Resolution-equivalent *D** for SPRITE detectors

Per Fredin and Glenn D. Boreman

It is desirable for design purposes to model a signal-processing-in-the-element (SPRITE) detector simply as a discrete-element detector with an integration-enhanced D^*. We present a method for normalization of measured D^* for SPRITE detectors to yield an equivalent-discrete D^*. The multiplicative factor is the square root of the ratio of two noise-equivalent bandwidths: one is that of the SPRITE detector with no boost filter, and the other is that of the SPRITE detector with a boost filter that approximately compensates for carrier diffusion, yielding a spatial resolution that approaches that of a discrete detector the same size as the readout. This approach allows a resolution-equivalent D^* comparison of SPRITE detectors with discrete-element detectors and facilitates such comparisons among SPRITE detectors. We find that, to obtain the D^* of an equivalent-discrete detector, a measured SPRITE D^* should typically be multiplied by a factor ranging from 0.85 to 0.57 for 8- to 12-μm SPRITE detectors and by a factor ranging from 0.50 to 0.23 for 3- to 5-μm SPRITE detectors.

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1. Introduction

Signal-processing-in-the-element (SPRITE) detectors[1–6] are widely used in high-performance thermal imagers. The primary advantage of a SPRITE detector is an increased signal-to-noise ratio (SNR), obtained by means of a time-delay-and-integration (TDI) operation. The TDI operation is implemented in the SPRITE detector itself, which avoids the external delay-line electronics that would be required for implementing TDI if discrete-element detectors were used. However, SPRITE detectors have limited spatial resolution, caused by charge-carrier diffusion. It is convenient for conceptual and design purposes to be able to model a SPRITE detector simply as a discrete-element detector with a TDI-enhanced D^*. However, this equivalent-discrete D^* is not the D^* that is typically measured and quoted at the component level. We present a method for normalization of the measured D^* for a SPRITE detector, with respect to carrier diffusion, to yield the equivalent-discrete D^*. This facilitates resolution- and noise-equivalent comparisons between SPRITE detectors and discrete-element detectors.

The primary figure of merit for discrete-element infrared detectors is D^*, which specifies a SNR normalized with respect to a detector area and a temporal measurement bandwidth. The spatial resolution of a discrete-element detector is defined by the dimension of the photosensitive area. Thus the SNR and the spatial resolution are independent quantities for a discrete-element detector. A SPRITE detector is more complex because spatial resolution and SNR are not independent. For example, a longer SPRITE detector allows a longer dwell time, which yields a better SNR. However, this increase in SNR is obtained at the expense of spatial resolution[2,3] because a longer dwell time gives the charge carriers more time to diffuse, degrading the spatial resolution.

SPRITE detectors are typically used with boost filters[4] that sharpen the image by partially compensating for the modulation transfer function (MTF) reduction caused by carrier diffusion. However, increasing the spatial resolution in this way results in an increased noise level, because the noise-equivalent bandwidth of the detector channel is increased by the boost filter. The details of any individual boost-filter implementation are at the discretion of the system designer, so component-level measurements of D^* are usually made without a boost filter.

D^* is calculated from a measured narrow-band

P. Fredin is with Infrared Systems, Saab Missiles AB, P.O. Box 13045, S-402 51 Gothenburg, Sweden; G. D. Boreman is with the Department of Electrical Engineering, Center for Research and Education in Optics and Lasers, University of Central Florida, Orlando, Florida 32816.

SNR, SNR_{NB}, which uses a lock-in amplifier with a narrow bandwidth, $\Delta f_{\text{lock-in}}$:

$$D^* = \frac{\text{SNR}_{\text{NB}}}{\phi_{\text{det}}} (A)^{1/2} (\Delta f_{\text{lock-in}})^{1/2}, \qquad (1)$$

where A is the detector area and ϕ_{det} is the signal flux on the detector. Flood illumination is used with a chopping frequency that is less than the inverse of the carrier lifetime. The D^* is normalized with respect to bandwidth because the rms noise is proportional to the square root of the noise-equivalent bandwidth. Thus the D^* can also be written in terms of the full-bandwidth SNR and the noise-equivalent bandwidth of the detector channel:

$$D^* = \frac{\text{SNR}}{\phi_{\text{det}}} (A)^{1/2} (\Delta f_{\text{channel}})^{1/2}. \qquad (2)$$

