

Marriage of frequency modulation reticles to focal plane arrays

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Abstract. A technique for segmenting focal planes to complement spinning frequency modulation (FM) reticles in tracking applications is presented. The technique involves the relaxation of two parameter constraints in the FM reticle equation. The advantages are a reduction in susceptibility to countermeasures and increased resolution. The disadvantages are increased electronics loading and a reduction in the maximum trackable target size.

Subject terms: trackers; reticles; frequency modulation.

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1. INTRODUCTION

Spinning FM reticle trackers have been limited in many applications due to the presence of a single detector element, as shown in Fig. 1. In infrared trackers especially, countermeasures such as flares tend to saturate the detector signal so that target location determination is difficult. An alternative to the single detector design is a segmented focal plane array. Since the advantage of reticle trackers is simplicity and cost, the segmented focal plane array must be comprised of a small number of detectors (say 20 to 30) so as not to become as complex and costly as an imaging system with a full strength focal plane array.

There have been a number of spinning FM reticle studies in tracking applications.¹⁻³ However, recently it has been shown that spinning FM reticles⁴ can be described with three frequency parameters: frequency versus angle $f(\theta)$, frequency versus radius $m(r)$, and phase $\rho(r)$, where $f(\theta)$, $m(r)$, and $\rho(r)$ provide angular target location, radial target location, and target correlation, respectively. Certain constraints must be met for each parameter to construct a useful tracking FM reticle with a single detector. A simple, but effective design method for determining shapes of focal plane arrays is guided by relaxing these constraints in a manner that complements a reticle design.

2. FOCAL PLANE SEGMENTATION AND FREQUENCY VERSUS ANGLE

The reticle transmission function of a reticle can be generated directly from the frequency parameters⁴ by the equation

$$T(r, \theta) = \frac{1}{2} + \frac{1}{2} \cos \left[m(r) \int_{\theta + \rho(r)} f(\alpha) d\alpha \right], \quad (1)$$

where α is a dummy variable. In Eq. (1), r and θ are spatial coordinate variables with limits of 0 to R and $-\pi$ to π , respectively. Now, consider a sinusoidal variation in the frequency versus angle parameter such that the transmission function is

$$T(r, \theta) = \frac{1}{2} + \frac{1}{2} \cos\{30[\theta + 0.4 \cos(\theta)]\}. \quad (2)$$

The reticle described by this transmission function is shown in Fig. 2(a). When the reticle spins, a point source image produces a signal with a frequency found by taking the derivative of the first cosine function with respect to theta

$$\text{freq}(\theta) = 30[1 - 0.4 \sin(\theta)]. \quad (3)$$

The frequency content is not a function of radial location. Hence, the $m(r)$ function is constant and is chosen to be 30. The frequency versus angle function is $1 - 0.4 \sin\theta$ and has a variation

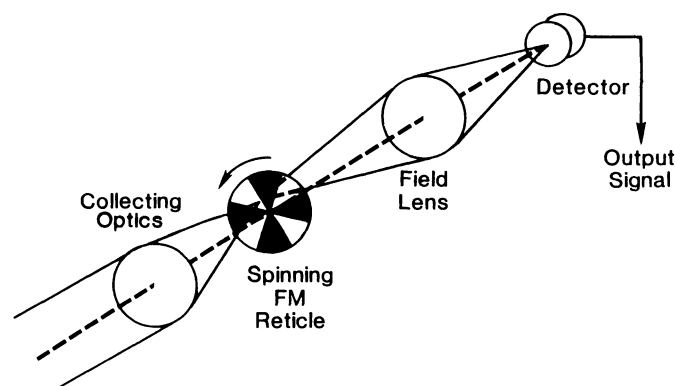


Fig. 1. Typical FM reticle tracker.

