Spreading of partially coherent beams in random media

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Some published computational work has suggested that partially coherent beams may be less susceptible to distortions caused by propagation through random media than fully coherent beams. In this paper this suggestion is studied quantitatively by examining the mean squared width of partially coherent beams in such media as a function of the propagation distance. The analysis indicates under what conditions, and to what extent, partially coherent beams are less affected by the medium.

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1. INTRODUCTION

The propagation of waves through random media is a topic that has been of considerable theoretical and practical interest for a long time, as is evident from the number of books and papers written on the subject (see, for instance, Refs. 1–3). In particular, it is of interest in connection with work in optical communications, imaging systems, and targeting systems to further the understanding of the effects of turbulence on the propagation of beams, and it would be useful in such applications to utilize beams that are distorted as little as possible by the presence of a turbulent medium.

In this context, the behavior of partially coherent beams in random media has received little attention (some relevant work is described in Ref. 4). Some published computational work[5,6] has suggested that beams that are partially coherent are less sensitive to the effects of turbulence than fully coherent ones. However, the criterion "insensitivity to turbulence" is somewhat vague in these papers, and it is not clear under what circumstances a partially coherent beam will be less affected by turbulence than a coherent one. Furthermore, it is not entirely clear what the reasons for it may be.

In this paper we consider these questions, and we derive a simple formula for the spreading of partially coherent beams in turbulent media. The formula may be considered a generalization of those previously derived for the spreading of coherent beams in a turbulent medium[7] and the spreading of partially coherent beams in free space.[8,9] Our results clarify under what circumstances a partially coherent beam will be less affected by turbulence than a fully coherent one.

2. PROPAGATION THROUGH TURBULENCE

We consider a quasi-monochromatic field of mean frequency ω propagating from the plane $z = 0$ into the half-space $z > 0$ (see Fig. 1). It is assumed that the index of refraction $n(\mathbf{r})$ in this half-space is a random function of position and that the turbulence is weak, so that

$$n(\mathbf{r}) = 1 + n_1(\mathbf{r}), \tag{1}$$

where $n_1 \ll 1$. The quantities $n(\mathbf{r})$ and $n_1(\mathbf{r})$ are taken to be real. For sufficiently weak turbulence, the state of polarization of an electromagnetic field will not be significantly changed on propagation, and we may then consider only a single component of the incident field, which we denote $U(\mathbf{r})$. The field may then be described by the scalar wave equation [Ref. 2, Eq. (2.19)],

$$\{\nabla^2 + k^2[1 + 2n_1(\mathbf{r})]\}U(\mathbf{r}) = 0, \tag{2}$$

where $k = \omega/c$ is the wave number of the incident radiation and c is the vacuum speed of light. Provided that the incident field $U_0(\mathbf{r})$ propagates close to the z axis, the solution of Eq. (2) may be expressed in the form [Ref. 2, Eq. (2.46)]

$$U(\mathbf{r}) = \frac{2ik}{z}\int U_0(\boldsymbol{\rho}')\exp[\psi(\boldsymbol{\rho}', \mathbf{r})]G_0(\boldsymbol{\rho}', \mathbf{r})\mathrm{d}^2\rho', \tag{3}$$

where G_0 is the paraxial free-space propagator,

$$G_0(\boldsymbol{\rho}', \mathbf{r}) = \exp\left[ikz + ik\frac{(x - x')^2 + (y - y')^2}{2z}\right], \tag{4}$$

and $\psi(\boldsymbol{\rho}', \mathbf{r})$ is a (generally complex) phase function that depends on the properties of the medium. Formula (3) is often referred to as the extended Huygens–Fresnel principle.

The intensity $I(\boldsymbol{\rho}, z)$ of the field at any point $\mathbf{r} \equiv (\boldsymbol{\rho}, z)$ is given by the squared modulus of the field at that point, i.e.,

$$\begin{aligned} I(\boldsymbol{\rho}, z) &\equiv |U(\boldsymbol{\rho}, z)|^2 \\ &= \frac{4k^2}{z^2}\iint U_0^*(\boldsymbol{\rho}_1')U_0(\boldsymbol{\rho}_2')\exp[\psi^*(\boldsymbol{\rho}_1', \boldsymbol{\rho}, z) \\ &\quad + \psi(\boldsymbol{\rho}_2', \boldsymbol{\rho}, z)]G_0^*(\boldsymbol{\rho}_1', \boldsymbol{\rho}, z) \\ &\quad \times G_0(\boldsymbol{\rho}_2', \boldsymbol{\rho}, z)\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \end{aligned} \tag{5}$$

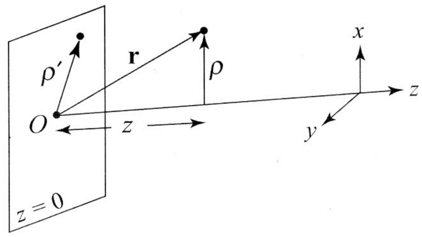

Fig. 1. Illustration of the notation relating to the propagation of a beam.