Equation (2) can be inverted to yield the SNR expected from a detector under conditions of actual use:

$$\text{SNR} = \frac{\phi_{\text{det}} D^*}{(A)^{1/2} (\Delta f_{\text{channel}})^{1/2}}. \qquad (3)$$

Substituting Eq. (1) into Eq. (3), we find the expected in-use SNR in terms of the bandwidths of the lock-in amplifier and the detector channel:

$$\text{SNR} = \frac{\text{SNR}_{\text{NB}} (\Delta f_{\text{lock-in}})^{1/2}}{(\Delta f_{\text{channel}})^{1/2}}. \qquad (4)$$

2. Equivalent- Discrete D^*

Given the interdependence of resolution and SNR in SPRITE's, a resolution-equivalent comparison requires the use of a boost filter to make the MTF of the boosted SPRITE as close as possible to that of a discrete element of dimension equal to the along-scan readout length of the SPRITE.

From Eq. (4) we form the ratio of the expected SNR of the equivalent-discrete SPRITE used at the full boosted bandwidth to the expected SNR of the SPRITE used at full bandwidth but without boost:

$$\frac{\text{SNR}_{\text{equivalent-discrete}}}{\text{SNR}_{\text{SPRITE}}} = \frac{\dfrac{\text{SNR}_{\text{NB}} (\Delta f_{\text{lock-in}})^{1/2}}{(\Delta f_{\text{with-boost}})^{1/2}}}{\dfrac{\text{SNR}_{\text{NB}} (\Delta f_{\text{lock-in}})^{1/2}}{(\Delta f_{\text{no-boost}})^{1/2}}}$$

$$= \frac{(\Delta f_{\text{no-boost}})^{1/2}}{(\Delta f_{\text{with-boost}})^{1/2}}. \qquad (5)$$

We then define $D^*_{\text{equivalent-discrete}}$ as the D^* of the ficticious single-element detector that would operate into the full (but unboosted) channel bandwidth of

the SPRITE and yield the same SNR as the equivalent-resolution boosted SPRITE:

$$D^*_{\text{equivalent- discrete}} \equiv \frac{\text{SNR}_{\text{equivalent-discrete}}}{\phi_{\text{det}}} (A)^{1/2} (\Delta f_{\text{no-boost}})^{1/2}. \qquad (6)$$

We use Eqs. (2), (5), and (6) to define η, the ratio of the D^* of the equivalent-discrete element to the measured D^* of the unboosted SPRITE:

$$\eta \equiv \frac{D^*_{\text{equivalent-discrete}}}{D^*_{\text{SPRITE}}} = \frac{\dfrac{\text{SNR}_{\text{equivalent-discrete}}}{\phi_{\text{det}}} (A \Delta f_{\text{no-boost}})^{1/2}}{\dfrac{\text{SNR}_{\text{SPRITE}}}{\phi_{\text{det}}} (A \Delta f_{\text{no-boost}})^{1/2}}$$

$$= \frac{\dfrac{\text{SNR}_{\text{NB}} (\Delta f_{\text{lock-in}})^{1/2}}{(\Delta f_{\text{with-boost}})^{1/2}}}{\dfrac{\text{SNR}_{\text{NB}} (\Delta f_{\text{lock-in}})^{1/2}}{(\Delta f_{\text{no-boost}})^{1/2}}} = \frac{(\Delta f_{\text{no-boost}})^{1/2}}{(\Delta f_{\text{with-boost}})^{1/2}}. \qquad (7)$$

The factor η is the square root of the ratio of two noise-equivalent bandwidths: that of the SPRITE with no boost filter and that of the SPRITE with the boost filter in place. Thus η is proportional to the ratio of the rms noise levels for the two cases. Larger amounts of carrier diffusion will require more boost for correcting the MTF and, consequently, the equivalent-discrete D^* will drop. The equivalent-discrete D^* defined in Eq. (7) is a simple model for the SPRITE that is both resolution equivalent and SNR equivalent. The equivalent-discrete reference condition facilitates comparison of SPRITE's with different D^*'s and resolutions, in addition to the comparison of SPRITE's with discrete-element detectors.

