Analytical fitting model for rough-surface BRDF

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Abstract: A physics-based model is developed for rough surface BRDF, taking into account angles of incidence and scattering, effective index, surface autocovariance, and correlation length. Shadowing is introduced on surface correlation length and reflectance. Separate terms are included for surface scatter, bulk scatter and retroreflection. Using the FindFit function in Mathematica, the functional form is fitted to BRDF measurements over a wide range of incident angles. The model has fourteen fitting parameters; once these are fixed, the model accurately describes scattering data over two orders of magnitude in BRDF without further adjustment. The resulting analytical model is convenient for numerical computations.

OCIS codes: (290.1483) BRDF; (290.5880) Rough surfaces.

References and links

1. J. Stover, *Optical Scattering, Measurement and Analysis* (SPIE Press, 1995).
2. J. Ogilvy, *Theory of Wave Scattering from Random Rough Surfaces* (Hilger, 1991).
3. J. Sánchez-Gil and M. Nieto-Vesperinas, “Light scattering from random rough dielectric surfaces,” J. Opt. Soc. Am. A **8**, 1270-1286 (1991).
4. P. Beckman and A. Spizzichino, *The Scattering of Electromagnetic Waves from Rough Surfaces*, (Pergamon, 1963).
5. M. Saillard and A. Sentenac, “Rigorous solutions for electromagnetic scattering from rough surfaces,” Waves Random Media **11**, R103-R137 (2001).
6. T. Elfouhaily and C. Guérin, “A critical survey of approximate scattering wave theories from random rough surfaces,” Waves Random Media **14**, R1-R40 (2004).
7. Y. Sun, “Statistical ray method for deriving reflection models of rough surfaces,” J. Opt. Soc. Am. A **24**, 724-741 (2007).
8. J. Goodman, *Statistical Optics* (Wiley, 1985).
9. B. Hoover and V. Gamiz, “Coherence solution for bidirectional reflectance distributions of surfaces with wavelength-scale statistics,” J. Opt. Soc. Am. A **23**, 314-328 (2006).
10. J. Greffet and M. Nieto-Vesperinas, “Field theory for generalized bidirectional reflectivity: derivation of Helmholtz´s reciprocity principle and Kirchhoff´s law,” J. Opt. Soc. Am. A **15**, 2735-2744 (1998).
11. J. Harvey, C. Vernold, A. Krywonos and P. Thompson, “Diffracted radiance: a fundamental quantity in non-paraxial scalar diffraction theory, Appl. Opt. **38**, 6469-6481 (1999).
12. J. Jafolla, D. Thomas, J. Hilgers, B. Reynolds, and C. Blasband, “Theory and measurement of bidirectional reflectance for signature analysis,” Proc. SPIE **3699**, 2-15 (1999).
13. A. Ngan, F. Durand, and W. Matusik, “Experimental analysis of BRDF models,” in *Eurographics Symposium on Rendering*, K. Bala and P. Dutré, eds., (Eurographics Association, 2005), pp. 117-126.
14. R. Watson and P. Raven, “Comparison of measured BRDF data with parameterized reflectance models,” Proc. SPIE **4370**, 159-168 (2001).
15. J. Bennett and L. Mattsson, *Introduction to Surface Roughness and Scattering* (Optical Society of America, 1999).
16. G. Rasigni, F. Varnier, M. Rasigni, J. Palmari, and A. Llebaria, “Autocovariance functions for polished optical surfaces,” J. Opt. Soc. Am. **73**, 222-233 (1983).
17. S. Nee, “Ellipsometric analysis for surface roughness and texture,” Appl. Opt. **27**, 2819-2831 (1998).
18. D. Jordan, G. Lewis, and E. Jakeman, “Emission polarization of roughened glass and aluminum surfaces,” Appl. Opt. **35**, 3583-3590 (1996).
19. L. Wolff, “Diffuse-reflectance model for smooth dielectric surfaces,” J. Opt. Soc. Am. A **11**, 2956-2968 (1994).

