Comparison of two frame noise calculations for infrared linescanners

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Abstract. Averaging line noise standard deviations and averaging line noise variances are two commonly used techniques for determining frame noise or noise equivalent temperature difference (NETD). The two techniques are equivalent if all lines in the frame have the same noise statistics, that is, if the noise process is ergodic. However, if a few lines contain a larger amount of noise, the process is not ergodic and the two techniques yield different results. A simple theoretical relationship for the difference between the two is developed and compared to a computer simulation under a variety of conditions. Results of the simulation closely matched the theoretical relationship. We recommend the use of the variance average as a description for noise in an NETD calculation, since it is more sensitive to lines that exhibit excess noise.

Subject terms: frame noise; image noise; noise equivalent temperature difference.

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CONTENTS

1. INTRODUCTION

The amount of noise in a frame of imagery from an infrared imager is called the frame noise and is often characterized by the noise equivalent temperature difference (NETD). NETD is defined[1] as the blackbody target-to-background temperature difference in a standard test pattern that produces a peak-signal to rms-noise ratio (S/N) of one at the output of a reference electronic filter when the system views the test pattern. An NETD measurement is usually accomplished by using the equivalent relationship

$$\mathrm{NETD} = \frac{V_{\mathrm{noise}}\ [\mathrm{V}]}{\mathrm{SITF}\ [\mathrm{V/^\circ C}]}\ , \tag{1}$$

where the two parameters, V_{noise} and SITF (system intensity transfer function), are aggregate system measures of frame noise and system response, respectively. Thus, NETD[3,4] quantifies the noise response of a system in a measure (i.e., degrees Celsius) that is readily comparable to features in the object of interest.

The SITF curve of an imager is measured by providing a number of temperature differences as stimuli and measuring the video output voltage as the response. The response is plotted as a function of input temperature difference (ΔT) and is usually similar to the curve shown in Fig. 1. The SITF value is taken as the slope of the curve for small ΔT values with units of volts per degrees Celsius. A slope measurement needs to involve more than one data point. Using the single data point assumption that the SITF curve passes through the origin usually gives incorrect results since the measurement apparatus (including blackbody sources, temperature probes, and A/D converters) induces SITF offset errors.

The V_{noise} value is computed using a combination of detector output voltages while the system observes a scene of uniform temperature. As an illustration, Fig. 2 shows a linear detector array using a one-dimensional scan pattern. This is currently the most common thermal imaging system configuration and is the starting point for our analysis. Detector noise output can be characterized by either variance or standard deviation. The frame

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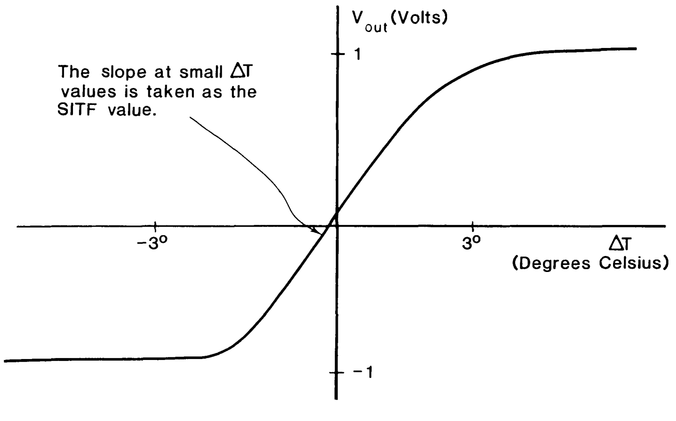

Fig. 1. Typical SITF curve.

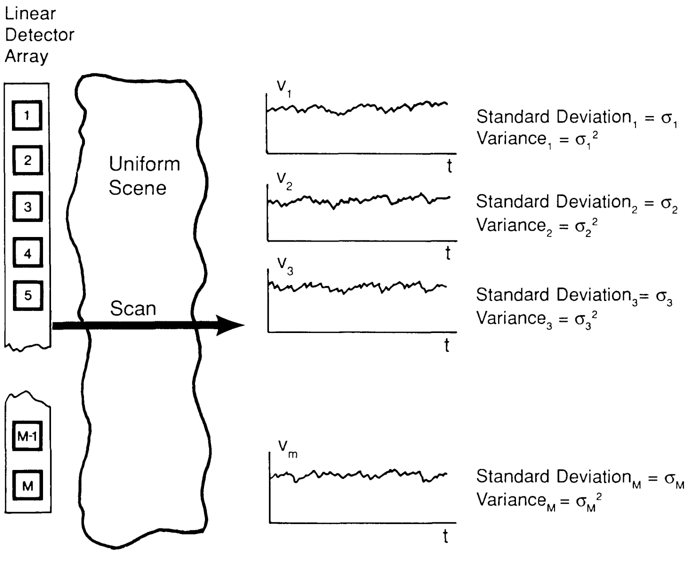

Fig. 2. Scanned detector array with output voltages.

noise of the imager, V_{noise} used in the NETD calculation, is then taken to be either the average of the line standard deviations or the root of the average line variance. In practice, a representative sample of the lines is used to provide this characterization of the frame noise.

