

# Effect of phasing-sector angular extent in FM reticles

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A technique is presented for increasing the useful power of the FM modulation signal from a spinning FM reticle with a phasing sector. Spinning FM reticles often determine a target location by using a combination of a phasing sector to establish the angular target location and a radial frequency variation to establish the radial target location. Typically, the phasing sector of this reticle type consists of a semicircular transmissive sector with the other semicircular sector which provides FM modulation. The power of the FM modulation signal is increased by matching the phased-sector geometry to the size that corresponds to the period of the modulation frequency. This type of reticle is compared and contrasted with the more typical semicircular phasing-sector reticle in both the time and frequency domains.

*Key words:* Reticles, frequency modulation, phase modulation.

## 1. Introduction

The introduction of a phasing sector into an FM spinning reticle permits complete target location determination<sup>1</sup> (i.e., radial and angular coordinates). This hybrid form of angle modulation produces frequency modulation (FM) and phase modulation and is analyzed here. A spectral analysis of the FM-PM spinning-reticle modulator is presented. A particular case of this reticle modulator was designed and patented by Lovell in 1960<sup>2</sup> and is shown in Fig. 1. This reticle is divided into two semicircular halves; the top half encodes the radial location (FM), and the bottom half encodes the angular location (PM). A general case of the FM-PM concept is presented in which the phasing sector occupies a number of the FM modulating bars. Also, the spectral power is shown at the modulating frequency for the case in which the phasing sector occupies one modulating bar pair.

## 2. Signal Description

Consider the signal for a point of light imaged upon the spinning Lovell reticle, as shown in Fig. 2. The

transmitted temporal signal can be thought of in terms of three functions. The first function,  $x_1(t)$ , is the frequency modulation of the reticle, and it encodes the radial target location. It can be seen from the reticle that the modulating frequency  $f_s$  is dependent on the radial target location. Although it does not appear so, an arc on a particular radius that traverses through the FM sector of the reticle intersects a perfect square wave of a single fundamental frequency, as shown in Fig. 2(b). The signal shown in Fig. 2(b) can be described as

$$x_1(t) = \left[ \sum_{i=-\infty}^{\infty} \delta(t - iT_s) \right] * \text{rect}\left(\frac{2t}{T_s}\right) - \frac{1}{2}, \quad (1)$$

where \* denotes the one-dimensional convolution, the  $\delta$  function corresponds to the impulse function,<sup>3</sup> and  $T_s$  is  $1/f_s$ . The second function, shown in Fig. 2(c), corresponds to the window in which the frequency sector is modulating the signal. The function has a value of zero when the phasing sector is traversing the target. The function can be written as

$$x_2(t) = \left[ \sum_{k=-\infty}^{\infty} \delta(t - kmT_s - \phi) \right] * \text{rect}\left(\frac{2t}{mT_s}\right), \quad (2)$$

where the  $\delta$  function positions the window at one time per reticle rotation, i.e., a time equivalent to that for traversing  $m$  modulating bars. The window, as in the Lovell reticle case, is shown to be  $m/2$  bars wide. For informative purposes we take the general case of a window that includes  $n$  modulating bar

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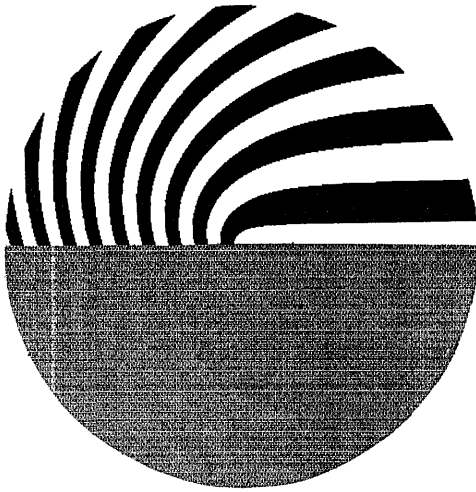


Fig. 1. Lovell reticle.

pairs:

$$x_2(t) = \left[ \sum_{k=-\infty}^{\infty} \delta(t - kmT_s - \phi) \right] * \text{rect}\left(\frac{t}{nT_s}\right). \quad (3)$$

