Elimination of threshold-induced distortion in the power spectrum of narrow-band laser speckle

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The distortion in the power spectrum of narrow-band laser speckle that results from irradiance thresholding is quantified. A method for compensation of this distortion is presented. An optimal threshold level is presented that simplifies the compensation method.

Key words: Modulation transfer function, laser speckle, thresholding.

1. Introduction

In a previous paper,[1] we investigated the effects of irradiance thresholding on speckle data generated during modulation transfer function (MTF) testing of detector arrays. A threshold value that minimized the error introduced into the MTF measurement was determined. In this paper, an expression for the distortion in the power spectral density caused by irradiance thresholding is derived. An optimal threshold value is found that eliminates the error in the MTF measurement caused by this distortion. The results of a MTF test implementing the optimal threshold value are presented.

2. Development of Expression for Distortion Magnitude

In laser-speckle MTF testing of detector arrays,[2] an integrating sphere is used to illuminate uniformly a two-slit aperture (see Fig. 1) with randomly phased monochromatic light of wavelength λ. The slits are spaced by distance L, and they have width l. The resulting laser-speckle irradiance at a distance z from the two-slit aperture has a power spectral density (PSD) in the x-dimension spatial frequency (ξ) direction as shown in Fig. 2 (solid curve). The PSD of the speckle irradiance is proportional to the autocorrelation of the aperture.[3] The height of the outer triangle is used as the input signal for the measurement of the modulation transfer of a detector array at $\xi = L/\lambda z$.

A speckle pattern is thresholded in irradiance by the use of

$$I^{(b)}(x, y) = \begin{cases} 1 & \text{if } I(x, y) \geq b\langle I(x, y)\rangle \\ 0 & \text{if } I(x, y) < b\langle I(x, y)\rangle, \end{cases} \tag{1}$$

where b is the irradiance threshold value and $\langle I(x, y)\rangle$ is the mean irradiance of the speckle pattern. When a speckle pattern is thresholded the nonlinearity distorts the PSD, thereby increasing the height of the outer triangles (Fig. 2, dotted curve). This distortion introduces error into the MTF test. To quantify this distortion, we derive an expression for the height of the outer triangle in the thresholded PSD. This will permit the removal of the distortion-induced error in the measurement.

We begin with an expression for the autocorrelation of the speckle irradiance for the two-slit aperture in the x dimension,

$$R_I(x) = \langle I\rangle^2\left[1 + \mathrm{sinc}^2\left(\frac{lx}{\lambda z}\right)\cos^2\left(\frac{\pi L x}{\lambda z}\right)\right]\cdot \tag{2}$$

Equation (2) is analogous to Eq. (2.83) in Ref. 3, modified for the case of the two-slit aperture of Fig. 1. To simplify Eq. (2) we eliminate, in two steps, all terms that do not contribute to the height of the outer triangles at $\xi = L/\lambda z$ in the PSD shown in Fig. 2. First, the dc bias is subtracted from Eq. (2) and the mean irradiance is set equal to one, yielding a normalized ac component of the autocorrelation,

$$[R_I(x)]_{ac} = \mathrm{sinc}^2\left(\frac{lx}{\lambda z}\right)\cos^2\left(\frac{\pi L x}{\lambda z}\right)\cdot \tag{3}$$

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Received 11 November 1994; revised manuscript received 3 March 1995.
0003-6935/95/286538-04$06.00/0.

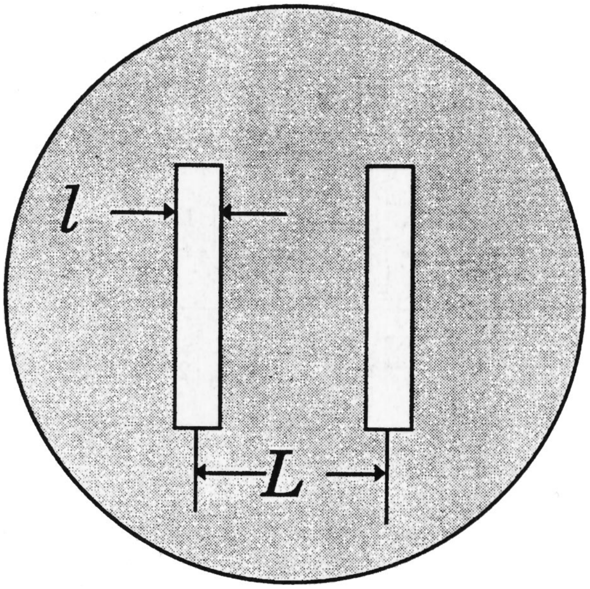

Fig. 1. Two-slit aperture configuration used for MTF testing.