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of one period per reticle rotation, as shown in Fig. 2(b). With a rotating reticle, this periodic increase and decrease in modulated target frequency can be compared to the chopper sync signal to achieve angular target location. This periodic frequency versus angle function cannot have a period smaller than one reticle rotation if the reticle is to provide a unique signal for every angular target location. For instance, if a frequency versus angle function had two periods per reticle rotation, there would be two angular target locations that would give the detector identical signals.

Using a number of segmented wedge detectors allows for useful frequency versus angle functions in single detector tracking reticles. The number of wedges desired corresponds to the number of periods in the frequency versus angle function per one reticle rotation. One must be careful in how the transmission function is selected. Consider the transmission function

$$T(r, \theta) = \frac{1}{2} + \frac{1}{2} \cos\{30[\theta + 0.4 \cos(4\theta)]\} \quad (4)$$

It may appear that simply placing a multiple of four in the cosine argument of the inside cosine would give a frequency versus angle function that contained four periods per reticle rotation with a modulation of 40% of the average frequency. It is surprising that the reticle does not look at all as predicted [see Fig. 2(c) for the actual reticle pattern]. The derivative of the outer cosine argument was taken to determine the frequency content of the spinning reticle and was found to be $30[1 - 1.6 \sin(4\theta)]$. The derivative of the inner cosine, 4, was multiplied by the 0.4 modulation factor to give a modulation percentage of 160%. This means frequency versus angle function is negative at certain angles. A reverse in the transmission amplitude occurs at negative frequencies (i.e., opaque turns transparent and transparent turns opaque). The frequency versus angle function is shown in Fig. 2(d). This problem can be avoided by selecting

a non-negative frequency versus angle parameter and evaluating it with Eq. (1) to provide a transmission equation.

In using the reticle equation, it can be shown that a FM spinning reticle with a sinusoidal $f(\theta)$ and a 40% modulation can be found by the equation

$$T(r, \theta) = \frac{1}{2} + \frac{1}{2} \cos\left\{30\left[\theta + \frac{0.4}{p} \cos(p\theta)\right]\right\}, \quad (5)$$

where p is the number of angular cycles of the frequency function in one reticle rotation. Figure 3(a) illustrates the case where p is equal to 2 and $f(\theta)$ has two cycles over the reticle angular dimension. For useful tracking to occur, this reticle would require a focal plane that was segmented into two angular detectors. Each sector would contain only one spatial period as the reticle rotated and modulated targets would provide a unique signal for any angular location on a sector. However a reference signal (electrical or optical) would be required at a period of twice the reticle rotation rate. One more example is shown in Fig. 3(b), where p is equal to 4 and a focal plane must be segmented into four detectors.

It is important to note that the function $f(\theta)$ need not be a sinusoidal form for useful tracking. The function can be any function that does not approach zero or negative values. However, if the function has a multiple (p) periods from $\theta = -\pi$ to $\theta = \pi$, then the detector must be segmented into p angular sectors. One last important note pertains to the reticle sync signal. Since the angular target location within a detector wedge is determined by comparing the phase of the signal frequency phase to a reference sync signal, the tracker must deliver p equally spaced sync signals per reticle rotation to each detector electronics. Otherwise, p periods of the signal frequency would occur between reference sync signals.