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1. Introduction

The bidirectional reflectance distribution function (BRDF) has been extensively studied and surveyed in various ranges of validity [1-6]. However, there are fundamental problems in developing a complete description of surface scattering, whether using geometrical- or physical-optics modeling. Incident radiation can be directly scattered from the rough surface, or can be refracted into the surface and then scattered. The phenomenology is complex, including specular and diffuse reflection, high-angle forward scattering, and retro-reflection. A comprehensive model based on first principles would be very beneficial, but this has not been achieved thus far. Raytracing approaches [7] have been used to describe processes of refraction, reflection, scattering, absorption, masking and shadowing from statistically characterized surfaces and subsurface particles. However, raytracing does not give an accurate description of the scattering processes when diffraction is important. Diffraction problems are very difficult to solve rigorously for general rough surfaces; there has been progress using the perturbation method [2, 8, 9] for surfaces that are smooth compared to the wavelength. If the scale of the surface roughness is of the same order of magnitude as the wavelength, analytical BRDF models have limited accuracy and are difficult to implement. In these cases, numerical models are an alternative; however, the computational complexity limits the calculation to very small surface areas. Analytical forms for BRDFs are needed in many applications of interest, especially for use in modeling and simulation software.

In this paper, concepts from geometrical and physical optics are combined to model general opaque-surface scattering properties with a finite number of parameters that can be functionally related to physical observables. An analytical framework for the solution is obtained by assuming that the autocovariance function of the field at the surface can be represented by a phase screen using a Gaussian or exponential form [8]. Shadowing and masking are introduced from a geometrical context, and the influence of the surface roughness is modified using a shielding function that depends on the angle of incidence and the angle of scattering. BRDF data is fitted to the model using the nonlinear fitting routine FindFit in Mathematica. Once the fitting parameters are thus determined, our model yields excellent agreement to measured BRDF data over a wide range of angles of incidence, even while the BRDF magnitude itself varies by two orders of magnitude.

Our approach is significant in that, to the best of our knowledge, a method for fitting rough-surface BRDF measurements to a single fixed set of physically-interpretable parameters has not been previously presented in the literature.

2. Development of analytical form for BRDF model

The connection between the electromagnetic field autocovariance function and the corresponding surface autocovariance function is very difficult to formulate. Using a phase-screen approach generally leads to a Gaussian or exponential autocovariance with a corresponding Gaussian or Lorentzian BRDF [9]. The surface statistics and autocovariance functions link the surface properties to the scattering behavior through the BRDF. We let the surface autocovariance be denoted as $g(x,y)$, and its Fourier transform, corresponding to the surface power spectrum as $G(\alpha,\beta)$. To insure proper radiometric behavior of the results, the parameters α and β are interpreted as a direction cosines [10, 11], in which case $G(\alpha,\beta)$ is proportional to the BRDF of the surface. The direction cosines are defined with respect to the angles shown in Fig. 1 as: $\alpha_0 = \sin(\theta_i)$, $\alpha = \sin(\theta)\cos(\phi)$, and $\beta = -\sin(\theta)\sin(\phi)$. A Gaussian autocovariance function is given by

$$g(x,y) = \sigma_g^2 \, e^{-\left(x^2+y^2\right)\rho^2} \tag{1}$$

with the corresponding Fourier transform

$$G(\alpha,\beta) = \frac{\sigma_g^2}{\sqrt{2}\,\rho} e^{-\frac{1}{4}\left(\alpha^2+\beta^2\right)/\rho^2} \tag{2}$$

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where σ_g is the root-mean-square surface roughness, and ρ is the inverse of the surface correlation length. Similarly, an exponential autocovariance function is given by

$$g(x,y)=\sigma_g^2\, e^{-\left(|x|+|y|\right)\rho} \tag{3}$$

with a Lorentzian form for the Fourier transform

$$G(\alpha,\beta)=\frac{2}{\pi}\frac{\rho^2\sigma_g^2}{\alpha^2+\beta^2+\rho^2}\,. \tag{4}$$

