Int J Infrared Milli Waves (2008) 29:179–187
DOI 10.1007/s10762-007-9307-8

Measurement of the Mutual Coherence Function of an Incoherent Infrared Field with a Gold Nano-wire Dipole Antenna Array

Christopher Middlebrook · Michael Roggemann · Glenn Boreman · Nikola Subotic · Kyle Cooper · William Buller · Weidong Yang · Javier Alda

Received: 3 August 2007 / Accepted: 31 October 2007 /
Published online: 4 December 2007

Abstract The first direct measurement of the mutual coherence function of a spatially incoherent infrared beam was performed at 10.6 μm using a pair of infrared dipole nano-wire antennas that were connected to a common bolometer in the center of the pair by short lengths of coplanar strip transmission line. A spatially incoherent source was constructed by

C. Middlebrook (✉) · M. Roggemann · W. Yang
Department of Electrical and Computer Engineering, Michigan Technological University, Houghton, MI, USA
e-mail: ctmiddle@mtu.edu

M. Roggemann
e-mail: mroggema@mtu.edu

W. Yang
e-mail: weyang@mtu.edu

G. Boreman
College of Optics & Photonics/CREOL, University of Central Florida, Orlando, FL, USA
e-mail: boreman@creol.ucf.edu

N. Subotic · K. Cooper · W. Buller
Michigan Tech Research Institute (MTRI), Ann Arbor, MI, USA

N. Subotic
e-mail: nikola.subotic@mtu.edu

K. Cooper
e-mail: kyle.cooper@mtu.edu

W. Buller
e-mail: William.buller@mtu.edu

J. Alda
Applied Optics Complutense Group, University Complutense of Madrid. School of Optics, Av. Arcos de Jalon s/n., Madrid 28037, Spain
e-mail: j.alda@opt.ucm.es

dithering a BaF2 diffuser near the focus of a CO2 laser beam. The distance from the diffuser to the nano-wire antenna pair was held constant while the distance from the focus of the laser beam to the diffuser was varied to control the effective diameter of the source. The measured bolometer signal was proportional to the magnitude of the mutual coherence function at the plane of the antennas. The experimental results were found to match the predicted performance closely. If this technology can be extended to large arrays, a form of synthetic aperture optical imaging based on the Van Cittert-Zernike theorem is possible, similar to that performed at microwave frequencies now by astronomers. This has the potential to greatly increase the angular resolution attainable with optical instruments.

Keywords Coherence · Remote sensing · Infrared · Coherence imaging · Infrared imaging

1 Introduction

In this paper we describe the use of an optical nano-wire dipole antenna pair to directly measure the mutual intensity function [1–3] of a spatially incoherent infrared scene. The success of this measurement suggests the possibility of imaging at infrared wavelengths based on the Van Cittert-Zernike theorem, which shows that the mutual intensity of the field from an initially incoherent source is proportional to the two dimensional Fourier transform of the intensity distribution of the object [1–3]. Radio astronomers have long used this technique at microwave frequencies [4], where measurement, recording, and temporal synchronization equipment allow widely separated observatories to measure the complex field arriving at each antenna from the object of interest, and save this information for later use. The required computations consist of synchronizing the data, computing cross correlations between the fields recorded at all of the observatories participating in the measurement, and arranging this data properly in the Fourier domain (often referred to as (u,v) space by radio astronomers). Reconstructing images from this data essentially involves computing the inverse Fourier transform of the (u,v) space data in conjunction with some kind of regularizing scheme [5] required due to the sparsity of the measurements in (u,v) space [4].

The Van Cittert-Zernike theorem is also valid at optical frequencies. However, using light detection devices sensitive to the intensity associated with the incident field eliminates the required phase information from the measurements, and the signal-to-noise ratio considerations for heterodyne detection of incoherent radiation are very unfavorable [6]. As a result, using the Van Cittert-Zernike theorem to image at optical frequencies presently requires complicated and expensive beam splitting and combining hardware to display the required cross-correlations as fringe patterns on intensity-sensing detectors [7].