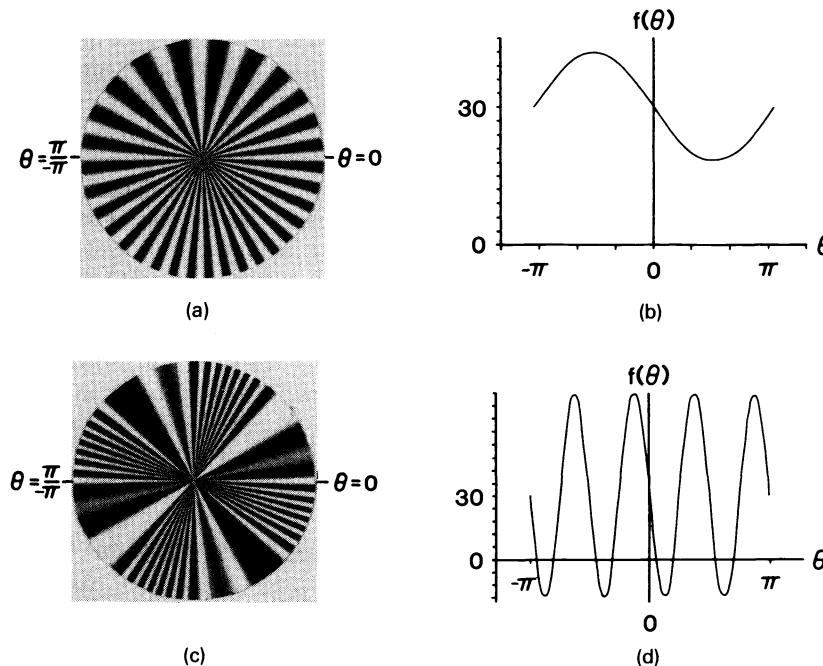


Fig. 2. Frequency versus angle reticles with their corresponding $f(\theta)$ curves: (a) reticle one, (b) $f(\theta)$ for reticle one, (c) reticle two, and (d) $f(\theta)$ for reticle two.

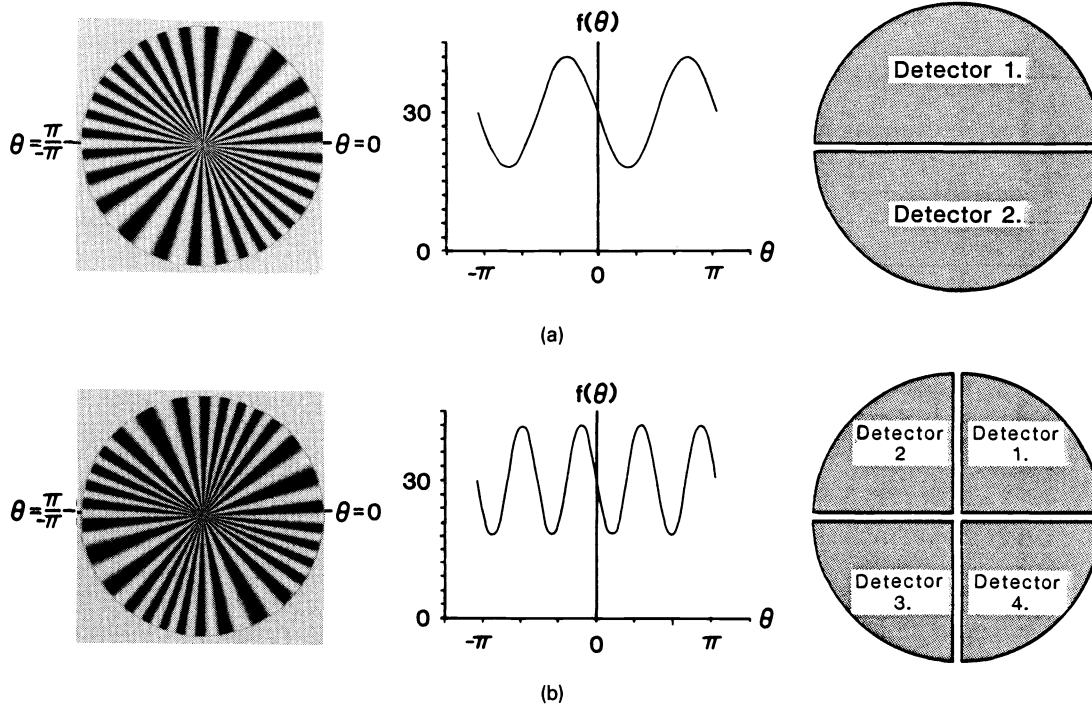


Fig. 3. Frequency versus angle reticles with complementary focal planes: (a) reticle, $f(\theta)$, and focal plane for $p = 2$; and (b) reticle $f(\theta)$, and focal plane for $p = 4$.