There are many semi-empirical BRDFs proposed and to some extent tested, based on these two basic forms [12, 13]. The specific approach varies with the type of surfaces being fitted, the interpretation of the variables and any *ad hoc* modifications introduced to account for additional phenomenology [14]. A number of parameters are normally fitted separately for each angle of incidence, including a separate parameter for forward scattering. In our development, we will pursue a modified Lorentzian power spectrum for illustrative purposes. Most random surfaces are fairly well described by exponential statistics; but only occasionally with a Gaussian distribution [15, 16]. If needed, the Gaussian model could be developed in a similar fashion to the approach presented here. Surfaces with semi-deterministic or anisotropic roughness, or multimodal statistics would require a corresponding modification to the autocorrelation function.

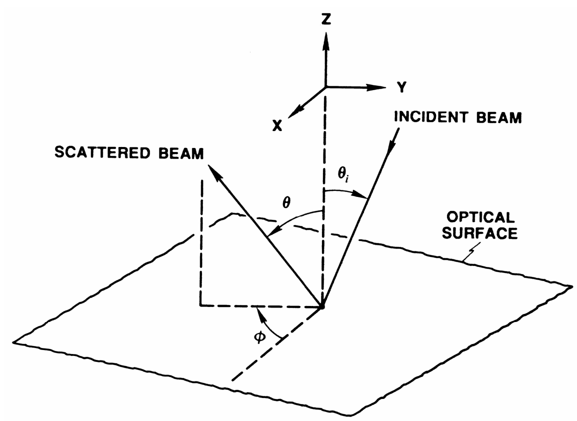

Fig. 1. Definition of coordinates for scattering geometry (adapted from [15]).

Taking the angle of incidence into account, the two-dimensional Lorentzian BRDF is given by

$$\mathrm{BRDF}=\sigma N\,\frac{1}{(\alpha-\alpha_0)^2+\beta^2+\rho^2} \tag{5}$$

where σ is the integrated reflectance. In this special case, an analytical normalization can be found that ensures the two-dimensional integral of the BRDF to be unity

$$N=\frac{1}{\pi\log\left[\dfrac{-\alpha_0^2+\rho^2+\sqrt{\rho^4+2\alpha_0^2\rho^2+2\rho^2+\left(1-\alpha_0^2\right)^2}+1}{2\rho^2}\right]}\,. \tag{6}$$

In conventional trigonometric coordinates, the BRDF is given by

$$\mathrm{BRDF} = \sigma N \frac{1}{\sin^2(\theta_i) - 2\cos(\phi)\sin(\theta)\sin(\theta_i) + \rho^2 + \sin^2(\theta)} \tag{7}$$

with corresponding normalization is given by

$$N = \frac{1}{\pi \log\left[\frac{-\sin^2(\theta_i) + \rho^2 + \sqrt{\rho^4 + 2\sin^2(\theta_i)\rho^2 + 2\rho^2 + \left(1 - \sin^2(\theta_i)\right)^2} + 1}{2\rho^2}\right]} \ . \tag{8}$$

To account for geometrical foreshortening, the surface correlation parameter ρ will vary with the angles of incidence and scattering, being replaced by the expression $½\rho[(1-\alpha_0^2)^{1/2} + (1-\alpha^2-\beta^2)^{1/2}]$ or $½\rho\left(\cos\theta_i + \cos\theta\right)$ in direction-cosine or trigonometric coordinates, respectively. Including this foreshortening effect, the BRDF is given by

$$\mathrm{BRDF} = \sigma N \frac{1}{\left[(\alpha-\alpha_0)^2 + \beta^2 + \frac{1}{4}\left(\sqrt{1-\alpha_0^2} + \sqrt{1-\alpha^2-\beta^2}\right)^2 \rho^2\right]} \tag{9}$$

where the normalization parameter *N* now depends on incidence and scattering angles, but for which no analytical expression has been found.