In this paper we describe an experiment to measure the mutual coherence function of an incoherent field using a pair of gold nano-wire dipole antennas connected by a coplanar strip transmission line to a bolometer centrally located between the two antennas [7, 8]. The bolometer output is sensitive to the modulus squared of the sum of the two fields arriving from the transmission lines. Hence, the output is proportional to the mutual intensity function of the field falling on the antenna pair at the (u,v) space component defined by the vector separation of the antennas, the wavelength, and the distance between the object and the plane containing the antenna pair [1–3]. Our central motivation for examining this technology is to determine if remote sensing systems made from large arrays of these devices may be feasible in the future. The potential advantages of this technology include the fact that nano-wire antenna arrays could potentially be made in much larger effective sizes than practical glass mirror designs allow, enabling better resolution, albeit at the expense of requiring post detection processing to recover images from the pupil plane measurements. As presently envisioned, nano-wire arrays would also likely be lighter than a similarly sized conventional mirror, and with proper signal processing, nano-wire arrays could have a larger field of view for a given aperture size than the competing glass lens design.

Previous research has shown that dipole-coupled antennas can be fabricated and are sensitive to 10.6 μm infrared radiation [8, 9]. Building from this research, antenna arrays for the infrared are currently being developed and investigated. A pair of gold nano-wire dipole antennas is shown in Fig. 1. The nano-wire antennas are connected to a common microbolometer using two-wire coplanar strip transmission lines [10] to experimentally demonstrate phase preservation and array properties. The bolometer outputs a voltage that is a function of the sum of the power collected by both antennas, which is a bias term, added to a term proportional to the cross correlation of the fields falling on the dipoles, which is the desired information.

In this paper we describe the first experiment that we know of to demonstrate the measurement of the mutual intensity of the source using a pair of gold nano-wire dipole antennas. The measurements are shown to agree well with the theoretical prediction of the performance. As a result, the basic feasibility of synthetic aperture infrared imaging using the Van Cittert-Zernike theorem has been established. Significant technical challenges must be overcome to realize synthetic aperture imaging with gold nano-wire antenna arrays. Perhaps the most crucial of these technical challenges is losses in the infrared transmission lines connecting the antennas to the bolometer. Research has started in this area [9], however, the losses in the coplanar strip lines are sufficiently strong that the geometries and numbers of dipoles which can presently be envisioned is quite limited. There are other configurations such as infrared microstrip lines for which the losses appear to be more favorable [11].

The remainder of this paper is organized as follows. In Section 2 we review the theoretical foundations of synthetic aperture imaging based on the Van Cittert-Zernike theorem. The experiment design is discussed in Section 3. In Section 4 the experimental measurements are compared to theoretical calculations of performance. Conclusions are drawn in Section 5.

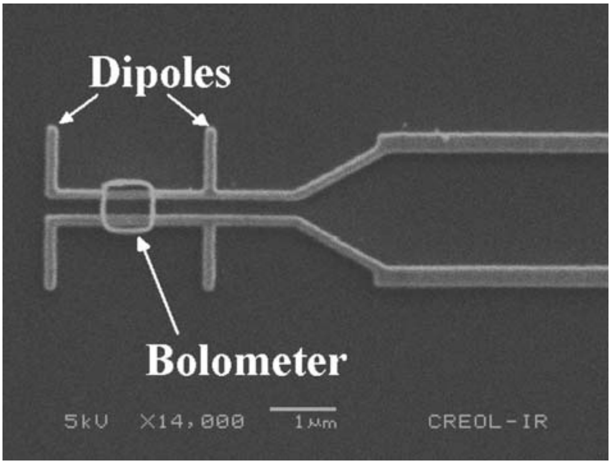

Fig. 1 SEM Image of a Dual Dipole IR Antenna.

2 Imaging based on the Van Cittert-Zernike Theorem

The Van Cittert-Zernike theorem demonstrates that under certain geometrical conditions a Fourier transform relationship exists between the mutual intensity $J(x_1, y_1; x_2, y_2)$ of a quasimonochromatic, spatially incoherent source, and the spatial intensity distribution of the source $I(\xi, \eta)$ [1–3]. The mutual intensity is the cross correlation of the quasimonochromatic field from the source falling on the measurement plane at the locations given by the coordinates (x_1, y_1) and (x_2, y_2). The Van Cittert-Zernike theorem is given by