We assume that the medium into which the beam propagates is spatially random; furthermore, we are interested in beams that are partially coherent. In taking into account the randomness of both the field and the medium, we must take the two corresponding ensemble averages, which, for weak scattering, may be considered to be independent. Hence

$$\langle I(\boldsymbol{\rho}, z)\rangle = \frac{4k^2}{z^2}\int\int W_0(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2')C_\psi(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \boldsymbol{\rho}, \boldsymbol{\rho}, z) \times G_0^*(\boldsymbol{\rho}_1', \mathbf{r})G_0(\boldsymbol{\rho}_2', \mathbf{r})\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2', \tag{6}$$

where

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \langle U_0^*(\boldsymbol{\rho}_1)U_0(\boldsymbol{\rho}_2)\rangle \tag{7}$$

is the cross-spectral density [Ref. 10, Sec. 4.3.2] of the field at points $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ in the plane $z = 0$ and

$$C_\psi(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2'; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \langle \exp[\psi^*(\boldsymbol{\rho}_1', \boldsymbol{\rho}_1, z) + \psi(\boldsymbol{\rho}_2', \boldsymbol{\rho}_2, z)]\rangle. \tag{8}$$

The angle brackets in Eq. (7) denote averaging over the field ensemble, while those in Eq. (8) denote averaging over the ensemble of the random medium.

If the scattering medium is statistically homogeneous and isotropic, as we will assume, it can be shown that [Ref. 3, Sec. 12.2.1]

$$C_\psi(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) = \exp[2E_1(z) + E_2(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, z)], \tag{9}$$

where

$$E_1(z) = -2\pi^2k^2z\int_0^\infty \kappa\Phi_n(\kappa)\mathrm{d}\kappa, \tag{10}$$

$$E_2(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, z) = 4\pi^2k^2z\int_0^1\int_0^\infty \kappa\Phi_n(\kappa) \times J_0[\kappa|(1-\xi)(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1) + \xi(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1')|]\mathrm{d}\kappa\mathrm{d}\xi. \tag{11}$$

In these formulas, $\Phi_n(\kappa)$ is the spatial power spectrum of the refractive-index fluctuations of the random medium, and J_0 is the Bessel function of the first kind and zero order. Formula (9) is an approximation that is valid if the turbulence is sufficiently weak; it is exact only if the quantity $\psi^* + \psi$ is a Gaussian random variable [Ref. 3, Chap. 6].

It is to be noted that C_ψ depends only on the difference between $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$, so we may write

$$C_\psi(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2', \boldsymbol{\rho}_1, \boldsymbol{\rho}_2, z) \equiv C_\psi(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', \boldsymbol{\rho}_2 - \boldsymbol{\rho}_1, z). \tag{12}$$

Let us now suppose that the partially coherent field at the plane $z = 0$ is of the Schell-model type [Ref. 10, Sec. 5.3.2], i.e., that its cross-spectral density has the form

$$W_0(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2') = \sqrt{I_0(\boldsymbol{\rho}_1')}\sqrt{I_0(\boldsymbol{\rho}_2')}\mu_0(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1'), \tag{13}$$

where $I_0(\boldsymbol{\rho})$ is the averaged intensity and $\mu_0(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1')$ the spectral degree of coherence of the light in the plane $z = 0$. It follows from Eqs. (6), (12), and (13) that the intensity of the field in the half-space $z > 0$ may then be expressed as

$$\langle I(\boldsymbol{\rho}, z)\rangle = \frac{4k^2}{z^2}\int\int \sqrt{I_0(\boldsymbol{\rho}_1')}\sqrt{I_0(\boldsymbol{\rho}_2')}\mu_0(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1') \times C_\psi(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1', 0, z)G_0^*(\boldsymbol{\rho}_1', \boldsymbol{\rho}, z) \times G_0(\boldsymbol{\rho}_2', \boldsymbol{\rho}, z)\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \tag{14}$$