Including the angle of incidence in the expression for the surface correlation represents an improvement, but this alone does not account for the forward-scattering behavior observed at large angles of incidence, which increases the amount of specular-reflected radiation at the expense of the diffuse component. This effect can be understood physically, because at high angles of incidence and scattering, not all parts of the rough surface are illuminated. To account for this, a masking/shadowing factor *V* that multiplies ρ can be introduced for both the incident and scattered radiation. We assume a flattened-Gaussian form for this function, using the expression

$$V(\chi, w, n) = \frac{\Gamma\left(1+n, \frac{(1+n)\chi^2}{w^2}\right)}{n!} \tag{10}$$

where Γ is the incomplete Gamma function and *n* is a flatness parameter corresponding to the number of Gaussians included in a series expansion. In this formulation *w* is the 1/*e* width of the Gaussian for *n* = 0, since the limiting form of Γ for *n* = 0 is

$$V(\chi, w, 0) = \exp\left(-\frac{\chi^2}{w^2}\right) \ . \tag{11}$$

Because of reciprocity, this factor behaves similarly for incident and scattered radiation; hence for one multiplier of ρ we set $\chi = \alpha_0$; for the other we set $\chi = (\alpha^2 + \beta^2)^{1/2}$. Introducing this factor results in an improved description of the scattered intensity and the beam spread for large angles of incidence. The BRDF is now given by

$$\mathrm{BRDF} = \frac{\sigma N}{(\alpha-\alpha_0)^2+\beta^2+\frac{1}{4}\left(\sqrt{1-\alpha_0^2}+\sqrt{1-\alpha^2-\beta^2}\right)^2 V(\alpha_0,w,n)V\left(\sqrt{\alpha^2+\beta^2},w,n\right)\rho^2} \; . \tag{12}$$

Finally, another masking/shadowing function is added which affects the reflectance primarily at large angles of incidence and scattering. The model developed in [7] is adopted, but we have made an *ad hoc* modification to accommodate decreased shadowing in the forward-scattering direction at large angles. This effect is a result of the increasingly smooth appearance of the surface in the limit of grazing incidence. Our modified masking/shadowing function applied to reflectance is given by

$$\begin{aligned} V_m(\zeta,\zeta_0,w_a,w_b,\gamma,\delta) &= \frac{1}{2}\left\{1-\exp\left(-\gamma|\zeta_0|\,e^{-\frac{\delta}{\zeta_0^2}}\right)\right\}[\mathrm{Sgn}(\zeta)+1] \\ &+\exp\left(-w_a|\zeta|\,e^{-\frac{w_b}{\alpha^2}}\right)\left\{\frac{1}{2}[1-\mathrm{Sgn}(\zeta)]+\frac{1}{2}\exp\left(-\gamma|\zeta_0|\,e^{-\frac{\delta}{\zeta_0^2}}\right)[\mathrm{Sgn}(\zeta)+1]\right\} \end{aligned} \tag{13}$$

where a dependence on angle of incidence has been added compared to the original model in [7]. The variable ζ is related to the direction cosine α by $\zeta = \alpha/(1-\alpha^2)^{1/2}$. The parameters w_a and w_b represent surface roughness, while γ and δ determine the onset of the transition to a specular surface in the forward-scattering direction. An example of this function is illustrated below in Fig. 2, for two different angles of incidence.

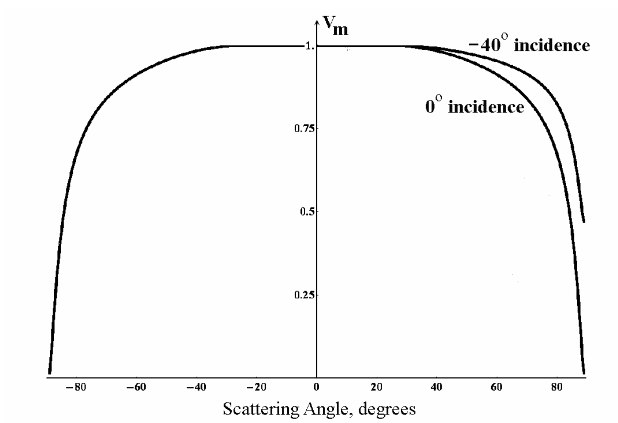

Fig. 2. Plot of masking/shadowing function V_m as a function of scattering angle, for two angles of incidence.