The two techniques are equivalent for an ergodic process in which the noise characteristics of all the lines are similar. However, if a few noisy lines are present, there is a difference between the results of the two techniques. The difference in the two techniques is dependent on three parameters: the total number of lines used in the calculation, the number of noisy lines, and a noise factor (a noise amplitude ratio of noisy lines to normal lines). A simple theoretical relationship for the difference in the results of the two calculations is developed in the next section and compared to a computer simulation for a variety of combinations of these parameters.

2. THEORETICAL DESCRIPTION

2.1. Variance for a single detector

The variance of a particular scanned detector output voltage around its mean is defined[2] as

$$\sigma_y^2 = E[(S_x - \mu_y)^2] \ , \tag{2}$$

where E is the expected value, S_x is the detector output voltage when the linear array is located at position x in the scan, and μ_y is the average response of all the samples contained in line y. The variance as given in Eq. (2) can be written in summation form as

$$\sigma_y^2 = \left(\frac{\sum_{x=1}^{n} S_x^2}{n}\right) - \left(\frac{\sum_{x=1}^{n} S_x}{n}\right)^2 \ , \tag{3}$$

where n is the number of samples taken in line y.

2.2. Root of the average line variance

An average line variance of m lines can be given by

$$\overline{\sigma_y^2} = \frac{\sum_{y=1}^{m} \sigma_y^2}{m} \ . \tag{4}$$

The frame noise content (rms) using the root of the average line variance is

$$N_v = \left(\frac{\sum_{y=1}^{m} \sigma_y^2}{m}\right)^{1/2} \ . \tag{5}$$

Expanding the square of the frame noise provides a means of finding a relationship between the two types of calculations:

$$N_v^2 = \frac{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2}{m} \ . \tag{6}$$

2.3. Average of line standard deviations

The frame noise as given by an average of line standard deviations for m lines is

$$N_s = \frac{\sum_{y=1}^{m} \sigma_y}{m} \ . \tag{7}$$

Expanding the square of the average standard deviation frame noise yields

$$N_s^2 = \frac{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2}{m^2} + \frac{\sum_{i=1}^{m} \sum_{j=1}^{m} \sigma_i \sigma_j}{m^2} \ , \quad i \neq j \ . \tag{8}$$

The first term in N_s^2 is N_v^2/m and the second term contains $m(m-1)$ unique crossterms. From practical considerations reflecting the types of noise data commonly seen in forward looking infrared systems, two cases are considered in this paper: the case where the noise statistics of all lines in the calculations are equivalent and the case where a few lines are excessively noisy.

2.4. Case 1: similar line statistics

When the noise statistics for all the detectors are equivalent, the following holds:

$$\sigma_1 \approx \sigma_2 \approx \ldots \approx \sigma_m \ . \tag{9}$$

The standard deviation for each line can now be estimated with an average line standard deviation $\overline{\sigma}$. With this substitution, Eq. (8) becomes

$$N_s^2 = \frac{N_v^2}{m} + \frac{m(m-1)\overline{\sigma}^2}{m^2} = \frac{N_v^2}{m} + \frac{(m-1)N_v^2}{m} = N_v^2 \ . \tag{10}$$

Hence, under conditions of similar line noise statistics, the two frame noise calculations are equivalent.

2.5. Case 2: lines with excess noise are introduced

The case where the line statistics are not the same usually occurs when a few detectors are excessively noisy. Starting with Eq. (8) gives

$$N_s^2 = \frac{N_v^2}{m} + \frac{1}{m^2}\left\{\sum_{i=1}^{m}\sum_{j=1}^{m}\sigma_i\sigma_j\right\} \ , \quad i \neq j \ . \tag{11}$$

The crossterms $\sigma_i\sigma_j$ where $i \neq j$ can be broken into three different groups: the crossterms that contain two excessively noisy terms (doubly noisy crossterms), the crossterms that contain one excessively noisy term (noisy crossterms), and the typical crossterms that do not contain excessively noisy terms. Equation (11) can now be written

$$N_s^2 = \frac{N_v^2}{m} + \frac{1}{m^2}\left(\sum \text{doubly noisy crossterms} + \sum \text{noisy crossterms} + \sum \text{typical crossterms}\right) \ . \tag{12}$$

Consider k noisy lines from the total number of lines, m. Since $i \neq j$ in the summation of Eq. (12), the total number of crossterms must sum to $m(m-1)$. The number of doubly noisy crossterms is $k(k-1)$, the number of noisy crossterms is $2k(m-k)$, and the number of typical crossterms is $(m-k)^2 - (m-k)$ for a total of $m(m-1)$.

