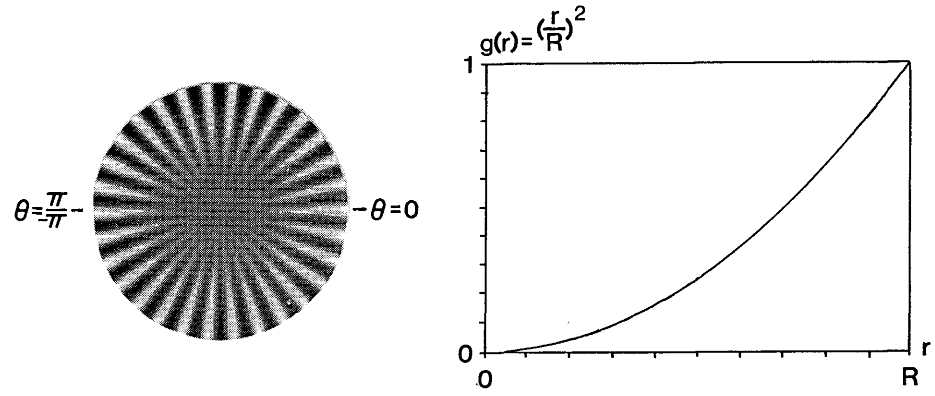
with $f(\theta) = \cos\theta$. The reticle corresponding to the transmission function is shown in Fig. 3(a) with a spinning impulse response for a point source located at $\theta = 0$ shown in Fig. 3(b). There appears to be two periods of modulation in one rotation of the reticle. The modulation envelope of the carrier is the magnitude of the low frequency cosine function as shown in Fig. 3(b). Note there is a phase change (black becomes white and white becomes black) that occurs at $\omega t = \pi/2$ and $\omega t = 3\pi/2$. The phase change occurs when $\cos\theta$ becomes negative causing the amplitude of the carrier to reverse. Unless the tracking system electronics can utilize the phase change for angular target location, the reticle is not useful as an AM tracking reticle. Targets separated by π give identical spinning impulse responses. For this reason, Eq. (5) is not a useful transmission equation for amplitude modulation.

If amplitude modulation in useful tracking reticles is strictly described by Eq. (1), we can place two simple constraints on the equation variables. The magnitude of $V[1 + mf(\theta)]$ cannot be $>1/2$ since this would cause the transmission function to have a value >1. Second, $mf(\theta)$ cannot have a value < -1 since phase reversal of the carrier occurs at this point. If phase reversal is not a problem for tracker electronics, this constraint can be relaxed provided the magnitude of $f(\theta)$ does not contain a period $<2\pi$ as in the case of the reticle shown in Fig. 3(a). To illustrate the results of these constraints, consider the reticle shown in Fig. 3(c). The transmission equation for the reticle is

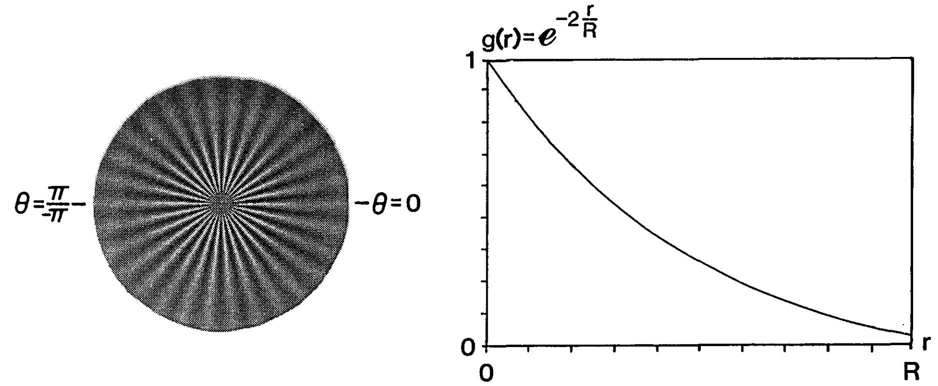
$$T(\theta) = \tfrac{1}{2} + \tfrac{1}{4}[1 + \cos(\theta)]\cos(30\theta), \tag{6}$$

where V is 1/4 and m is 1. The equation satisfies the constraints described above. Hence, the amplitude envelope contains one period per reticle rotation and no phase reversal occurs.