3. FOCAL PLANE SEGMENTATION AND FREQUENCY VERSUS RADIUS

The frequency versus radius parameter is easier to manipulate in the transmission equation since the derivative of the cosine argument is with respect to Θ . Therefore, $m(r)$ is plugged directly into the transmission equation. The only restriction on $m(r)$ for the parameter to be useful in tracking is a one-to-one mapping constraint. That is, for one radial target location, there exists only one radial modulation frequency. Reticles have been designed⁴ that have linear, squared, and exponential $m(r)$ parameters. The linear case shown in Fig. 4, a modified version of the Lovell² reticle, is the simplest case in single detector tracking. As the radial target location increases, the spinning reticle modulates the target with an increased number of bars (i.e., a larger single frequency). It is not obvious from the reticle geometry, but this increase in frequency is linear with radial target location. We will use this linear mapping case to illustrate the segmentation of the focal plane with the $m(r)$ parameter. Keep in mind that any one-to-one mapping function could have been used.

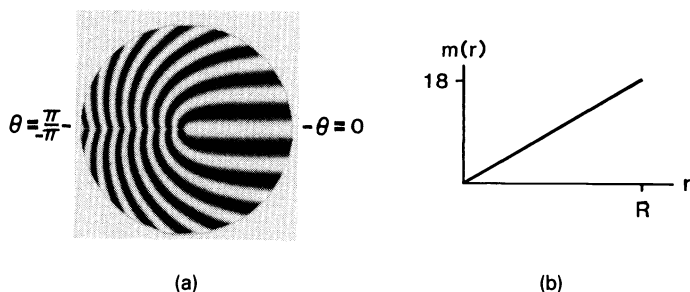


Fig. 4. Lovell reticle with radial dependence: (a) reticle and (b) $m(r)$ versus r .

In relaxing the one-to-one mapping constraint, more than one detector would be needed. For example, if

$$m(r) = \begin{cases} \left(\frac{2r}{R}\right) & 0 \leq r < \frac{R}{2} \\ m_{\max} \left(\frac{2r}{R} - 1\right) & \frac{R}{2} \leq r \leq R \end{cases}$$

as shown in Fig. 5(a), a two-to-one mapping is imposed on the reticle and two detector sections would be needed. One section would range from $r=0$ to $r=R/2$ and the other would range from $R/2$ to R . This requires a focal plane segmented as a circular disk and an annulus. The frequency versus radius function and the focal plane for this situation are also shown in Fig. 5(a). A similar configuration is shown in Fig. 5(b) for a reticle with a four-to-one mapped $m(r)$ function requiring four detectors.

One last example shown for the case of two required detectors, is an $m(r)$ function that does not include low frequencies. This may be an important consideration for ac coupled tracking systems. The $m(r)$ function begins at 15 modulating bars at r equal to 0 and $R/2$ and increases to 30 modulating bars at r equal to $R/2$ and R . The reticle, the $m(r)$ function, and the focal plane are shown from left to right in Fig. 6.

4. COMBINATIONS OF WEDGES AND RINGS

The transition to combinations of frequency versus angle and frequency versus radius is a simple one using Eq. (1). Consider a reticle that is designed to have p periods in frequency over the reticle angular range and to have a q -to-one mapping of reticle radial range to the frequency versus radius parameter. This reticle would require a focal plane of p wedges and q rings (the inner

