Phase conjugation with random fields and with deterministic and random scatterers

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The theory of distortion correction by phase conjugation, developed since the discovery of this phenomenon many years ago, applies to situations when the field that is conjugated is monochromatic and the medium with which it interacts is deterministic. In this Letter a generalization of the theory is presented that applies to phase conjugation of partially coherent waves interacting with either deterministic or random weakly scattering nonabsorbing media.

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Since the publication of a pioneering paper by Zel'dovich and collaborators,[1] the possibility of correcting various distortion effects imparted on a beam that is incident on a scattering medium by the use of the technique of phase conjugation has been confirmed by many experiments (see, for example, Refs. 2–4). The early theoretical analyses of this effect were based on the paraxial approximation[5] or on the first Born approximation.[6] Later treatments were based on more accurate analyses that took into account higher-order terms in the perturbation expansion for phase conjugation involving a broad class of scatterers.[7,8]

In all of the theoretical investigations of this subject, it was assumed that the incident field is monochromatic and that the scattering medium is deterministic. The question arises whether these assumptions can be relaxed, i.e., whether it is possible to eliminate by phase conjugation the effects of distortion imparted by the medium on the incident wave when the wave is partially coherent or when the medium is random.[9] The purpose of this Letter is to elucidate these questions.

We begin with a simpler problem, concerning the effect of phase conjugation on a partially coherent wave in free space. Consider a statistically stationary, partially coherent wave field that is incident on a phase-conjugate mirror, located in the plane $z = z_1$. For simplicity we assume that the incident field does not contain any evanescent components and that the mirror occupies the whole plane.

According to coherence theory in the space-frequency domain (Ref. 10, Sec. 4.7, especially Sec. 4.7.2), we may represent each temporal frequency component of the field in terms of an ensemble, denoted by curly brackets, $\{U(\mathbf{r},\omega)\exp(-i\omega t)\}$, of monochromatic fields, each with the same frequency ω. The phase-conjugate mirror generates a conjugate field on it surface at $z = z_1$, represented by an ensemble $\{U^{(c)}(\mathbf{r},\omega)|_{z=z_1}\exp(-i\omega t)\}$, with

$$U^{(c)}(\mathbf{r},\omega)|_{z=z_1} = \eta(\omega)U^*(\mathbf{r},\omega)|_{z=z_1}\,, \tag{1}$$

where $\eta(\omega)$ denotes the reflectivity of the phase-conjugate mirror.

It will be convenient to decompose the position vector $\mathbf{r}$ into a two-dimensional transverse vector component $\boldsymbol{\rho}$ and a longitudinal component z so that $\mathbf{r} \equiv (\boldsymbol{\rho}, z)$ (see Fig. 1). Equation (1) then becomes

$$U^{(c)}(\boldsymbol{\rho}, z_1;\omega) = \eta(\omega)U^*(\boldsymbol{\rho}, z_1;\omega)\,. \tag{2}$$

According to a theorem derived in Ref. 11 (Theorem 6, p. 1315), one has for all $z \le z_1$,

$$U^{(c)}(\boldsymbol{\rho}, z;\omega) = \eta(\omega)U^*(\boldsymbol{\rho}, z;\omega)\,. \tag{3}$$

The physically significant quantities are, however, not the individual members U and $U^{(c)}$ of the statistical ensembles which represent the fluctuating incident and the conjugated fields, respectively, but rather their correlation functions, such as their cross-spectral densities [Ref. 10, Sec. 4.3.2 and Sec. 4.7.3, Eq. (4.7–60)],

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle\,, \tag{4a}$$

$$W^{(c)}(\mathbf{r}_1,\mathbf{r}_2,\omega) = \langle U^{(c)*}(\mathbf{r}_1,\omega)U^{(c)}(\mathbf{r}_2,\omega)\rangle\,, \tag{4b}$$

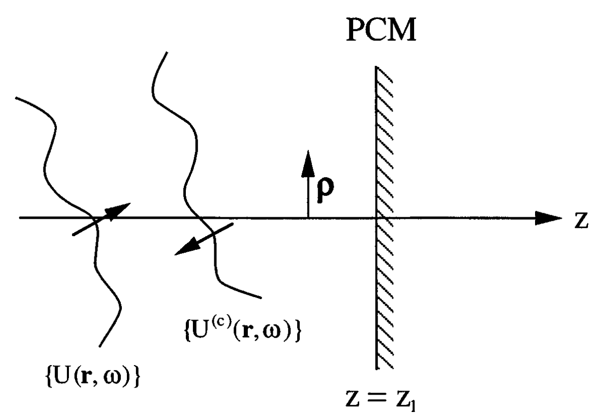

Fig. 1. Illustrating the notation relating to the behavior of waves in the vicinity of a phase-conjugate mirror (PCM). $\{U(\mathbf{r},\omega)\}$ and $\{U^{(c)}(\mathbf{r},\omega)\}$ represent the statistical ensembles characterizing the incident and the phase-conjugate field, respectively.