$$J(x_1, y_{1;}; x_2, y_2) = \frac{\kappa e^{-j\Psi}}{(\lambda z)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} I(\xi, \eta) e^{\left[j\frac{2\pi}{\lambda z}(\Delta x\xi + \Delta y\eta)\right]} d\xi d\eta \tag{1}$$

where κ is a constant, ψ is a quadratic phase factor, $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ represent the vector separation of the points at which the field is sampled, λ is the mean wavelength, and ξ, η are coordinates in the object plane. In words, Eq. (1) can be stated as follows: the mutual intensity of an incoherent source is proportional to the two dimensional Fourier transform of the intensity distribution of the object evaluated at the spatial frequencies $f_x = \frac{\Delta x}{\lambda z}, \quad f_y = \frac{\Delta y}{\lambda z}$. The geometrical conditions relating the source and observation planes required for the Van Cittert-Zernike theorem are [3]: (i) the maximum extent of the source and the region over which the observation points (x_1, y_1) and (x_2, y_2) are allowed to vary must be much smaller than the distance z between the source and the observation plane; and (ii) only small angles are involved. A normalized form of the mutual intensity, referred to as the complex coherence factor $\mu(x_1, y_1; x_2, y_2)$, is generally used to analyze performance since $|\mu(x_1, y_1; x_2, y_2)|$ is the visibility V of the interference fringe arising from the squared modulus of the sum of the fields at (x_1, y_1) and (x_2, y_2). The relationship between $J(x_1, y_1; x_2, y_2)$, $\mu(x_1, y_1; x_2, y_2)$, and V are thus given by

$$\mu(x_1, y_1; x_2, y_2) = \frac{J(x_1, y_1; x_2, y_2)}{J(x_1, y_1; x_1, y_1)} \tag{2}$$

and

$$V = |\mu(x_1, y_1; x_2, y_2)| \tag{3}$$

The Van Cittert-Zernike theorem written in terms of the complex coherence factor is

$$\mu(x_1, y_1; x_2, y_2) = \frac{e^{-j\psi} \int\int_{-\infty}^{\infty} I(\xi, \eta) \exp\left[j\frac{2\pi}{\lambda z}(\Delta x\xi + \Delta y\eta)\right] d\xi d\eta}{\int\int_{-\infty}^{\infty} I(\xi, \eta)\, d\xi d\eta} \tag{4}$$

where the quadratic phase term ψ is given by

$$\psi = \frac{\pi}{\lambda z}\left[\left(x_2^2 + y_2^2\right) - \left(x_1^2 + y_1^2\right)\right] \tag{5}$$

It should be noted that if z is sufficiently large, and the maximum distances of the observation points from the axis of the system are sufficiently small, ψ is quite small and can be neglected.

For the case of the source being an on-axis, incoherent, uniformly bright circular source of radius a the theoretical calculation of the complex coherence has been presented elsewhere [3]. The result of this analysis is

$$|\mu(D)| = \frac{2J_1\left(\frac{2\pi aD}{\lambda z}\right)}{\frac{2\pi aD}{\lambda z}} \tag{6}$$

where D is the dipole separation. We now describe an experiment designed to demonstrate that we can directly measure $|\mu(D)|$ in the infrared with the nano-wire antenna and bolometer system.

As will be discussed in greater detail in section 4, if the source has a Gaussian distribution, the modulus of the coherence factor also shows Gaussian dependence with the variables of the problem.

3 Experiment

A schematic drawing of the experimental system is shown in Fig. 2. The experiment was designed to allow the radius of the illuminated disk, represented by a in Eq. (6), to be varied in a controlled manner, allowing easy comparison between theory and experiment. We note that a real sensing system based on this technology would measure $\mu(D)$ over a wide range of antenna spacings D. However, it should be noted that inspection of Eq. (6) shows that varying the disk radius a while keeping the antenna separation D constant is functionally equivalent to keeping a constant and varying D. Since we had only a single antenna separation to work with in the present case, we chose to vary the disk radius a for our experiments.

A linearly polarized 30 W CO_2 laser operating at 10.6 μm wavelength was used as the source in the experiment. After transmission through the optical elements of the experiment 22 W was available at the focus. A diffuser was constructed from a piece of Barium Fluoride (BaF_2) which was roughened by sandblasting both sides of the sample.