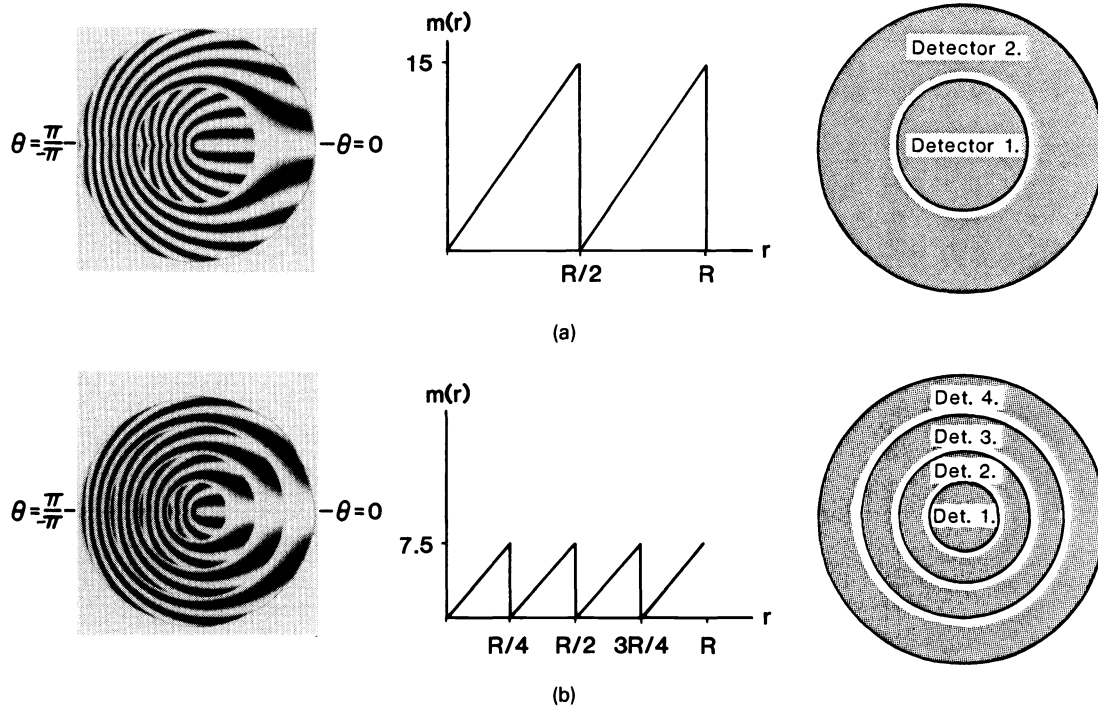


Fig. 5. Frequency versus radius reticles with complementary focal planes: (a) reticle, $m(r)$, and focal plane for two-to-one mapping; and (b) reticle, $m(r)$, and focal plane for four-to-one mapping.

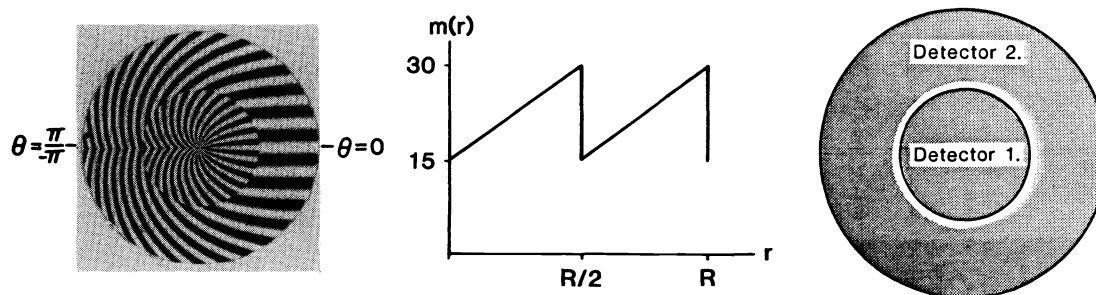


Fig. 6. Frequency versus radius reticle with offset frequency.

ring being the center, circular shaped detector) for useful target tracking.