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with the angle brackets denoting the ensemble average. It immediately follows on substituting from Eq. (3) into Eq. (4b) and using Eq. (4a) that

$$W^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = |\eta(\omega)|^2 W^*(\mathbf{r}_1, \mathbf{r}_2, \omega) \quad \text{when } z \le z_1 . \tag{5}$$

This relation shows that *the cross-spectral density of the conjugated field throughout the half-space $z < z_1$ is proportional to the complex conjugate of the cross-spectral density of the field incident on the phase-conjugate mirror.*[12] We note that because the cross-spectral density is Hermitian with respect to the interchange of $\mathbf{r}_1$ and $\mathbf{r}_2$ Eq. (5) may also be expressed in the form

$$W^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = |\eta(\omega)|^2 W(\mathbf{r}_2, \mathbf{r}_1, \omega) . \tag{6}$$

From Eqs. (5) and (6) we may also derive a simple relation between the spectral degree of coherence (Ref. 10, Sec. 4.3.2) of the incident field,

$$\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{W(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{W(\mathbf{r}_1, \mathbf{r}_1, \omega)}\sqrt{W(\mathbf{r}_2, \mathbf{r}_2, \omega)}} , \tag{7}$$

and the spectral degree of coherence of the conjugated field,

$$\mu^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{W^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{W^{(c)}(\mathbf{r}_1, \mathbf{r}_1, \omega)}\sqrt{W^{(c)}(\mathbf{r}_2, \mathbf{r}_2, \omega)}} \cdot \tag{8}$$

It follows at once by making use of Eqs. (5) and (6) that

$$\mu^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \mu^*(\mathbf{r}_1, \mathbf{r}_2, \omega) \quad \text{when } z \le z_1 \tag{9}$$

or, equivalently,

$$\mu^{(c)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \mu(\mathbf{r}_2, \mathbf{r}_1, \omega) \quad \text{when } z \le z_1 . \tag{10}$$

At frequencies for which $\eta(\omega)$ is nonzero, Eqs. (9) and (10) are evidently independent of the mirror reflectivity. The degree of coherence of the field is, therefore, restored frequency by frequency by the phase conjugation, even though the cross-spectral density function $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ may be significantly altered in some cases.

Suppose next that a partially coherent incident field is scattered by a deterministic medium and is then phase conjugated. The conjugated field propagates back toward the medium and is also scattered by it. We will examine the relationship between the conjugated field after it has been scattered and the incident field prior to scattering. Because of the complexity of the problem we will assume that the incident and the scattered fields do not contain evanescent components and that the following additional conditions are satisfied:

(1) The scatterer is weak in the sense that the scattered field may be described, to a good approximation, by the first-order Born approximation.
(2) The scatterer is nonabsorbing.
(3) Backscattering of both the incident and the conjugated fields is negligible.

As before, we represent the incident and the conjugated field by ensembles of monochromatic realizations $\{U(\mathbf{r}, \omega)\exp(-i\omega t)\}$ and $\{U^{(c)}(\mathbf{r}, \omega)\exp(-i\omega t)\}$, respectively. Let us suppose that the scattering medium is located in the strip $0 < z < L$ and that the phase-conjugate mirror, again taken to be infinite, is located in the plane $z = z_1 > L$ (see Fig. 2). It was shown in Ref. 6, Eq. (3.4), that under the assumptions stated above, the conjugated field $U^{(c)}(\mathbf{r}, \omega)$ at any point in the half-space $z < 0$, produced from an incident monochromatic field $U(\mathbf{r}, \omega)$, are related by the equation

$$U^{(c)}(\mathbf{r}, \omega) = \eta(\omega) U^*(\mathbf{r}, \omega) , \tag{11}$$

where $\eta(\omega)$ is again the reflectivity of the phase-conjugate mirror. This equation is identical with Eq. (3), with $(\boldsymbol{\rho}, z) \equiv \mathbf{r}$, and hence the same conclusion can be derived from it. Consequently, Eqs. (5), (6), (9), and (10) hold in the half-space $z < 0$. This result implies that *under the assumptions stated above, the presence of a deterministic scatterer in the strip $0 < z < L$ has no influence whatsoever on the partially coherent conjugated field in the half-space $z < 0$.*

Finally, let us consider the situation when the incident field is partially coherent and the scatterer is spatially random rather than deterministic but again satisfies the assumptions stated earlier. The random scatterer may be characterized by an ensemble of deterministic scatterers, and the result that we just derived will hold for each realization of the ensemble of the scatterer. Consequently, the result just stated holds not only for phase conjugation with deterministic scatterers but also with *random scatterers,* irrespective of the state of coherence of the incident field.

In the first part of this Letter we considered the effect of a phase-conjugate mirror on a partially coherent field in free space. We found simple relations between the cross-spectral density and the spectral degree of coherence of the incident field and of the conjugated field.

In the second part we considered correction by phase conjugation of distortions imparted on a partially coherent wave by scattering on a deterministic or on a random medium. We found that in both cases, provided that the scatterer is weak and nonabsorbing and that backscattering is negligible, the distortion

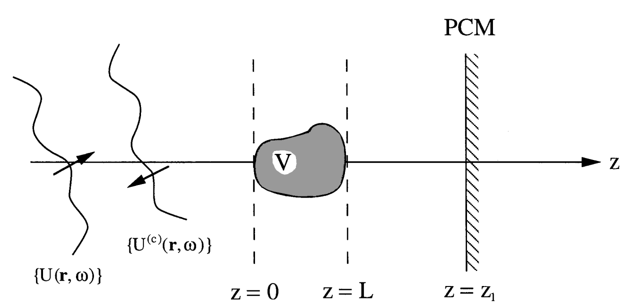

Fig. 2. Illustrating phase conjugation of a partially coherent wave scattered by a medium occupying a volume V. $\{U(\mathbf{r}, \omega)\}$ and $\{U^{(c)}(\mathbf{r}, \omega)\}$ have the same meaning as in Fig. 1.

of the incident field is completely canceled by phase conjugation.

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