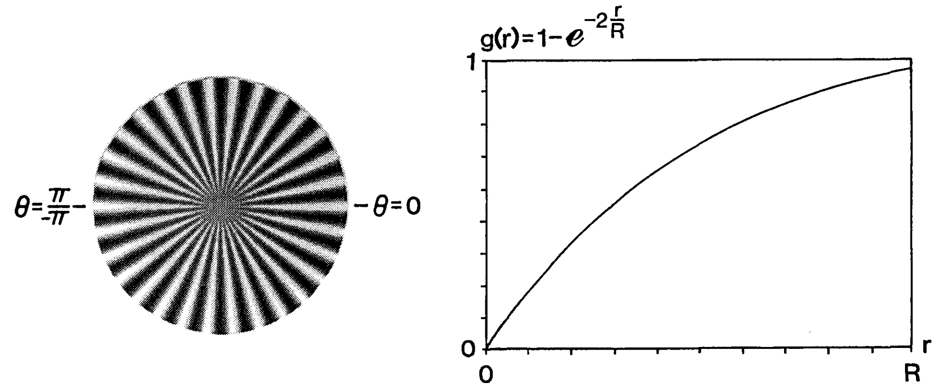
Defining $f(\theta)$ described in Eq. (1) as the amplitude vs angle parameter will allow insight into AM reticles through a few examples. Consider the reticles shown in Fig. 4. Reticle 4a includes a linear increase in modulation as θ varies from $-\pi$ to π. V, m and $f(\theta)$ for the reticle are 1/4, 1, and θ/π, respectively. The reticle shown in Fig. 4(b) has an $f(\theta)$ that is proportional to the



Reticle a.



Reticle b.



Reticle c.

Fig. 6. Various amplitude vs radius reticles.

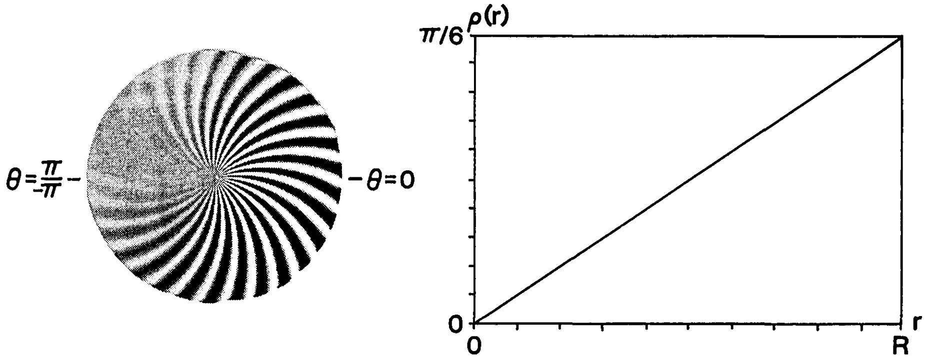
square of the reticle angle. V, m, and $f(\theta)$ for this reticle are 1/8, 3, and $(\theta/\pi)^2$, respectively. The last example shown, reticle 4c, has a Gaussian amplitude vs angle parameter. Since the range of $f(\theta)$ is -1 to 0 and m is 1, the value for V is set at its maximum constraint value of 1/2.

It should be noted that $f(\theta)$ for useful tracking reticles can be any function with the limitations described in this section. These are the minimum requirements for a useful amplitude vs angle parameter. Other considerations include image size, reticle fabrication, and tracker electronics.

III. Amplitude vs Radius

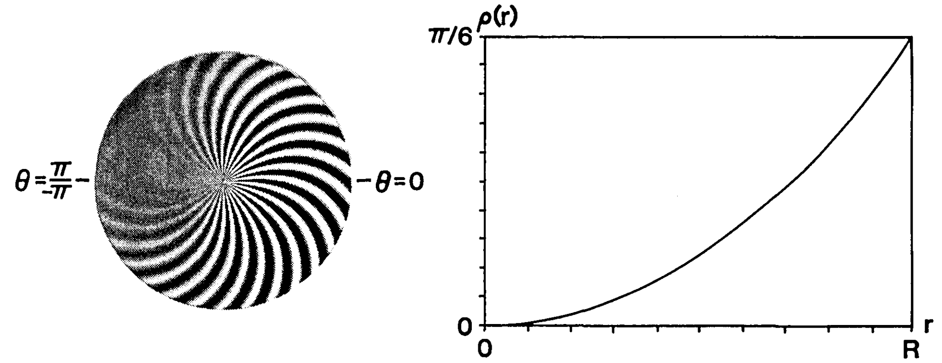
Radial target location is provided by the amplitude vs radius parameter, $g(r)$. Consider the transmission equation