The complete BRDF is now given by

$$\mathrm{BRDF} = \frac{\sigma N V_m(\zeta,\zeta_0,w_a,w_b,\gamma,\delta)}{(\alpha-\alpha_0)^2+\beta^2+\frac{1}{4}\left(\sqrt{1-\alpha_0^2}+\sqrt{1-\alpha^2-\beta^2}\right)^2 V(\alpha_0,w,n)V\left(\sqrt{\alpha^2+\beta^2},w,n\right)\rho^2} \; . \tag{14}$$

For surface scattering, the reflectance σ can be calculated as the geometric mean of the Fresnel reflectance at the incident and scattered angles [1] as $\sigma_{\text{surface}} = [R(\alpha_0)\, R(\alpha,\beta)]^{1/2}$. Considering linearly polarized incident radiation, we use the effective complex index of refraction ($n_{\text{eff}} + ik_{\text{eff}}$) for the rough surface [17, 18] to obtain the reflection coefficients for *s*- and *p*-polarized radiation:

$$R_s(\alpha) = \left| \frac{\sqrt{1-\alpha^2} - (n_{\text{eff}} + ik_{\text{eff}})\sqrt{1 - \dfrac{\alpha^2}{(n_{\text{eff}} + ik_{\text{eff}})^2}}}{\sqrt{1-\alpha^2} + (n_{\text{eff}} + ik_{\text{eff}})\sqrt{1 - \dfrac{\alpha^2}{(n_{\text{eff}} + ik_{\text{eff}})^2}}} \right|^2$$

$$R_p(\alpha) = \left| \frac{-(n_{\text{eff}} + ik_{\text{eff}})\sqrt{1-\alpha^2} + \sqrt{1 - \dfrac{\alpha^2}{(n_{\text{eff}} + ik_{\text{eff}})^2}}}{(n_{\text{eff}} + ik_{\text{eff}})\sqrt{1-\alpha^2} + \sqrt{1 - \dfrac{\alpha^2}{(n_{\text{eff}} + ik_{\text{eff}})^2}}} \right|^2 . \tag{15}$$

By a summation of the various scattered radiation components, the BRDF model form as presented is used to separately account for specular and diffuse surface scattering, as well as bulk scattering and retroreflection. The total BRDF is now modeled as the sum

$$\text{BRDF}_{\text{total}} = \text{BRDF}_{\text{surface}} + \text{BRDF}_{\text{bulk}} + \text{BRDF}_{\text{retro}} \tag{16}$$

where the reflectance σ_{surface} is as given above for surface scattering; with

$$\sigma_{\text{bulk}} = \left[1 - \sqrt{R(\alpha_0)\, R(\alpha,\beta)}\right]\Sigma_{\text{bulk}} = (1 - \sigma_{\text{surface}})\Sigma_{\text{bulk}} \tag{17}$$

for the bulk scattering; and

$$\sigma_{\text{retro}} = \left[1 - \sqrt{R(\alpha_0)\, R(\alpha,\beta)}\right]\Sigma_{\text{retro}} = (1 - \sigma_{\text{surface}})\Sigma_{\text{retro}} \tag{18}$$

for the retroreflective scattering. The multiple-scattering processes where radiation is scattered from the surface after bulk scattering in the material are not considered further, since they are largely masked by the shadowing effect on the reflectance [19].

The BRDF model formulation could be extended to include wavelength as a variable to yield the bidirectional spectral reflectance distribution function (BSRDF). However, we will assume that the BRDF varies slowly with wavelength, similar to the spectral dependence of index of refraction. The parameters determined by the fitting process are therefore applicable within a given wavelength region. It should be noted that the wavelength dependence may well be different for the scattering parameters related to surface and bulk effects.

3. Fitting of experimental data to the analytical model

The total scattered radiation is modeled as the BRDF given above, which is a highly nonlinear function of the fitting parameters and variables. Given a data set of BRDF measured at various angles of incidence, a fitting routine is performed using the FindFit function in Mathematica, which uses a nonlinear iterative minimum-squared error procedure to determine optimum values of the fitting parameters.