The final component of the signal is shown in Fig. 2(d), which shows the dc component that is added to the signal shown in Fig. 2(a). The integrated signal shown in Fig. 2(a) can be written in terms of the component signals described above:

$$x(t) = x_1(t)x_2(t) + x_3(t), \quad (4)$$

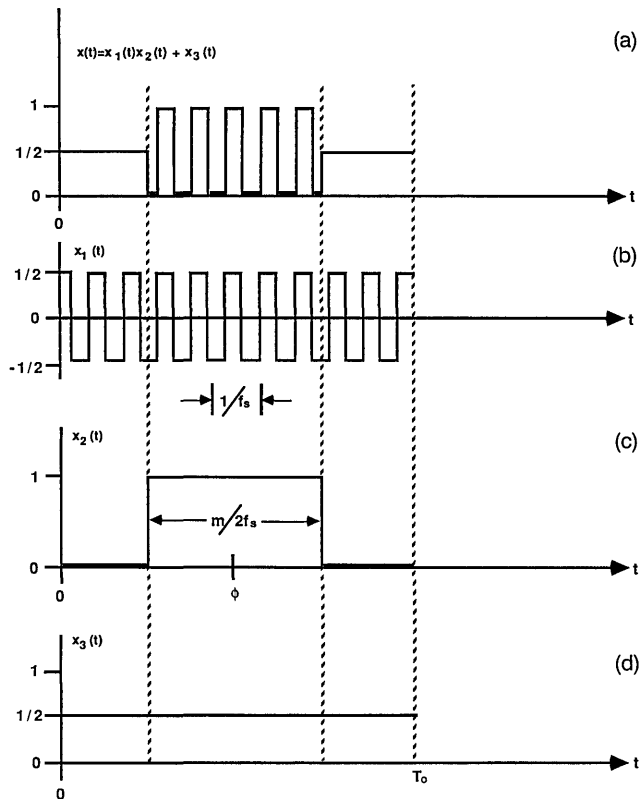


Fig. 2. Lovell reticle detector output.

where  $x_3$  is equal to  $1/2$ . We give the spectral analysis of this signal in Section 3.

### 3. Spectral Analysis

The signal spectrum of the reticle can also be written in terms of the signal components of Eq. (4) by using the Fourier superposition and convolution theorems:

$$X(\xi) = X_1(\xi) * X_2(\xi) + X_3(\xi), \quad (5)$$

where the components shown are the spectral components for each of the three signals. Also,  $X_3(\xi)$  is a  $\delta$  function of half the unit area and is located at the origin of the frequency spectrum. This term represents the dc component of the temporal signal and is present for any reticle signal. Since this term is always present given any reticle configuration, it is not of interest in a comparison of reticles. It is the power at higher frequencies that is of interest.

The spectral terms of interest are  $X_1(\xi)$  and  $X_2(\xi)$ . The temporal signal  $x_1(t)$  can be written in terms of the comb function:

$$x_1(t) = \frac{1}{T_s} \text{comb}\left(\frac{t}{T_s}\right) * \text{rect}\left(\frac{2t}{T_s}\right) - \frac{1}{2}, \quad (6)$$

where the scaled comb function is equivalent to the previously written sum of unit-area delta functions located  $T_s$  apart. The spectrum of Eq. (6) is

$$\mathcal{F}\{x_1(t)\} = X_1(\xi) = [T_s \text{comb}(\xi T_s)] \times \left[ \frac{1}{2} \text{sinc}\left(\frac{\xi T_s}{2}\right) \right] - \frac{1}{2} \delta(\xi), \quad (7)$$

where the  $\mathcal{F}$  operator denotes the Fourier transform. The spectrum of this signal is shown in Fig. 3. Note that the comb component at the origin of the spectrum cancels with the negative delta function toward the end of Eq. (7) for the case of no dc signal component. The components left in the spectrum are the fundamental, the third harmonic, and so on. This is exactly the spectrum expected for a square wave with no dc component.