Two simple examples are presented to illustrate the combination process. Both examples include sinusoidal variations in $f(\theta)$ and linear variations in $m(r)$. Again $f(\theta)$ and $m(r)$ can be any functions desired with p periods and q -to-one mappings, respectively. The first example is shown in Fig. 7, where p is 2 and q is 2. Hence, the focal plane is segmented in two wedges and two rings. The reticle is shown in Fig. 7(a) and the focal plane is shown in Fig. 7(b). The parameters $f(\theta)$ and $m(r)$ for the reticle can be seen in Figs. 7(c) and 7(d), respectively. The transmission function for the reticle becomes

$$T(r, \theta) = \frac{1}{2} + \frac{1}{2} \cos\{m(r)[\theta + 0.2 \sin(2\theta)]\}, \quad (6)$$

where $m(r)$ is the piecewise continuous function shown in Fig. 7(d).

Note that $m(r)$ has a constant offset of 10. This offset provides the reticle with a larger number of bars than the reticles shown in Fig. 5 (zero offset). The larger number of bars are required since one period of frequency must be shown over a shorter angular length. If one bar is close to the size of a wedge, then a change in frequency cannot be shown. That is, a smoother variation in frequency requires a larger number of bars. However, one must be careful not to design the bars too small. As the target size becomes larger than the bars, the modulation depth starts to decrease since the target overlaps into the next transmissive area. Hence, there is a trade-off in target size tracking capabilities and the smoothness of the reticle angular frequency variation.

The second example is shown in Fig. 8, where p is 6 and q is 4. In this case, an $m(r)$ offset of 15 is used since the wedges are smaller. Hence, the targets that can be tracked with this configuration must be smaller than those that can be tracked with the first example. However, the accuracy in angular target lo-

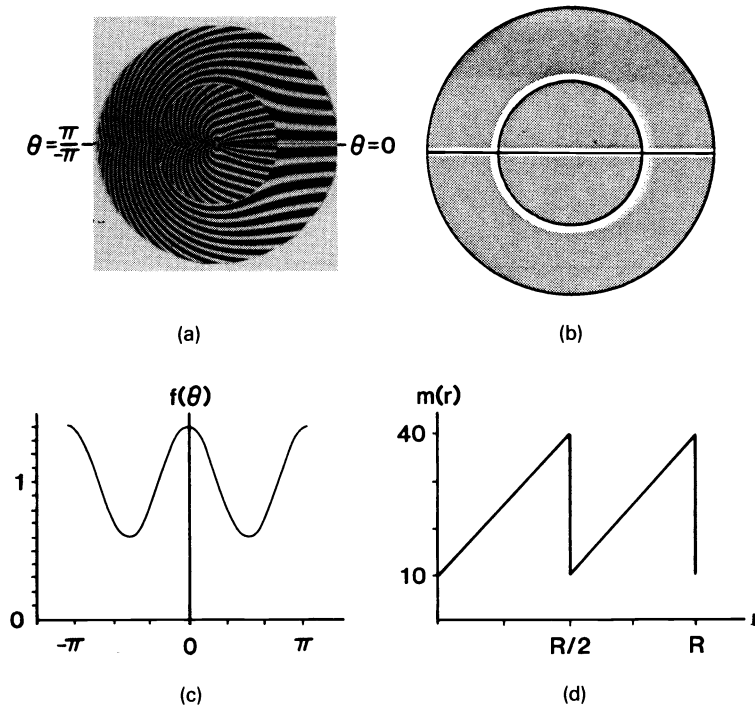


Fig. 7. Reticle with four detector segmented focal plane: (a) reticle, (b) focal plane, (c) $f(\theta)$, and (d) $m(r)$.