Let p be an excess noise factor. That is, p is the ratio of the standard deviations of an excessively noisy line to a typical line. The doubly noisy and noisy crossterms can be written as p^2 and p times the typical noise crossterms. Let $\overline{\sigma_t^2}$ be the average of the typical crossterm variances. Equation (12) becomes

$$N_s^2 = \frac{N_v^2}{m} + \frac{1}{m^2}[p^2k(k-1) + p2k(m-k) + (m-k)^2 - (m-k)]\overline{\sigma_t^2} \ . \tag{13}$$

The average typical variance $\overline{\sigma_t^2}$ can be estimated as the average of the typical line variances. Mathematically, this estimation is

$$\overline{\sigma_t^2} = \frac{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_m^2}{m-k} \ , \tag{14}$$

where the noisy $(m-k)$ line variances are not included. If the noisy line variances are included, then

$$\overline{\sigma_t^2} = \frac{mN_v^2}{p^2k + (m-k)} \ , \tag{15}$$

where the k noisy line variances are all doubly noisy. With this substitution and algebraic manipulation, Eq. (14) becomes

$$N_s = N_v\left\{\frac{1}{m}\left[\frac{(m-k)(m+2kp-k) + k^2p^2}{kp^2 + (m-k)}\right]\right\}^{1/2} \ . \tag{16}$$

TABLE I. N_s and N_v simulation data.

TOTAL LINES (m)	NOISY LINES (k)	NOISE FACTOR (p)	N_S	N_V	Actual Diff. %	Predict. Diff. %
10-40	0	0	1.0	1.0	0	0
10	1	2	1.09	1.13	3.48	3.52
	2		1.20	1.27	5.48	5.13
	4		1.40	1.49	5.83	5.61
	8		1.78	1.83	2.49	2.38
	1	3	1.19	1.32	10.4	10.5
	2		1.40	1.62	13.6	13.2
	4		1.80	2.05	12.4	12.2
	8		2.58	2.70	4.52	4.42
	1	5	1.38	1.82	23.9	24.1
	2		1.80	2.43	25.7	25.2
	4		2.61	3.27	20.3	20.1
	8		4.17	4.46	6.63	6.55
20	1	2	1.05	1.07	2.28	2.09
	4		1.20	1.26	5.28	5.13
	8		1.40	1.48	5.89	5.61
	12		1.60	1.68	4.59	4.38
	1	3	1.10	1.19	7.43	7.03
	4		1.40	1.62	13.4	13.2
	8		1.80	2.06	12.5	12.2
	12		2.21	2.42	8.89	8.65
	1	5	1.20	1.50	19.8	19.1
	4		1.80	2.41	25.5	25.3
	8		2.61	3.28	20.4	20.1
	12		3.42	3.95	13.5	13.4
40	1	2	1.03	1.04	1.24	1.14
	8		1.20	1.26	5.09	5.13
	18		1.45	1.53	5.31	5.41
	24		1.59	1.67	4.34	4.38
	1	3	1.05	1.10	4.32	4.14
	8		1.40	1.61	13.1	13.2
	18		1.90	2.14	11.3	11.4
	24		2.19	2.40	8.61	8.64
	1	5	1.10	1.27	13.4	13.0
	8		1.80	2.40	25.1	25.2
	18		2.79	3.42	18.4	18.5
	24		3.39	3.91	13.3	13.3

In the next section, we compare the results of this equation with a direct calculation of noise using simulated data sets.

3. COMPUTER SIMULATION

In order to illustrate the implications of Eq. (16), a computer program was written to simulate the output of a linear detector array scanned across a uniform scene. A random number generator with a programmable magnitude was used to simulate line noise. The program was written with three input parameters: total number of lines in the calculations, number of lines with excess noise, and an excess noise factor. Each line contained 200 data points. The simulation was run with a number of different parameters, and the frame noise was calculated using both techniques N_s and N_v, using Eqs. (7) and (5), respectively. The difference between the two frame noise values as calculated from the simulation data was compared to the predicted difference using only m, k, p, and Eq. (16). The results of the simulation are shown in Table I.