3. Boost-Filter Transfer Function

We calculate the noise-equivalent bandwidth of the SPRITE detector, with and without the boost filter, by using an analytical expression for the SPRITE MTF that accounts for the effects of charge-carrier diffusion and the finite along-scan dimension of the readout. Including only these two terms is consistent with standard modeling practice but gives a somewhat optimistic prediction of the MTF.[5] If desired, measured SPRITE data for MTF and noise power spectra can be used in the following bandwidth calculations.

The MTF caused by the diffusion of charge carriers during transfer along the SPRITE element is given by[3]

$$\text{MTF}_{\text{diffusion}}(k) = \frac{1 - \exp\left\{-\left[1 + (2\pi k Q)^2\right]\dfrac{L}{v\tau}\right\}}{\left[1 + (2\pi k Q)^2\right]\left[1 - \exp\left(-\dfrac{L}{v\tau}\right)\right]}, \qquad (8)$$

where k is the spatial frequency in cycles per unit length, Q is the charge-carrier diffusion length, L is the total length of the SPRITE element, v is the charge velocity along the element (the product of mobility and electric field), and τ is the charge-carrier lifetime. The term $L/v\tau$ is the integration time divided by the carrier lifetime.

We take the readout geometry for the SPRITE detector as a rectangle of along-scan dimension X. Tapered readouts[6] are sometimes used to provide a small MTF enhancement (reducing the required boost), but the assumption of a rectangular readout is particularly convenient for comparison with a discrete detector of the same size. The MTF for a rectangular readout is given by

$$\text{MTF}_{\text{readout}}(k) = \frac{\sin(\pi kX)}{\pi kX} \equiv \text{sinc}(\pi kX) . \quad (9)$$

The total MTF of the SPRITE is written as

$$\text{MTF}_{\text{SPRITE}}(k) = \text{MTF}_{\text{diffusion}}(k)\text{MTF}_{\text{readout}}(k) . \quad (10)$$

We want a boost filter that produces an MTF approximating just that of the readout. The desired boost-filter voltage transfer function $G(k)$ is the inverse of the diffusion MTF:

$$G(k) = \frac{[1 + (2\pi kQ)^2]\left[1 - \exp\left(-\dfrac{L}{v\tau}\right)\right]}{\left(1 - \exp\left\{-[1 + (2\pi kQ)^2]\dfrac{L}{v\tau}\right\}\right)} . \quad (11)$$

The diffusion MTF given by Eq. (8) approaches 0 for large k. Thus, to keep $G(k)$ bounded, the boost must be implemented over a finite range of frequencies. In our analysis of both the boosted and the unboosted channels, we use one cycle per readout length ($k = 1/X$) as the cutoff frequency. This is consistent with usual practice, because the MTF of the readout will force the SPRITE MTF to be 0 at $k = 1/X$, and the gain in image quality resulting from the use of a higher cutoff frequency is negligible. Implementation of the boost in Eq. (11) from $k = 0$ to $k = 1/X$ will approximately compensate the MTF of the

SPRITE to that of a discrete detector of dimension X. The boosted impulse response is then the desired rectangular impulse response (limited to a bandwidth of $1/X$) of a discrete detector that is the same length as the readout.

4. Noise-Equivalent Bandwidth

We can now calculate the noise-equivalent bandwidths of Eq. (7), both with and without the boost filter. We express these bandwidths in spatial frequencies, easily converted to the more familiar bandwidth units of hertz by use of the scan velocity, v. For the case of the unboosted SPRITE detector, the noise-equivalent bandwidth is just the integral of the power spectrum of the noise up to $k = 1/X$. We use the normalized noise power spectrum $S(k)$ presented in Ref. 1, which has been validated by measurements:

$$S(k) = \frac{\text{sinc}^2(\pi kX)}{1 + (2\pi kQ)^2} . \quad (12)$$

Using Eq. (12), we express the noise-equivalent bandwidth of the unboosted SPRITE

$$\Delta f_{\text{no-boost}} = \int_0^{1/X} S(k)\mathrm{d}k = \int_0^{1/X} \frac{\text{sinc}^2(\pi kX)}{1 + (2\pi kQ)^2} \, \mathrm{d}k. \quad (13)$$