Surprisingly, this expression for the expectation value of the intensity of a partially coherent field propagating in a random medium is of the same form as the formula for the intensity of a field with initial cross-spectral density $W_1(\boldsymbol{\rho}_1', \boldsymbol{\rho}_2') = I_1(\boldsymbol{\rho}_1')I_1(\boldsymbol{\rho}_2')\mu_1(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1')$ propagating in free space, for which

$$I_1(\boldsymbol{\rho}) = I_0(\boldsymbol{\rho}), \tag{15}$$

$$\mu_1(\boldsymbol{\rho}') = \mu_0(\boldsymbol{\rho}')C_\psi(\boldsymbol{\rho}', 0, z). \tag{16}$$

This result may be stated more concisely as follows:

The expectation value of the intensity at a given plane $z = \text{const.} > 0$ of the field generated by a partially coherent Schell-model source propagating through a homogeneous isotropic random medium is the same as that of the field generated on propagation in free space by the equivalent partially coherent source characterized by Eqs. (15) and (16).

Note, however, that the "equivalent source" itself depends on z, so that the z dependence of the intensity differs from that of a field propagating in free space. However, for a given z plane, we may calculate any quantities that depend on the intensity as if the "equivalent source" had produced them. We will use this property next.

3. SPREADING OF A PARTIALLY COHERENT BEAM IN TURBULENCE

Equation (14) already gives some indication of conditions under which a beam will be essentially unaffected by turbulence. If $\mu_0(\boldsymbol{\rho}')$ is a much narrower function than $C_\psi(\boldsymbol{\rho}', 0, z)$ (see Fig. 2), then the latter function may be approximated by its value at $\boldsymbol{\rho}' = 0$, and one then has

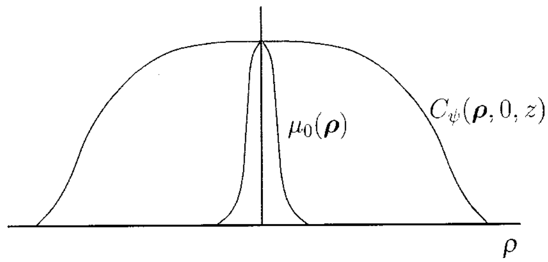

Fig. 2. Illustration of the condition for the validity of approximation (17).

$$\langle I(\boldsymbol{\rho}, z)\rangle \approx C_\psi(0, 0, z)\frac{4k^2}{z^2} \times \iint \sqrt{I_0(\boldsymbol{\rho}_1')}\sqrt{I_0(\boldsymbol{\rho}_2')}\mu_0(\boldsymbol{\rho}_2' - \boldsymbol{\rho}_1') \times G_0^*(\boldsymbol{\rho}_1', \boldsymbol{\rho}, z)G_0(\boldsymbol{\rho}_2', \boldsymbol{\rho}, z)\mathrm{d}^2\rho_1'\mathrm{d}^2\rho_2'. \tag{17}$$

Because $C_\psi(0, 0, z) = 1$, approximation (17) implies that under these circumstances the beam will propagate as if it were in free space. However, it can be seen from Eqs. (9), (10), and (11) that the function $C_\psi(\boldsymbol{\rho}', 0, z)$ becomes narrower with respect to $\boldsymbol{\rho}'$ as z increases, so at some distance sufficiently far from the plane $z = 0$, approximation (17) becomes invalid and the turbulence then starts to significantly affect the beam.

We can make these observations more quantitative by considering the normalized mean-squared width of the beam as a function of the propagation distance, i.e., by considering

$$\overline{\rho^2(z)} \equiv \frac{\int \rho^2 I(\boldsymbol{\rho}, z)\mathrm{d}^2\rho}{\int I(\boldsymbol{\rho}, z)\mathrm{d}^2\rho}. \tag{18}$$

We can express this quantity in a more explicit form by use of the formulas for free-space propagation [Ref. 9, Eq. (33)], using the effective source defined by Eqs. (15) and (16). We assume that the phase front of the wave is (on average) constant. It then follows that

$$\overline{\rho^2(z)} = a_2 + c_2(z)z^2, \tag{19}$$

where

$$a_2 = \frac{\int I_1(\boldsymbol{\rho})\rho^2\mathrm{d}^2\rho}{\int I_1(\boldsymbol{\rho})\mathrm{d}^2\rho}, \tag{20}$$

$$c_2(z) = \frac{\int \tilde{W}_1(-k\mathbf{s}_\perp, k\mathbf{s}_\perp)s_\perp^2\mathrm{d}^2s}{\int \tilde{W}_1(-k\mathbf{s}_\perp, k\mathbf{s}_\perp)\mathrm{d}^2s}, \tag{21}$$

and

$$\tilde{W}_1(\mathbf{K}_1, \mathbf{K}_2) = \frac{1}{(2\pi)^4}\int W_1(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \times \exp[-i(\mathbf{K}_1\cdot\boldsymbol{\rho}_1 + \mathbf{K}_2\cdot\boldsymbol{\rho}_2)]\mathrm{d}^2\rho_1\mathrm{d}^2\rho_2 \tag{22}$$

is the two-dimensional spatial Fourier transform of the effective cross-spectral density of the source. We note that c_2 depends on z through the z dependence of μ_1.