To create a spatially incoherent disk, the location of the diffuser transverse to the optical axis of the system was mechanically varied in time by directly attaching it to the cone of a speaker. The speaker was then driven using a signal generator with a sine wave input of

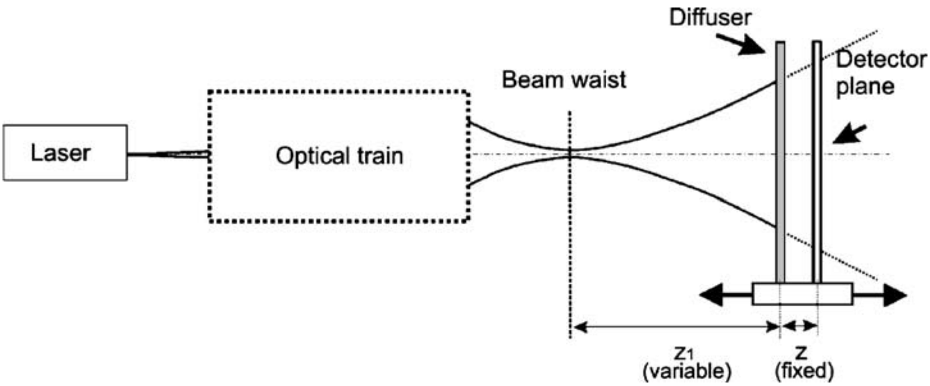

Fig. 2 Experimental set-up. (Note: The transverse dimension of the beam has been greatly exaggerated for better visualization).

182 Hz with amplitude of 50 mV. BaF_2 was chosen due to its high transmission at λ= 10.6 μm, which is greater than 80%. Scanning Electron Microscope (SEM) images depicting the surface roughness of the BaF_2 are shown in Fig. 3. Inspection of these images shows surface roughness on a spatial scale smaller than λ, which is generally taken as a sufficient condition to assure that the transmitted beam is spatially incoherent [3].

The laser beam was focused using an F/8 optical train, resulting in a nearly diffraction-limited spot with a Gaussian width at the beam waist of 150±20 μm. The half-angle of the cone formed by the converging beam is about 1.3 degrees. A detailed analysis of the propagation of the beam through the optical train has assured that the beam can be considered as a non-truncated beam. On the other hand, the quality parameter of the beam is quite good, having a measured value of about M^2=1.1 for the CO_2 laser used in this experiment. The antenna pair/bolometer device used here was biased at 100 mV, consistent with previous work with these devices [8, 9]. A mechanical chopper was used to modulate the laser at 2.5 kHz and the change in the voltage across the bolometer was recorded after a 10× pre-amplification stage using a computer coupled with a lock-in amplifier.

The incoming laser beam passed through the diffuser. The size of the source presented to the antenna detectors was controlled by varying length z_1 in Fig. 2. Light leaving the diffuser was propagated a distance z to the plane where the antennas were placed, as shown in Fig. 2. This distance z was fixed at 1.5±0.5 mm. The large value of the uncertainty of this parameter is due to the difficult accessibility of the space between these elements. The diffuser and the antenna/bolometer system were hard-mounted together, and translated along the optical axis in order to vary the size of the source, σ. This parameter follows a hyperbolic evolution of the type:

$$\sigma = \sigma_0 \sqrt{1 + \left(\frac{z_1}{z_R}\right)^2} \tag{7}$$

where σ_0 is the Gaussian width at the beam waist, and $z_R = \frac{\pi\sigma_0^2}{\lambda}$ is the Rayleigh range of the illuminating beam.

Measurements of σ were taken starting at a distance of z_1=19 mm, and then reduced in steps of 2 mm, until the focal point of the laser coincided with the first surface of the diffuser. By using Eq. 7 it is possible to calculate σ vs. z for all the measurement points. We