Two important results can be obtained from Table I: the N_v calculation is more sensitive than the N_s calculation to the presence of excessively noisy lines, and the difference in the actual N_s and N_v calculations can be accurately described by Eq. (16).

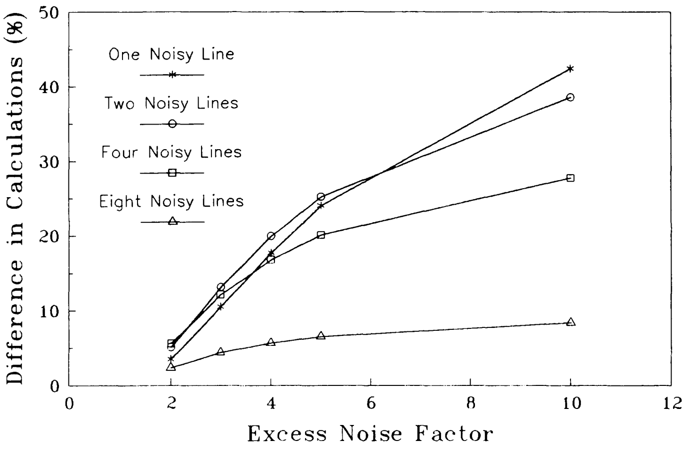

Fig. 3. Ten calculation lines.

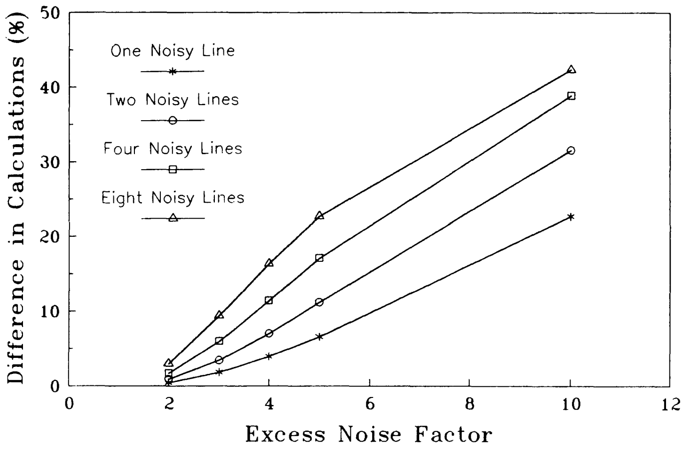

Fig. 5. One hundred calculation lies.

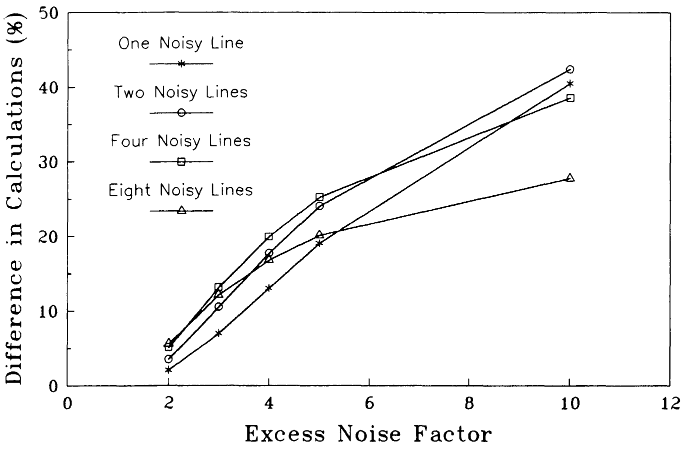

Fig. 4. Twenty calculation lines.

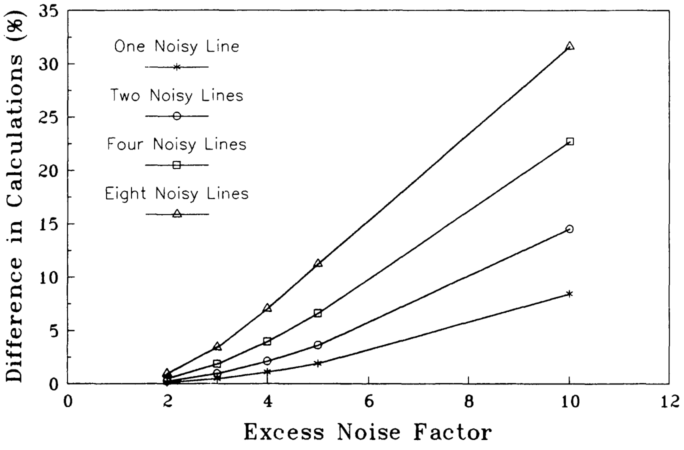

Fig. 6. Four hundred calculation lines.