Next, the sinc-squared term in Eq. (3) affects the width of the outer triangle but not the height. Considering this, we set the sinc-squared term to one by allowing slit width l to go to zero. The resulting autocorrelation function, $\mathscr{R}_I$, corresponds only to the height of the outer triangle in the PSD of the speckle irradiance,

$$\mathscr{R}_I(\Delta x) = \cos^2\left(\frac{\pi L x}{\lambda z}\right). \tag{4}$$

The continuous autocorrelation in Eq. (4) is converted to the thresholded autocorrelation by the use of Eq. (2.4) from Ref. 1 or Eq. (25) of Ref. 4,

$$\mathscr{R}_I^{(b)}(x) = \frac{\Gamma^2(\alpha, b\alpha)}{\Gamma^2(\alpha)} + \frac{(b\alpha)^2\exp(-2b\alpha)}{\Gamma^2(\alpha)} \times \sum_{n=1}^{\infty} \frac{[L_{n-1}{}^{(\alpha)}(b\alpha)]^2}{\binom{n+\alpha-1}{n} n^2} [\mathscr{R}_I(x)]^n, \tag{5}$$

where $\Gamma(\alpha, b\alpha)$ is the complementary incomplete gamma function and $L_{n-1}{}^{(\alpha)}$ is the associated Laguerre polynomial.[5] Variable α can be interpreted as the number of speckles seen by a single detector[6] and is given analytically[7] by

$$\alpha = \left[\int_{-\infty}^{\infty} \operatorname{sinc}^2\left(\frac{l_d x}{\lambda z}\right) \mathscr{A}(x) \mathrm{d}x\right]^{-2}, \tag{6}$$

where $\mathscr{A}(x)$ is the normalized autocorrelation function of the exitance at the speckle-generating aperture and l_d is the width of a single detector. Once a speckle pattern is obtained, α is calculated from the

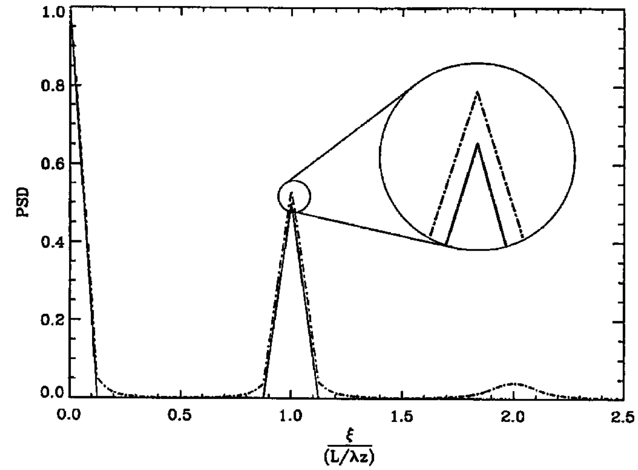

Fig. 2. PSD of laser-speckle irradiance for a two-slit aperture: continuous speckle (solid curve) and thresholded speckle (dashed curve).

mean and standard deviation of irradiance by the use of[8]

$$\alpha = \left(\frac{\langle I \rangle}{\sigma_I}\right)^2. \tag{7}$$

Equations (6) and (7) converge to the same value of α for large data-set sizes.

Fourier transforming Eq. (5) yields the spatial frequency power spectrum of the laser speckle,

$$\mathscr{S}_I^{(b)}(\xi) = \frac{\Gamma^2(\alpha, b\alpha)}{\Gamma^2(\alpha)} \delta(\xi) + \frac{(b\alpha)^2\exp(-2b\alpha)}{\Gamma^2(\alpha)} \times \sum_{n=1}^{\infty} \frac{[L_{n-1}{}^{(\alpha)}(b\alpha)]^2}{\binom{n+\alpha-1}{n} n^2} \int_{-\infty}^{+\infty} \cos^{2n}\left(\frac{\pi L x}{\lambda z}\right) \times \exp(-j2\pi\xi x)\mathrm{d}x. \tag{8}$$

The delta function in Eq. (8) is removed (because it does not effect the height of the outer triangles) to yield

$$\mathscr{S}_I'^{(b)}(\xi) = \frac{(b\alpha)^2\exp(-2b\alpha)}{\Gamma^2(\alpha)} \sum_{n=1}^{\infty} \frac{[L_{n-1}{}^{(\alpha)}(b\alpha)]^2}{\binom{n+\alpha-1}{n} n^2} \times \int_{-\infty}^{+\infty} \cos^{2n}\left(\frac{\pi L x}{\lambda z}\right) \exp(-j2\pi\xi x)\mathrm{d}x. \tag{9}$$