The spectrum of the second signal  $x_2(t)$  can also be written in terms of the comb function:

$$x_2(t) = \frac{1}{mT_s} \text{comb}\left(\frac{t - \phi}{mT_s}\right) * \text{rect}\left(\frac{2t}{mT_s}\right). \quad (8)$$

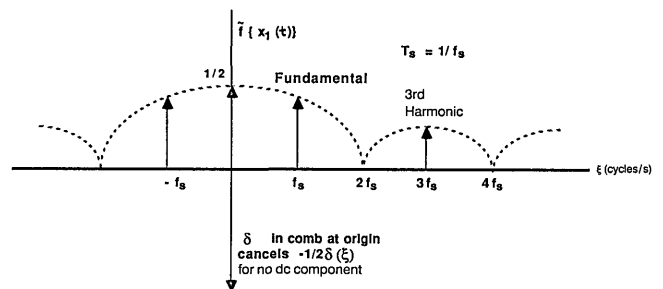


Fig. 3. Spectrum of  $x_1(t)$ .

However, instead of letting the window include half a reticle rotation (i.e.,  $m/2$  modulating bar pairs), let the window take the general case of including  $n$  modulating bar pairs:

$$x_2(t) = \frac{1}{mT_s} \text{comb}\left(\frac{t - \phi}{mT_s}\right) * \text{rect}\left(\frac{t}{nT_s}\right). \quad (9)$$

The spectrum of the signal  $x_2(t)$  is

$$X_2(\xi) = [mT_s \text{comb}(mT_s\xi)] \times \left(\frac{n}{m}\right) \text{sinc}(nT_s\xi) \exp(-j2\pi\phi\xi). \quad (10)$$

Note that the maximum amplitude of this function is  $n/m$  and occurs at the origin of the spectrum. The effect of this function on the original FM spectrum (i.e.,  $X_1$ ), especially on the amplitude of the fundamental harmonic, is considered in the following three cases.

For Case 1 ( $n = m$ ) the window includes all modulating cycles. Since  $X_2(\xi)$  is convolved with the spectrum  $X_1(\xi)$ , we are interested in the effect that the window has on the fundamental modulating frequency of  $X_1(\xi)$ . When the window includes all modulating bars, or  $n = m$ , the spectrum of  $X_2(\xi)$  is found by Eq. (10) and is shown in Fig. 4(a). Note that the amplitude of the signal is one and that all of the comb spikes except for the spike at the origin are multiplied by zeros in the sinc function. That is,  $X_2(\xi)$  is a delta function of unity area located at the spectrum origin. When this unity-area delta func-

tion is convolved with  $X_1(\xi)$ , the function  $X_1(\xi)$  is obtained. This is the response that one would expect from a window function that includes the entire modulating signal. The fundamental harmonic is not affected (i.e., 100% of the fundamental-harmonic modulating power is available).

Case 2 ( $n = m/2$ ) is the Lovell reticle case. Here, the window includes half the modulating bar pairs, and the spectrum of  $X_2(\xi)$  becomes

$$X_2(\xi) = [mT_s \text{comb}(mT_s\xi)] \times \frac{1}{2} \text{sinc}\left(\frac{mT_s}{2}\xi\right) \exp(-j2\pi\phi\xi). \quad (11)$$

This spectrum is shown in Fig. 4(b). Note that this spectrum also includes only odd harmonics and that the even harmonics occur at the zeros of the sinc function. When this function is convolved with the  $X_1(\xi)$  spectrum, only half of the amplitude is left at the modulating frequency. Therefore only 25% of the modulating fundamental is retained in the power spectrum.

For Case 3 ( $n = m - 1$ ) the window includes all but one bar pair of the modulating signal. A modified Lovell reticle is proposed in which the phase sector includes only one cycle of the modulating signal, as shown in Fig. 5. Angular target location can still be determined since a phase section is retained, and modulating signal power is improved by transmitting the modulating signal for a longer time. The spectrum of  $X_2(\xi)$  becomes

$$X_2(\xi) = [mT_s \text{comb}(mT_s\xi)] \left(\frac{m-1}{m}\right) \times \text{sinc}[(m-1)T_s\xi] \exp(-j2\pi\phi\xi). \quad (12)$$

and is shown in Fig. 4(c). Notice that the higher-cost harmonics are close to the zeros of the sinc function. In fact, the larger  $m$  becomes, the closer these harmonics are positioned to the zeros. Also, when this spectrum is convolved with the  $X_1(\xi)$