$$T(r,\theta) = \tfrac{1}{2} + Vg(r)[1 + mf(\theta)]\cos(k\theta), \tag{7}$$



a. Reticle one.

b. Phase for reticle one.



c. Reticle two.

d. Phase for reticle two.

Fig. 7. Two phase AM reticles.

where $g(r)$ has a 0–1 range. The transmission function is now a function of both r and θ. Also, $g(r)$ weights the reticle modulation as a function of radius. Since this section describes the radial parameter, let $mf(\theta)$ equal one and let V equal 1/4 for the reticles in this section. Now, for illustrative purposes, a linear increase in modulation as a function of radius will be imposed on reticle Eq. (7). That is, we will let $g(r)$ equal r/R. With a carrier frequency of thirty cycles per rotation, the transmission equation becomes

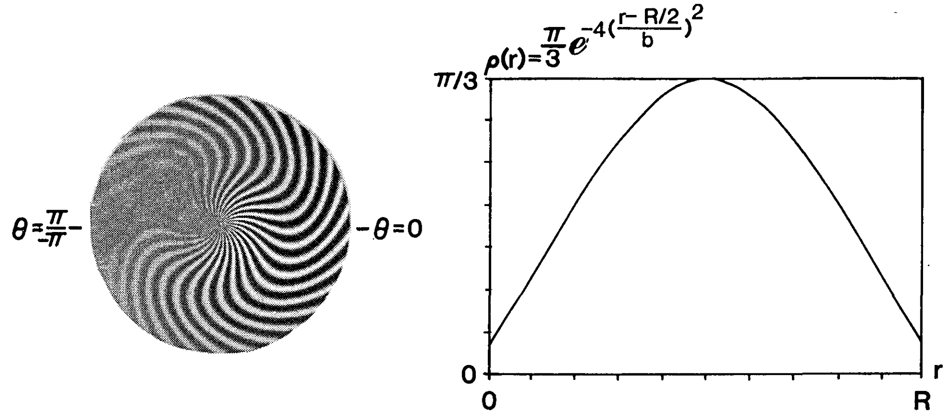
$$T(r,\theta) = \frac{1}{2} + \frac{1}{2}\frac{r}{R}\cos(30\theta). \tag{8}$$

The reticle corresponding to the transmission equation is shown along with its $g(r)$ in Figs. 5(a) and (b). The one-to-one linear mapping of r to $g(r)$ allows an increase in the amplitude modulated signal of a point source as its radial location increases. Hence, one signal amplitude corresponds to one radial target location. This one-to-one mapping is required since the function is nonperiodic contrary to the case of $f(\theta)$. A case where two radial target locations can give the same amplitude modulated signal is shown by the reticle and $g(r)$ function in Figs. 5(c) and (d), respectively. The amplitude vs radius function $g(r)$ is a Gaussian function centered on $R/2$. For every radial target location on one side of $R/2$, there is another radial target location on the other side of $R/2$ that corresponds to an identical amplitude modulated signal. Therefore, the reticle would not be useful as a radial target locater when coupled to a single detector.