We evaluate the integral in Eq. (9), which we call Ω, by expressing the cosine as a sum of two exponentials,

$$\Omega = \int_{-\infty}^{+\infty} \frac{1}{2^{2n}} \left[\exp\left(j\frac{\pi L x}{\lambda z}\right) + \exp\left(-j\frac{\pi L x}{\lambda z}\right)\right]^{2n} \times \exp(-j2\pi\xi x)\mathrm{d}x. \tag{10}$$

Using the binomial series identity,[5]

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k, \tag{11}$$

yields

$$\Omega = \int_{-\infty}^{+\infty} \frac{1}{2^{2n}} \sum_{k=0}^{2n} \binom{2n}{k} \exp\left[j \frac{\pi L x}{\lambda z} (2n - k) \right] \times \exp\left(-j \frac{\pi L x}{\lambda z} k \right) \exp(-j 2\pi \xi x) \mathrm{d}x. \tag{12}$$

Simplifying and changing orders of integration and summation yield

$$\Omega = \frac{1}{2^{2n}} \sum_{k=0}^{2n} \binom{2n}{k} \int_{-\infty}^{+\infty} \exp\left[j \frac{2Lx}{\lambda z} (n - k) \right] \times \exp(-j 2\pi \xi x) \mathrm{d}x. \tag{13}$$

Finally, the integral in Eq. (13) is recognized as a shifted delta function,

$$\Omega = \frac{1}{2^{2n}} \sum_{k=0}^{2n} \binom{2n}{k} \delta\left[\xi - \frac{L}{\lambda z} (n - k) \right]. \tag{14}$$

Equation (14) can be substituted back into Eq. (9) to yield

$$\mathscr{S}_I'^{(b)}(\xi) = \frac{(b\alpha)^2 \exp(-2b\alpha)}{\Gamma^2(\alpha)} \sum_{n=1}^{\infty} \frac{[L_{n-1}^{(\alpha)}(b\alpha)]^2}{\binom{n + \alpha - 1}{n} n^2} \frac{1}{2^{2n}} \times \sum_{k=0}^{2n} \binom{2n}{k} \delta\left[\xi - \frac{L}{\lambda z} (n - k) \right]. \tag{15}$$

We define the triangle height, $T(\alpha, b)$, as

$$T(\alpha, b) \equiv \frac{S_I'^{(b)}(L/\lambda z)}{S_I'^{(b)}(0)}. \tag{16}$$

We find the numerator in Eq. (16) by evaluating Eq. (15) with $k = n - 1$,

$$\mathscr{S}_I'^{(b)}\left(\frac{L}{\lambda z} \right) = \frac{(b\alpha)^2 \exp(-2b\alpha)}{\Gamma^2(\alpha)} \sum_{n=1}^{\infty} \frac{[L_{n-1}^{(\alpha)}(b\alpha)]^2}{\binom{n + \alpha - 1}{n} n^2} \times \left[\frac{1}{2^{2n}} \binom{2n}{n - 1} \right]. \tag{17}$$

We find the denominator of Eq. (16) by evaluating Eq. (15) with $k = n$,

$$\mathscr{S}_I'^{(b)}(0) = \frac{(b\alpha)^2 \exp(-2\, b\alpha)}{\Gamma^2(\alpha)} \sum_{n=1}^{\infty} \frac{[L_{n-1}^{(\alpha)}(b\alpha)]^2}{\binom{n + \alpha - 1}{n} n^2} \times \left[\frac{1}{2^{2n}} \binom{2n}{n} \right]. \tag{18}$$

Substituting Eqs. (17) and (18) into Eq. (16) yields the final expression for the normalized triangle height,

$$T(\alpha, b) = \frac{\mathscr{S}_I^{(b)}\left(\dfrac{L}{\lambda z} \right)}{\mathscr{S}_I^{(b)}(0)} = \frac{\displaystyle\sum_{n=1}^{\infty} \frac{[L_{n-1}^{(\alpha)}(b\alpha)]^2}{\binom{n + \alpha - 1}{n} n^2} \left[\frac{1}{2^{2n}} \binom{2n}{n - 1} \right]}{\displaystyle\sum_{n=1}^{\infty} \frac{[L_{n-1}^{(\alpha)}(b\alpha)]^2}{\binom{n + \alpha - 1}{n} n^2} \left[\frac{1}{2^{2n}} \binom{2n}{n} \right]}. \tag{19}$$

3. Determination of Optimal Threshold

Equation (19) has been evaluated at four threshold values, over a range of α, and the results are shown in Fig. 3. The range of $\alpha = 1$ to $\alpha = 10$ was chosen because the measured α for all speckle data obtained during previous MTF testing have been within this range. These curves represent the height of the outer triangle that will be present in the PSD of a narrow-band speckle pattern, thresholded at $b = 1.0$, 1.132, 2.0, and 3.0. The value of $T(\alpha)$ corresponding to no thresholding is 0.5.