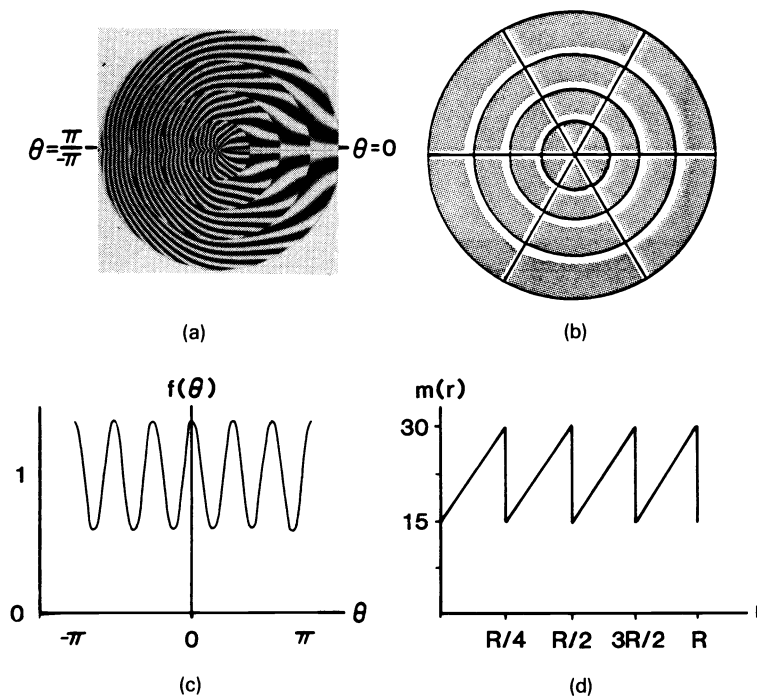


Fig. 8. Reticle with 24 detector segmented focal plane: (a) reticle, (b) focal plane, (c) $f(\theta)$, and (d) $m(r)$.

cation has increased by at least a factor of three over the example in Fig. 7 for small targets. The accuracy in radial target location has increased by at least a factor of two. The accuracy in angular target location can be further increased by making the offset even larger (smoothing the angular frequency variation even

more), but the maximum target size would continue to decrease. The accuracy in radial target location can be further increased by increasing the slope of the $m(r)$ curves. Again, maximum target sizes would decrease for larger $m(r)$'s and the angular accuracy decreases for smaller $m(r)$'s. Another advantage of

this configuration over the previous example is that there are six times the number of detectors in this focal plane, reducing the susceptibility of the tracker to countermeasures.

It takes a great deal of thought to design a reticle/focal plane combination with trade-offs of accuracy, target sizes, and countermeasures. These considerations do not include target shapes, background phenomenology, tracker electronics requirements, and others. Nevertheless, this technique allows some quantitative design of the reticle transmission function once some of the design constraints have been determined.

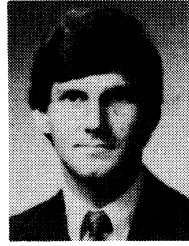
The target-reticle correlation and background decorrelation are performed with the reticle phase (sometimes referred to as spoke function) function.¹ Imposing a phase function on the reticle does not change the combination technique described in this paper. In fact, a phase function can be imposed on the focal plane, altering the shapes of the focal plane wedges without affecting the reticle/focal plane tracking capabilities. However, imposing a phase on the focal plane is probably not a good idea since the correlation/decorrelation of target and background to a tracker response requires movement. Naturally, the reticle is moving at a high rate relative to the target, whereas the correlation of the target to the focal plane depends on target motion with respect to each focal plane detector. For stationary or slow-moving targets, the correlation can be nonexistent. Hence, it is expected that phase is best imposed on reticles.

5. CONCLUSION

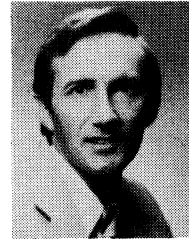
A technique for designing a FM spinning reticle/segmented focal plane pair that complement each other in target tracking has been presented. The technique utilizes the constraints on the frequency versus angle and frequency versus radius reticle parameters for useful tracking. The advantages of using a segmented focal plane over a single detector is a decrease in susceptibility to countermeasures and increases accuracy in target location. The disadvantage is that the maximum trackable target size is reduced with the large number of detectors for a given total field of view.

6. REFERENCES

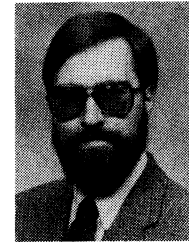
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