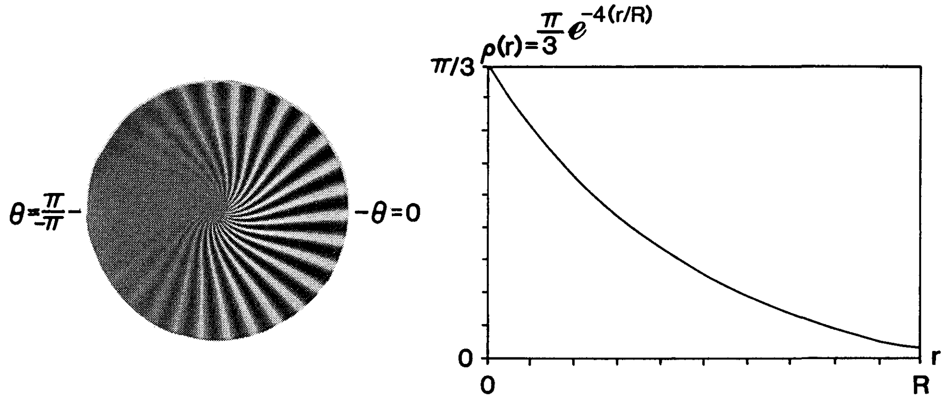
To provide more insight to the amplitude vs radius parameter, a number of examples are presented in Fig. 6. The reticle shown in Fig. 6(a) has an amplitude vs radius function that depends on the square of the reticle radius. The reticle shown in Fig. 6(b) has a negative exponential function for amplitude vs radius. Therefore, the modulation of a target towards the reticle center is larger than the modulation of a target near the reticle periphery. The last reticle example, shown in Fig. 6(c), has an increasing exponential amplitude vs radius function. The modulation increases sharply from the reticle center toward the reticle periphery.

IV. Phase

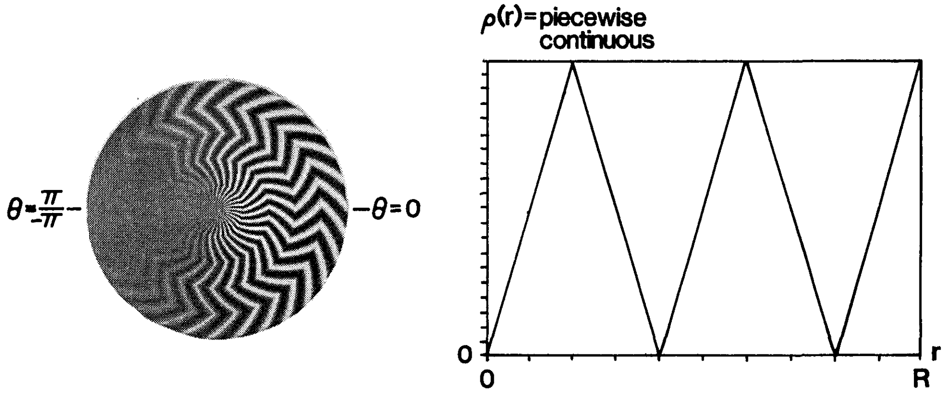
The final parameter associated with AM modulated reticles is the phase function. In FM reticles, this parameter is often referred to as the spoke function since it shapes the geometry of the reticle spokes (bars). The reticle transmission functions presented in this paper so far have been written with respect to some phase reference. This phase reference has been the $\theta = 0$ line on the reticle. The phase function can be considered the line from which the reticle transmission function is referenced. This line changes in angle as a function of radius. The phase, $\rho(r)$, is the line where the arguments of the angular functions are zero and $T(r,\theta) = 1$.



Reticle a.



Reticle b.

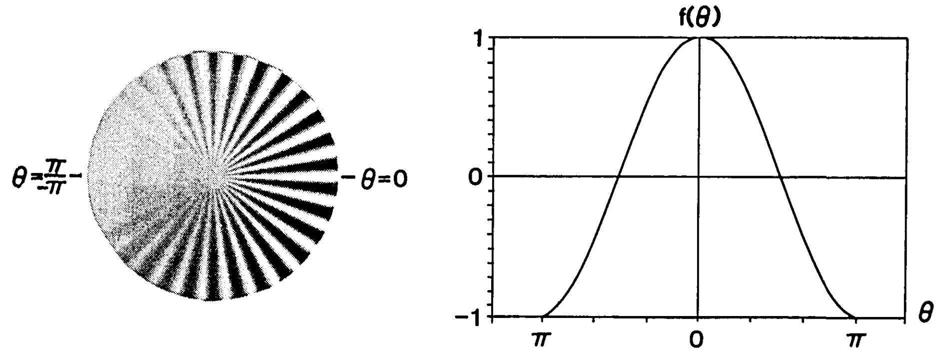


Reticle c.