The quantities defined by Eqs. (20) and (21) may be shown to be given by the expressions

$$a_2 = \sigma_I^2, \tag{23}$$

where

$$\sigma_I^2 = \frac{\int I_0(\boldsymbol{\rho})\rho^2\mathrm{d}^2\rho}{\int I_0(\boldsymbol{\rho})\mathrm{d}^2\rho}, \tag{24}$$

and

$$c_2(z) = \left[\sigma_J^2 - \frac{1}{k^2}\nabla_\rho^2 C_\psi(\rho, 0, z)|_{\rho=0}\right], \tag{25}$$

with

$$\sigma_J^2 = \frac{\int J(\mathbf{s})s_\perp^2\mathrm{d}^2s}{\int J(\mathbf{s})\mathrm{d}^2s}, \tag{26}$$

and

$$J(\mathbf{s}) = (2\pi k)^2\tilde{W}_0(-k\mathbf{s}_\perp, k\mathbf{s}_\perp) \tag{27}$$

is the radiant intensity of the field in free space. $J(\mathbf{s})$ is defined as the power radiated by the source per unit solid angle around the $\mathbf{s}$ direction [Ref. 10, Sec. 5.2.1]. The quantity σ_I is the normalized RMS width of the intensity in the plane $z = 0$, and σ_J evidently represents the normalized RMS width of J, i.e. it is a measure of the angular spread of the beam *in free space*.

By use of Eqs. (9) and (11), and the properties of Bessel functions, one may evaluate the second expression in the square brackets of Eq. (25), and one finds that

$$-\frac{1}{k^2}\nabla_\rho^2 C_\psi(\rho, 0, z)|_{\rho=0} = zF_2, \tag{28}$$

where

$$F_2 = \frac{2\pi^2}{3}\int_0^\infty \kappa^3\Phi_n(\kappa)\mathrm{d}\kappa. \tag{29}$$

On substitution from Eqs. (28) and (25) into Eq. (19), we may express the mean squared width of the beam propagating through turbulence in a physically more transparent form as

$$\overline{\rho^2(z)} = \sigma_I^2\left(1 + \frac{\sigma_J^2}{\sigma_I^2}z^2 + \frac{F_2}{\sigma_I^2}z^3\right). \tag{30}$$

This formula, which is the main result of this paper, is a generalization of known formulas for the spreading of coherent beams in turbulence[7] and the spreading of partially coherent beams in free space.[8,9]

It was shown in Ref. 9 that the first two terms on the right-hand side of Eq. (30) represent the diffractive spreading of a partially coherent beam in free space. Hence the effect of the turbulent medium is contained entirely within the third (cubic) term. With increasing z, the cubic term will eventually dominate the expression for beam spreading. Every beam, be it fully or partially coherent, succumbs at some distance to the deteriorating effects of the turbulence.

It is also to be noted that the initial state of coherence of the beam is represented in Eq. (30) entirely by the quantity σ_J^2, as can be seen from expression (27). Any improvement of beam propagation characteristics due to partial coherence will therefore be related to the magnitude of σ_J^2.

Finally, we note that there are potentially three distinct regions along the direction of propagation of such a beam:

1. A region in which the beam propagates essentially parallel, without spreading.
2. A region in which the beam spreads as a result of free-space diffraction.
3. A region in which the beam spreads rapidly owing to turbulence deterioration.

The sizes of these regions is determined by the relative sizes of the three terms in formula (30).