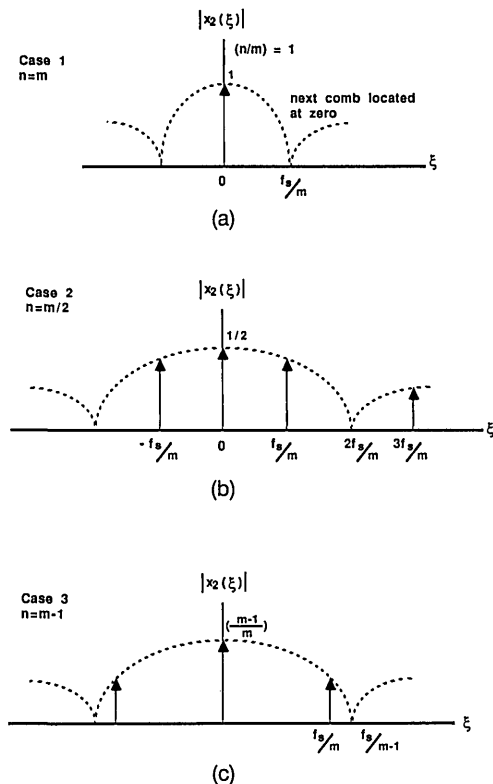


Fig. 4. Three cases of  $|X_2(\xi)|$ .



Fig. 5. Modified reticle.

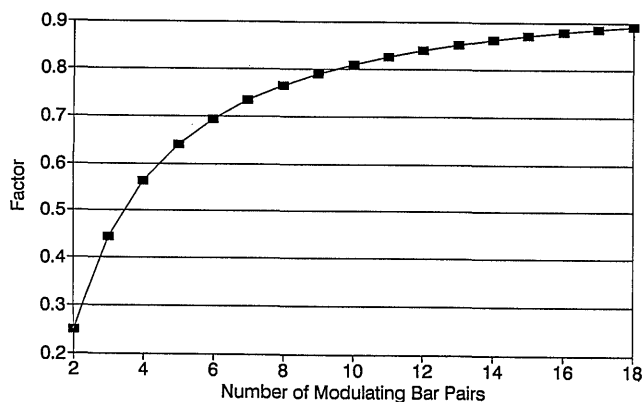


Fig. 6. Power of the modulating signal as affected by the phase sector.

spectrum, the modulating fundamental amplitude is decreased by  $(m - 1)/m$ .

#### 4. Power in the Fundamental

The effect of the window on the power of the modulating signal [i.e., the fundamental harmonic of  $x_1(t)$ ] is seen by convolving the spectrum of  $x_2(t)$  with the spectrum of  $x_1(t)$ , as discussed in Section 3. However, the power is proportional to the amplitude squared. Therefore the power of the fundamental  $x_1(t)$  signal is affected by the modified window, which is

$$P = P_{\text{fund}} \left( \frac{m - 1}{m} \right)^2, \quad (13)$$

where  $P_{\text{fund}}$  is the unaffected fundamental-harmonic power, i.e., the power of the fundamental harmonic of  $x_1(t)$  when only the modulating (no window) signal is present. The factor  $[(m - 1)/m]^2$  is shown in Fig. 6 for the reticle shown in Fig. 5. Note that as a target moves from an inner radius on the reticle to an outer radius, the number of modulating bar pairs increases; therefore the fundamental-harmonic power increases. The maximum number of bar pairs is 18 and occurs on the reticle periphery. The increase in fundamental power is in contrast to the constant 25% of fundamental power as seen with the Lovell reticle. Also, the maximum number of bars can be designed to well beyond 18 and is typically in the hundreds.

#### 5. Conclusion

The spectral analysis of the Lovell reticle has been presented. The modulating power of the fundamental harmonic of the reticle is shown to decrease by 25% owing to the size of the phasing sector. A modified reticle is presented in which the phasing sector occupies only one modulating bar pair. The spectral analysis of this reticle shows that as the number of modulating bar pairs increases, the power at the fundamental harmonic increases.

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