Fig. 8. Various phase reticles.

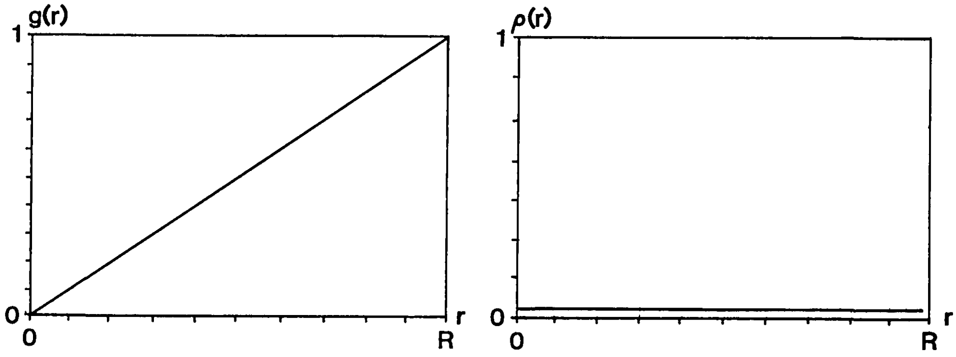
The purpose of imposing a phase on a reticle is that it changes the reticle bar shapes in such a manner that the reticle can correlate to various images well and other images poorly. For instance, a reticle can be designed so that it correlates with small targets well and correlates with large targets poorly. This type of reticle would be useful in tracking small objects in cloud clutter.

Consider the reticle shown in Fig. 7(a) along with its phase function shown in Fig. 7(b). The reticle has a cosine amplitude vs angle function with a linear phase. The cosine amplitude vs angle parameter was retained so that the variation in the angular modulation can be seen to follow the same curves as that of the carrier spokes. That is, the modulation in angle curves around in the same manner, as the reticle bars. To keep this in mind, all of the reticles presented in the Phase Sec. will have a cosine amplitude vs angle pa-



a. Reticle.

b. Amplitude versus angle.



c. Amplitude versus radius.

d. Phase.

Fig. 9. Reticle with two nonconstant AM parameters.

rameter. Since the amplitude vs radius parameter is not affected by phase, the parameter is not imposed on the reticle here.

To visualize the phase function of reticle 7a, consider the phase graph of Fig. 7(b). When the reticle radius is zero, the phase shift in angle is zero and the reticle is the same as the reticle shown in Fig. 3(c). Now, when the radius is $R/2$, the reticle transmission function is shifted by $\pi/(12)$ and the bars curve to meet the $T(r,\theta) = 1$ or $\theta + \rho(r) = 0$ point at the radius. At a radius equal to R, the transmission function has curved to $\pi/6$. In comparison, the reticle shown in Fig. 7(c) has a squared phase function as shown in Fig. 7(d). The squared function effect can be easily seen since the bars toward the center of the reticle do not curve as much as toward the periphery of the reticle.

The phase parameter can be imposed on the reticle in the following equation:

$$T(r,\theta) = \tfrac{1}{2} + Vg(r)[1 + mf(\alpha)]\cos(k\alpha)\big|_{\alpha=\theta+\rho(r)}. \tag{9}$$

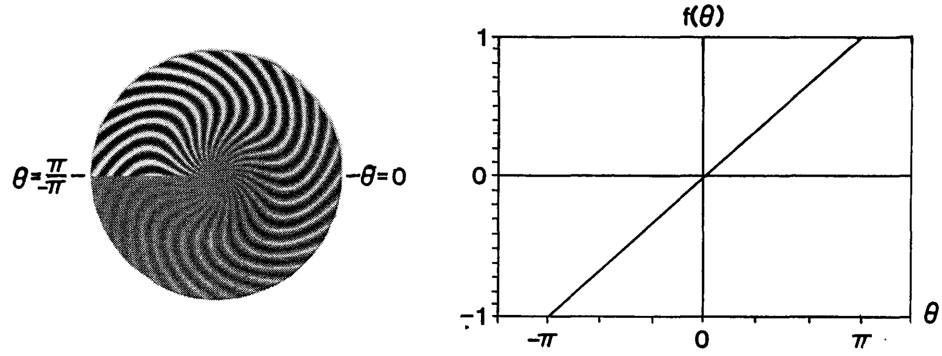
For the reticles shown in this section, V is 1/4, $g(r)$ is 1, m is 1, and $f(\theta) = \cos\theta$. To illustrate the impact of the phase function on the reticle, a few examples are shown in Fig. 8. The first reticle, Fig. 8(a), has a Gaussian phase function. A one-to-one mapping is often not of concern in phase functions since enhancement of the correlation of the spoke edge with target objects and suppression of the correlation of the spoke edge with background objects are the usual design goals. The reticle shown in Fig. 8(b) has a negative exponential phase function. This type of phase function may be useful in giving good correlation with straight line targets near the reticle periphery while discriminating against straight line targets near the reticle center. The reticle shown in Fig. 8(c) may be useful in filtering clouds, horizons, and other large objects while maintaining a good correlation with small objects.