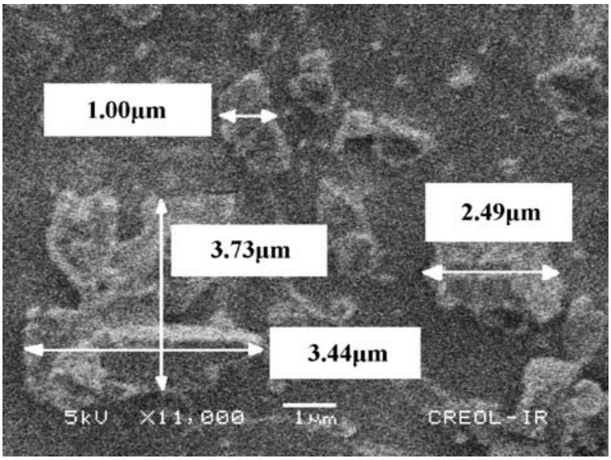

Fig. 3 SEM image of the sand-blasted surface of the barium fluoride diffuser taken at 11,000×magnification.

now present the results of this experiment, and compare them to the theoretically predicted performance.

4 Experimental results and comparison to theory

Due to the use of a laser source, the illuminated disk used in the experiment had a Gaussian intensity distribution. Hence, to accurately model the output of the device, the modulus of the complex coherence factor for this source must be calculated. The intensity distribution of the source in this case is

$$I(\xi,\eta) = \frac{I_0}{2\pi\sigma^2} e^{\frac{-1}{2\sigma^2}\left(\xi^2+\eta^2\right)} \tag{8}$$

where σ is the e^{-1} radius of the beam. Calculating the complex coherence factor requires computing the Fourier transform the intensity distribution in Eq. (8) using standard techniques [3], with the result

$$\mu(x_1,y_1;x_2,y_2) = e^{-j\Psi} e^{-\frac{\sigma^2}{2\lambda^2 z^2}\left(\Delta x^2+\Delta y^2\right)} \tag{9}$$

In the particular device used for this experiment the effective separation of the antennas was Δx=8 μm (taking into account the geometrical separation and the effect of the index of refraction of the substrate. the net effect is a multiplicative factor equal to the index of refraction of the media), and Δy=0, and hence

$$|\mu(\Delta x)| = Ke^{-\frac{z_1^2}{\Omega^2}} \tag{10}$$

Where the hyperbolic dependence of σ, is used to obtain an analytical expression for

$$\Omega_{analytical} = \frac{\sigma_0 z\sqrt{2}}{\Delta x} \tag{11}$$

The results of the experiment are shown in Fig. 4, where the measured and theoretical values of the modulus of the complex coherence factor are shown. The horizontal axis shows the variation in the distance z_1. The vertical axis represents the modulus of the

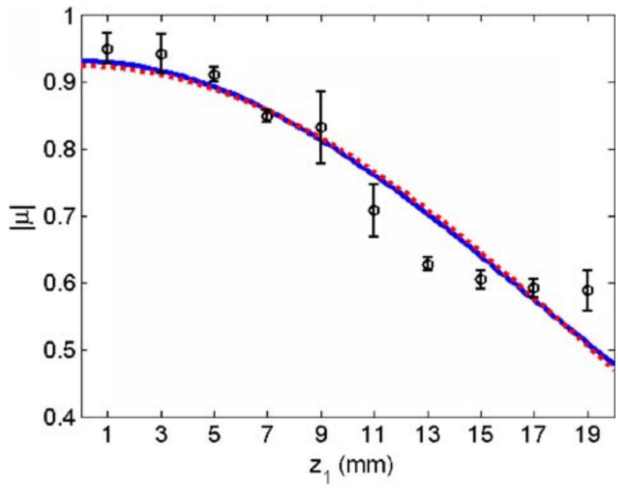

Fig. 4 Plot of the modulus of the coherence factor as a function of the distance between the waist of the beam and the diffuser. The experimental data, along with their uncertainties, are plotted as circles. The solid line represents the best fit of the experimental data with the modulus of the coherence function for a Gaussian distribution of irradiance. The dotted line is the best fitting of the data for an uniform distribution of irradiance.