An increase in triangle height with increased α is seen for $b = 2.0$ and $b = 3.0$. However, the triangle height for $b = 1.0$ decreases with increased α. This suggests that a threshold value exists that permits the triangle height to remain constant over the range of α evaluated, and thus not introduce error in the MTF measurement.

The standard deviation of triangle heights over a range of $\alpha = 1$ to $\alpha = 10$ was calculated for threshold values $b = 0.5$ to $b = 3.0$. The threshold value with the smallest standard deviation was $b = 1.132$. The corresponding $T(\alpha, 1.132)$ is plotted as a solid curve in Fig. 3. At this threshold value, the height of the triangle is nearly constant ($\sigma = 0.003$) and has a

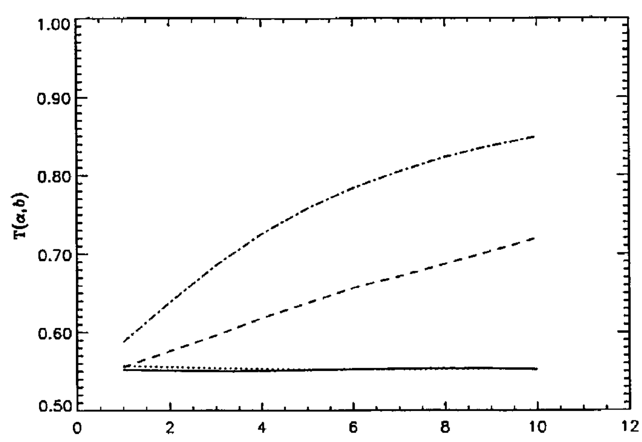

Fig. 3. Triangle height caused by irradiance thresholding of laser speckle at $b = 1.0$ (dotted curve), $b = 1.132$ (solid curve), $b = 2.0$ (dashed curve), and $b = 3.0$ (dashed–dotted curve).

mean value of 0.552. This threshold value is best because the distortion introduced in the PSD is not a function of α.

In a MTF test, speckle patterns are recorded at various distances. The mean and standard deviation of irradiance are calculated for each pattern and are used in Eq. (7) to calculate α. The pattern is then irradiance thresholded at $b\langle I\rangle$ and processed to obtain the PSD along ξ.

The PSD from each pattern is used to generate a single point on the MTF curve. The relationship between the PSD and MTF is[2]

$$\mathrm{MTF}\left(\frac{L}{\lambda z}\right) = \left[\frac{\mathrm{PSD}_{\mathrm{tri\ out}}(L/\lambda z)}{\mathrm{PSD}_{\mathrm{tri\ in}}(L/\lambda z)}\right]^{1/2}, \tag{20}$$

where z is varied to obtain the MTF at different spatial frequencies. For a nonthresholded speckle, $\mathrm{PSD}_{\mathrm{tri\ in}}$ is equal to 0.5 at $\xi = L/\lambda z$.

For each thresholded pattern, threshold value b and the calculated value of α are used in Eq. (19) to calculate $T(\alpha, b)$. The resulting value is used in Eq. (20) as $\mathrm{PSD}_{\mathrm{tri\ in}}$ to compensate for the threshold-induced distortion of the data. Thresholding all the speckle data at $b = 1.132$ eliminates the added step of evaluating Eq. (19) for each speckle pattern as well as measuring the standard deviation of irradiance. A value of $\mathrm{PSD}_{\mathrm{tri\ in}} = 0.552$ can be used in Eq. (20) for all data sets to eliminate distortion in the MTF measurement, because this offset is constant with spatial frequency.

4. Conclusions

An expression for the distortion to the PSD of irradiance that results from irradiance thresholding has been derived. A method for compensating for error in the MTF test caused by this distortion has been presented. An optimal threshold value of $b = 1.132$ has been determined, which eliminates the need for a spatial-frequency-dependent compensation technique to remove the threshold-induced distortion.

This research was supported by the U.S. Air Force, Wright Laboratory, under contract DAA-B07-88-C-F-405-ARPA-6324.

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