All of the reticles presented in this section have some type of phase parameter. With the insight into phase functions, it can now be shown that the three amplitude parameters can be combined using Eq. (9) to construct an AM reticle transmission function.

V. Combinations of AM Parameters

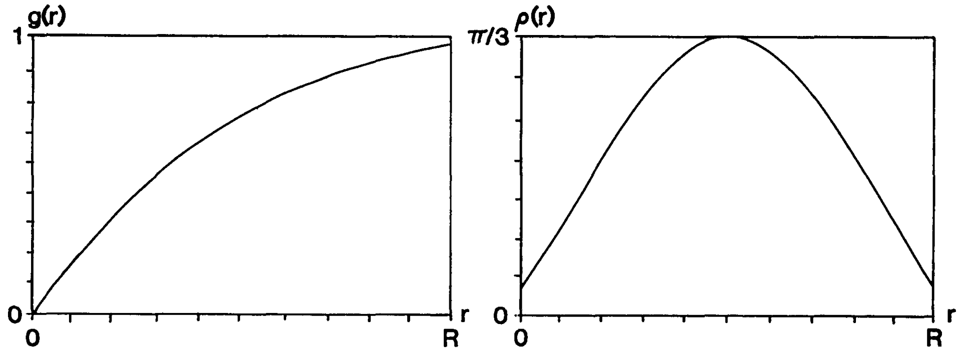
Equation (9) can be used to construct an AM reticle with multiple parameters. That is, a reticle transmission function can be developed that determines both angular and radial target locations while correlating well with targets and poorly with backgrounds using amplitude modulation. The envelope function can be any signal desired with the simple constraints described in the previous sections. Once the three parameters are known, the reticle transmission function can be written.

Two examples are presented in this section that show combinations of AM reticle parameters. The first example is the reticle shown in Fig. 9. The reticle has a cosine amplitude vs angle parameter and a linear amplitude vs radius parameter. The phase is held at a



a. Reticle.

b. Amplitude versus angle.



c. Amplitude versus radius.

d. Phase.

Fig. 10. Reticle with three nonconstant AM parameters.

constant zero value. The transmission function for the reticle was found using Eq. (9) to be

$$T(r,\theta) = \frac{1}{2} + \frac{1}{4}\frac{r}{R}[1 + \cos\theta]\cos(30\theta). \qquad (10)$$

In this case, target location is found by determining the phase offset of the cosine envelope for angular location and the magnitude of the AM envelope determines the radial target location. Therefore, the reticle is useful in determining target location. The detector output due to a point source located at (r_0,θ_0) imaged onto a spinning reticle with the transmission function described would be Eq. (9) with (r,θ) replaced by $(r_0,\omega t - \theta_0)$

The second example of AM parameter combinations is shown in Fig. 10. The reticle is a combination of three nonconstant AM parameters. The amplitude vs angle parameter is linear, the amplitude vs radius parameter is one minus a negative exponential, and the phase is Gaussian as shown in Figs. 10(b), (c), and (d), respectively. The angular target location is found by the phase offset of periodic ramp envelope and the radial target location is found by the magnitude of the ramp envelope. The phase decorrelates the reticle with straight lines, especially near the $R/2$ radial location. The reticle transmission function was found again using Eq. (9) to be

$$T(r,\theta) = \frac{1}{2} + \frac{1}{4}\left[1 - \exp\left(-2\frac{r}{R}\right)\right]\left\{1 + \frac{[\theta + \rho(r)]}{\pi}\right\} \times \cos\{30[\theta + \rho(r)]\}, \qquad (11)$$

where

$$\rho(r) = \frac{\pi}{3}\exp\left[-4\left(\frac{r - \frac{R}{2}}{b}\right)^2\right],$$

and b was $\sim R/5$.