complex coherence factor, $|\mu|$. These values have been obtained directly from the measured signals after applying equation (2). Since the signal from the antenna is not given in the same units as the signal obtained from a power meter that monitors the laser power, a normalization factor needs to be included in the calculation. This normalization factor is obtained after extrapolating the values to the case of $z_1=0$. The value of $|\mu|$ does not go to unity at the vertical axis when $z_1=0$ because the source does not collapse to a point source. The error bars have been obtained by calculating the standard deviation of a series of measurements for each one of the z_1 locations. Along with the experimental values, we have plotted a solid line representing the best fit to a Gaussian dependence of $|\mu|$, corresponding with a Gaussian illumination. The fitting value, $\Omega_{fit}=24.5$ mm, is smaller than the analytical value for Ω obtained from the measured data. However, when the uncertainties in the geometrical and beam parameters of the experiment ($\sigma_0=130$ μm, and $z=1.0$ mm) are accounted for, the value, $\Omega_{analytical}=23.0$ mm, matches reasonably well with the fitted value. The dotted line represents the dependence of $|\mu|$ for a uniform source having a radius equal to the Gaussian width of the Gaussian beam. This line is the best fitted curve for such dependence. The parameter of the fitting for the uniform illumination case also lies within the values obtained from the experimental set-up conditions when taking into account their uncertainties ($\sigma_0=130$ μm, and $z=1.2$ mm). The reason for this comparison is to take into account the effect of the double sided diffuser. The effect of the propagation from the first sandblasted surface to the second one would produce a more uniform distribution than the initial Gaussian one. The results show that this effect is unnoticeable within the experimental conditions shown here, making it difficult to distinguish between both cases.

5 Conclusions

In this experiment we used a gold nano-wire antenna pair coupled to a microbolometer to measure the complex coherence factor of an incoherent source. Good agreement between theory and experiment was obtained. This forms the experimental foundation for an optical synthetic aperture imaging system based on the Van Cittert-Zernike theorem, which does not, in principle at least, require a lens. Given that the signal loss on the transmission line can be reduced for a feasible sized array to be functional and 3-D lithographic construction is utilized in the design a lensless imager would have several advantages over its glass optical lens system counterpart. While weight and size reduction are the most intuitive of these advantages, radio frequency domain techniques that allow for pupil plane imaging can also be extended into the infrared. Completion of the aforementioned infrared antenna coupled micobolometer array experimentation shows promise towards the advancement of the creation of potentially large 2-D and 3-D antenna arrays that respond coherently to electromagnetic fields at infrared wavelengths.

Acknowledgements One of the authors, Javier Alda, acknowledges the financial support of the Programa de Movilidad PR2006-0006 and the Project TEC2006-01882 from the Ministerio de Educacion y Ciencia of Spain.

References

1. M. Born, and E. Wolf, *Principles of optics: electromagnetic theory of propagation, interference and diffraction of light* (Cambridge University Press, Cambridge; New York, 1999).

2. L. Mandel, and E. Wolf, Optical coherence and quantum optics (Cambridge University Press, Cambridge; New York, 1995).
3. J. W. Goodman, *Statistical optics* (Wiley, New York, 1985).
4. A. R. Thompson, J. M. Moran, and G. W. Swenson, *Interferometry and synthesis in radio astronomy* (Wiley, New York, 1986).
5. C.R. Vogel, *Computational methods for inverse problems* (Society for Industrial and Applied Mathematics, Philadelphia, 2002).
6. A. Yariv, *Optical electronics* (Holt Rinehart and Winston, New York, 1985).
7. P. Haguenauer, M. Severi, I. Schanen-Duport, K. Perraut-Rousselet, J-P Berger, Y. Duchene, M. Lacolle, P. Y. Kern, F. Malbet, P. Benech, Planar optics three-telescope beam combiners for astronomical interferometry. Proc. SPIE **4006**, 1107–1115 (2000).
8. I. Codreanu, F.J. González, and G.D. Boreman, Detection Mechanisms in microstrip dipole antenna-coupled infrared detectors. Infrared Physics & Technology, **44**, 155–163 (2003).
9. F. J. González, B. Ilic, J. Alda, and G. D. Boreman, Antenna-coupled infrared detectors for imaging applications, Selected Topics in Quantum Electronics. IEEE Journal **11**, 117–120, (2005).
10. T. Mandviwala, B. Lail, G. Boreman, Infrared-frequency Coplanar Striplines: Design, Fabrication, and Measurement. Microwave and Optical Technology Letters, **47**, 17–20 (2005).
11. T. Mandviwala, “Transmission lines for IR signal routing,” in *Electrical Engineering*, (University of Central Florida, Orlando, 2006), p. 108.