For the case of the SPRITE detector boosted by the voltage transfer function $G(k)$, the noise-equivalent bandwidth is given[7] by the integral from $k = 0$ to $k = 1/X$ of the product of $S(k)$ and $G^2(k)$:

$$\Delta f_{\text{with-boost}} = \int_0^{1/X} [1 + (2\pi kQ)^2]$$

$$\times \left[\frac{\text{sinc}(\pi kX)\left[1 - \exp\left(-\dfrac{L}{v\tau}\right)\right]}{\left(1 - \exp\left\{-[1 + (2\pi kQ)^2]\dfrac{L}{v\tau}\right\}\right)} \right]^2 \mathrm{d}k. \quad (14)$$

5. Comparison of Equivalent-Discrete *D** and Measured SPRITE *D**

Using Eqs. (7), (13), and (14), we find the ratio of the equivalent-discrete D^* to the measured SPRITE D^*:

$$\eta \equiv \frac{D^*_{\text{equivalent-discrete}}}{D^*_{\text{SPRITE}}} = \left\{ \frac{\displaystyle\int_0^{1/X} \frac{\text{sinc}^2(\pi kX)}{1 + (2\pi kQ)^2} \, \mathrm{d}k}{\displaystyle\int_0^{1/X} [1 + (2\pi kQ)^2]\left[\frac{\text{sinc}(\pi kX)\left[1 - \exp\left(-\dfrac{L}{v\tau}\right)\right]}{\left(1 - \exp\left\{-[1 + (2\pi kQ)^2]\dfrac{L}{v\tau}\right\}\right)} \right]^2 \mathrm{d}k} \right\}^{1/2} . \quad (15)$$
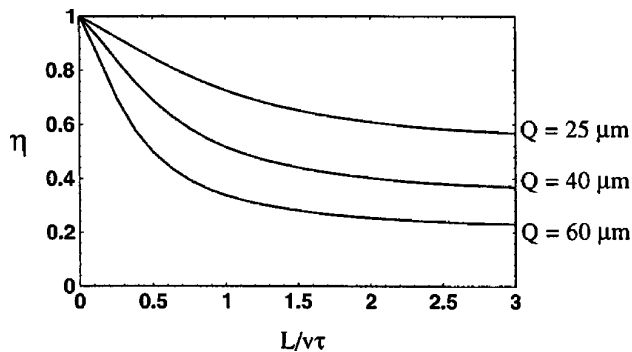
Fig. 1. Ratio η of equivalent-discrete D^* to measured SPRITE D^* for $Q = 25$, 40, and 60 µm, as a function of $L/v\tau$. A readout length of $X = 50$ µm is assumed.

Solving Eq. (15) numerically, we plot η as a function of $L/v\tau$ in Fig. 1 for three different values of Q: 25, 40, and 60 µm. In each case, we take the readout length X to be the typical value of 50 µm. A charge-carrier diffusion length Q of 25 µm is typical of 8- to 12-µm SPRITE's, and a Q of 60 µm is typical of 3- to 5-µm SPRITE's.[1] As expected, $\eta \to 1$ for small values of L and Q, and the resolution-equivalent D^* is reduced for larger values of diffusion length and for longer integration times.

To calculate a resolution-equivalent D^* for an 8- to 12-µm SPRITE, the measured D^* should be multiplied by 0.85 for $L/v\tau = 0.5$, by 0.73 for $L/v\tau = 1.0$, or by 0.57 for $L/v\tau = 3.0$. For a 3- to 5-µm SPRITE, the measured D^* should be multiplied by 0.50 for $L/v\tau = 0.5$, by 0.34 for $L/v\tau = 1.0$, or by 0.23 for $L/v\tau = 3.0$.

6. Conclusions

We have presented a method for the calculation of an equivalent-discrete D^* for SPRITE detectors, based on the use of a boost filter that approximately corrects the SPRITE MTF to that of a rectangular element the size of the readout. The method facilitates a simple and unbiased comparison of the performance of SPRITE detectors, taking into account both spatial resolution and noise. It also provides a comparison with conventional discrete elements. We find in typical cases for an 8- to 12-µm SPRITE that the measured D^* should be multiplied by a factor ranging from 0.85 to 0.57. For a 3- to 5-µm SPRITE, the measured D^* should be multiplied by a factor ranging from 0.50 to 0.23.

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