There are important considerations in designing an AM reticle that must be considered before selecting the AM parameters. A few of these considerations are target size, background geometries, electronics bandwidths, amplitude normalization, and others. However, once the AM parameters are determining using these considerations, the reticle transmission function can be found directly using Eq. (9).

VI. Combination of FM and AM Parameters

An alternative to the use of both angular and radial AM parameters involves a combination of AM and FM reticle parameters. Consider the case where the amplitude vs angle parameter is retained and the amplitude vs radius parameter is replaced with a frequency vs radius parameter. The phase parameter affects AM and FM reticles in the same manner. The use of this type of configuration allows the elimination of the magnitude of the AM envelope as a target location device. A change in target frequency (i.e., carrier frequency) will correspond to a change in target radial location. The angular target location is still determined by the phase of the AM envelope.

Consider the reticle shown in Fig. 11. The carrier

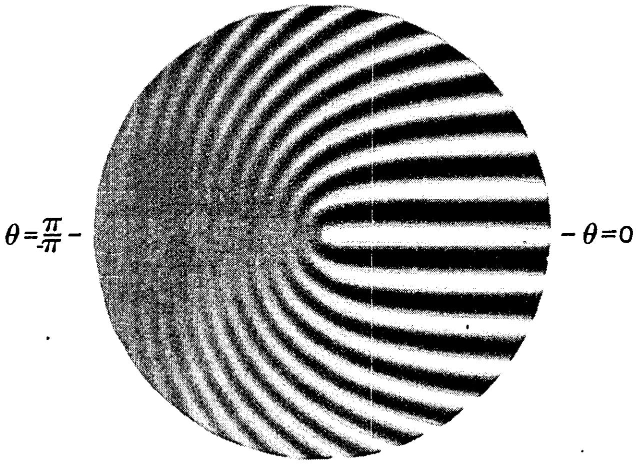


Fig. 11. Reticle with AM amplitude vs angle parameter and FM frequency vs radius parameter.

frequency increases linearly with radial target location and the amplitude vs angle parameter varies as a cosine. The phase imposed on the reticle is a constant (zero). The transmission function for the reticle is

$$T(r,\theta) = \frac{1}{2} + \frac{1}{4}(1 + \cos\theta)\cos\left(30\frac{r}{R}\theta\right). \tag{12}$$

The reticle appears to be extremely useful in that an increased accuracy can be obtained in the following manner. Once a point source is imaged onto the reticle, the radial target location can be determined from the value of the carrier frequency. A rough angular target location is then determined by comparing the phase of the amplitude modulated envelope to the reticle reference signal (chopper electronic signal or LED reference). The resolution in this rough angular target location is determined by the accuracy of electronic phase discriminators. Now, since the carrier frequency is known, the phase of the target carrier frequency is compared to the reference signal. It is known that using the phase of the carrier signal will give a number of ambiguous target locations equal to the number of cycles per reticle rotation at the target radius. However, since an approximate angular location has been determined, the high resolution carrier phase location used will correspond to the approximate angular location. The angular location resolution for the same electronic phase discriminator has increased by the number of cycles per rotation at the target radius. For example, for a target radius of $R/2$ for the reticle shown, the angular location resolution has increased by a factor of 15.

VII. Conclusion

It has been shown that AM spinning reticles can be described using the three amplitude modulation parameters: amplitude vs angle, amplitude vs radius, and phase. The amplitude vs angle parameter, the amplitude vs radius parameter, and the phase parameter give angular target location, radial target location, and correlation, respectively. Once the parameters have been determined, a reticle transmission function can be developed using Eq. (9). Also, it has been shown that amplitude modulation can be combined with frequency modulation in reticles to provide a signal without amplitude normalization while increasing angular target location resolution.

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