4. SENSITIVITY

It can be shown using Table I or Eq. (16) that the average line variance is more sensitive to the presence of excessively noisy lines than the average of line standard deviations. Therefore, the authors recommend the N_v calculation for use in the frame noise NETD calculation because of the increased sensitivity. However, use of the N_s calculation compared to the N_v calculation may be helpful as a frame noise diagnostic tool. That is, if the difference in N_v and N_s is calculated from noise data, information regarding the noise factor and the number of noisy lines may be determined from Eq. (16).

5. COMPARISON OF THE CALCULATIONS

Using Eq. (16), the differences in percent between the two calculations were plotted and are shown in Figs. 3 through 7. The excess noise factors and number of excessively noisy lines are 2, 3, 4, 5, and 10 and 1, 2, 4, and 8, respectively. As expected, all of the plots show the differences increasing monotonically with the excess noise factor.

Figure 3 shows the differences for a total number of 10 calculation lines. Note that 8 noisy lines in a 10-line calculation does not cause as large a difference as 1 noisy line. However, this is reasonable since the 8 noisy lines become the typical lines and the other two lines are "excessively quiet." Figures 4, 5, and 6 are plots of the differences where the total number of calculation lines are 20, 100, and 400. Figure 4 shows in 20 total calculation lines that 8 noisy lines cause a larger difference than a single noisy line for small values of noise factor. The crossover point in noise factor where 1 noisy line overtakes the

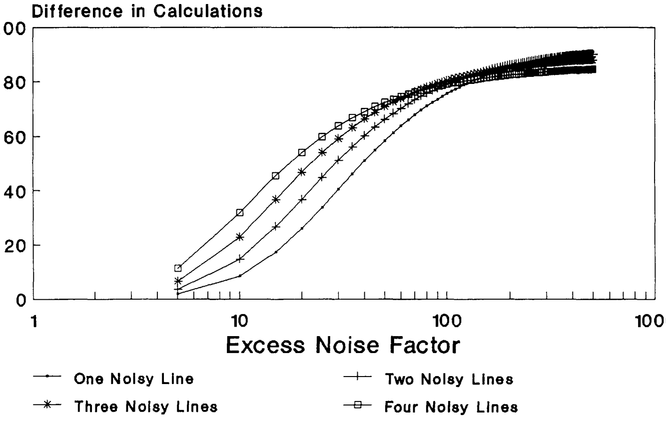

Fig. 7. Expanded plot of 400 calculation lines.

difference caused by 8 noisy lines increases with the number of total calculation lines. Figures 5 and 6 include enough total calculation lies that this crossover point is not seen. In all cases, as the noise factor increases, the 1 line difference will eventually surpass all other differences.

To illustrate this phenomenon we present an expanded 400 line calculation plot in Fig. 7. The single noisy line variance dominates the variances of all other lines when the noise factor increases past a certain value. The relatively small variances of the typical lines become a small perturbation in the noise calculation. A larger number of typical line variances corresponds to a larger perturbation (even though each typical variance is small compared to the noisy line variance). However, the expanded version of the 400-line calculation plot is not useful in practice since the excess noise factors shown are not realistic for a typical thermal imager. Typical excess noise factors range from 2 to 10.

6. CONCLUSION

Two techniques for calculating frame noise given a number of line variances have been compared. One technique involves taking an average of line standard deviations and the other involves taking the square root of the average line variance. The techniques were found to give identical frame noise values when the line statistics were approximately equivalent. However, the values differed when a number of lines were excessively noisy. A simple relationship was developed relating the two noise values for frames containing excessively noisy lines. Also, the line variance average technique was shown to be more sensitive to excessively noisy lines and would therefore be appropriate for NETD calculation. The difference between the two frame noise calculations would be a useful diagnostic tool to indicate the presence of anomalous lines in the data sets.

7. REFERENCES

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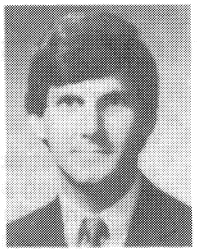

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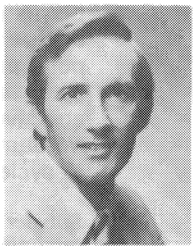

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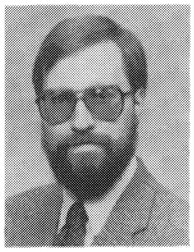

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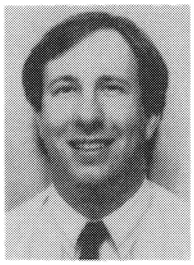

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