By this process, the following fourteen fitting parameters are used: {n_{eff}, k_{eff}, σ_{surface}, ρ_{surface}, Σ_{bulk}, ρ_{bulk}, Σ_{retro}, ρ_{retro}, w_a, w_b, γ, δ, w, n}, optimizing the fit of the model to the variables corresponding to the measured BRDF data at the various angles: {$\text{BRDF}_{\text{total}}$, α_0, α,

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$\beta\}$. It should be noted that, in order to properly fit the parameters, there need to be sufficient data available, measured in appropriate conditions which exercise the various dependences. For example, it is necessary to have large-angle-of-incidence data available in order to determine parameter values related to the shadowing/masking functions V and V_m.

We illustrate our approach using BRDF data for a painted rough surface, measured at 0.63 micrometer wavelength, over a range of angles of incidence from 0 to 80 degrees. Measurements were made at 0.5 degree increments of scattering angle in the plane of incidence ($\beta = 0$), over a range from –80 to 80 degrees. The surface was chosen as an example for the fitting technique because it showed a significant amount of forward scatter for high angles of incidence, which is governed by the variation of reflectance as a function of both angles of incidence and scattering. This is representative of the challenge in modeling rough-surface BRDF and is why this particular surface data was chosen for illustration. Smoother surfaces can generally be handled more easily, and would also be amenable to description by our analytical model.

Comparison of the measured data (shown as discrete points) to the fitted BRDF derived from the model (solid curves) is shown in Fig. 3. The modeled results, using a single fixed set of fitting parameters, are seen to provide an excellent fit to the measured data, even for conditions of large variation of angle of incidence, wherein the magnitude of BRDF varies by more than two orders of magnitude.

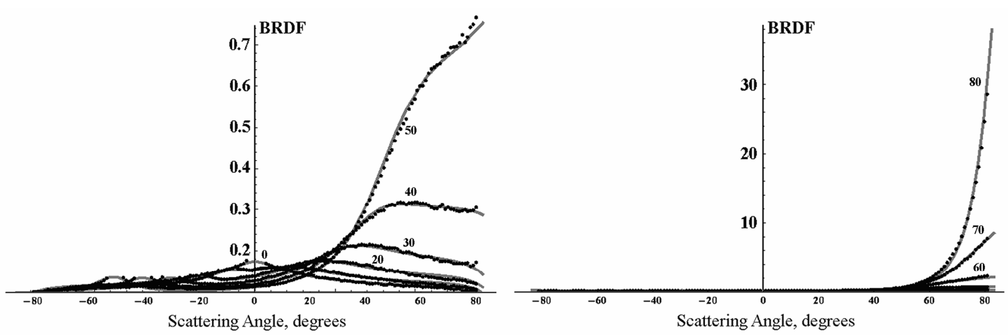

Fig. 3. Rough-surface BRDF as a function of in-plane scattering angle, parameterized on angle of incidence in degrees.

4. Discussion and conclusions

A BRDF model of this type, where the fitting parameters are related to the physical properties of the surface, is useful in several ways. First, the model can also be used to interpolate and extrapolate beyond measured data sets. Also, having fitted values for effective index of refraction, surface-scattering and bulk-scattering properties facilitates development of material databases that are derived from BRDF data. Further, the analytical form of the model as a function of the fitted parameters facilitates numerical calculation such as those required in image-simulation computer programs. This is particularly relevant to a description of forward-scattering behavior, which is significant in the data taken at large angles of incidence.

The general approach presented could be developed further by pursuing fitted relationships between scattering behavior and fundamental surface properties such as the surface-height and slope distributions. Comparative fittings to determine the best statistical surface description could also be pursued. It would also be useful from a design-of-experiments viewpoint to examine the variance of the fitted parameters to determine data-sufficiency criteria (especially in angular sampling and angular coverage) to ensure robust estimates of the various quantities of interest. Also, for the sake of convenience, we have postulated two shadowing/masking processes denoted by the functions V and V_m, which are likely to be related if a sufficiently detailed physics-based model were available.

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