4. RANGE OF TURBULENCE-INDEPENDENT PROPAGATION

The basic formula [Eq. (30)] may be written in a slightly different form by recalling that the ratio

$$z_R = \sigma_I/\sigma_J \tag{31}$$

represents the so-called Rayleigh range, defined as the distance at which the cross-sectional area of a beam propagating in free space doubles.[9] By use of Eq. (31), Eq. (30) may be expressed as

$$\overline{\rho^2(z)} = \sigma_I^2\left(1 + \frac{z^2}{z_R^2} + \frac{F_2}{\sigma_I^2}z^3\right). \tag{32}$$

We may introduce a quantity somewhat similar to the Rayleigh range to quantify the effect of the turbulent medium on a partially coherent beam. Let us define a turbulence distance z_T as the distance at which the third term of Eq. (32) accounts for 10% of the magnitude of $\overline{\rho^2(z)}$, i.e., such that

$$\frac{\overline{\rho^2(z_T)}_{\text{turb}} - \overline{\rho^2(z_T)}_{\text{free}}}{\overline{\rho^2(z_T)}_{\text{turb}}} = 0.1. \tag{33}$$

The quantity z_T thus represents the distance at which the spreading due to the turbulent medium accounts for 10% of the cross-sectional area of the beam. In Eq. (33), $\overline{\rho^2(z)}_{\text{turb}}$ is the mean squared width of the beam propagating in turbulence, given by Eq. (32). The quantity $\overline{\rho^2(z_T)}_{\text{free}}$ is the mean squared width of a beam with the same coherence function and the same intensity profile at $z = 0$ propagating in free space, given by the expression

$$\overline{\rho^2(z)}_{\text{free}} = \sigma_I^2 + \sigma_J^2 z^2. \tag{34}$$

It can be seen by substitution from Eq. (32) into Eq. (33) that the determination of z_T involves the solution of a cubic equation in z_T. Although this can be done, the solution is complicated and difficult to interpret. Under certain circumstances, however, the expression for z_T may be simplified.

Let us first suppose that the turbulence is so weak that it does not influence the beam until the beam has spread appreciably as a result of diffraction; then the first term on the right-hand sides of Eqs. (32) and (34) may be neglected, and one then has

$$\overline{\rho^2(z)}_{\text{free}} \approx \sigma_J^2 z^2, \tag{35}$$

$$\overline{\rho^2(z)}_{\text{turb}} \approx \sigma_J^2 z^2 + F_2 z^3. \tag{36}$$

On substitution of Eqs. (35) and (36) into Eq. (33), one finds that

$$z_T \approx \frac{1}{9}\frac{\sigma_J^2}{F_2}. \tag{37}$$

For a beam produced by a Schell-model source [whose cross-spectral density is of the form of Eq. (13)] as we are considering in this paper, it can be shown that[9]

$$\sigma_J^2 = [\sigma_J^2]_{\text{coh}} - \frac{1}{k^2}\nabla_\rho^2 \mu_0(\boldsymbol{\rho})|_{\boldsymbol{\rho}=0}, \tag{38}$$

where $[\sigma_J^2]_{\text{coh}}$ is the angular spread in free space of a fully coherent beam with initial intensity $I_0(\boldsymbol{\rho})$. The quantity $[\sigma_J^2]_{\text{coh}}$ may be expressed in the form

$$[\sigma_J^2]_{\text{coh}} = \frac{\displaystyle\int \tilde{W}_{\text{coh}}(-k\mathbf{s}_\perp, k\mathbf{s}_\perp)s_\perp^2 \mathrm{d}^2 s}{\displaystyle\int \tilde{W}_{\text{coh}}(-k\mathbf{s}_\perp, k\mathbf{s}_\perp)\mathrm{d}^2 s}, \tag{39}$$

with

$$W_{\text{coh}}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sqrt{I_0(\boldsymbol{\rho}_1)}\sqrt{I_0(\boldsymbol{\rho}_2)}, \tag{40}$$

and $\tilde{W}_{\text{coh}}$ denotes the fourfold Fourier transform of W_{coh}, defined with the same convention as in Eq. (22). For a beam that is partially coherent, the second term on the right-hand side of Eq. (38) is positive, indicating that σ_J^2 is always larger for a partially coherent beam than for a fully coherent beam with the same intensity distribution in the plane $z = 0$. As can be seen from approximation (37), this in turn suggests that a partially coherent beam propagates without distortion further in a turbulent medium than does the fully coherent one.

The approximation involved in Eqs. (35) and (36) may be used when the parameter z_T is much greater than the Rayleigh range, i.e., when $z_T/z_R \gg 1$, or, more explicitly, when

$$\frac{\sigma_J^3}{\sigma_I F_2} \gg 1. \tag{41}$$

We may also consider the case when the turbulence distorts the beam well before free-space diffraction is appreciable. In this case, we may write

$$\overline{\rho^2(z)}_{\text{free}} \approx \sigma_I^2, \tag{42}$$

$$\overline{\rho^2(z_T)}_{\text{turb}} \approx \sigma_I^2 + F_2 z^3. \tag{43}$$

On substituting from Eqs. (42) and (43) into Eq. (33), it follows that

$$z_T \approx \sqrt[3]{\frac{1}{9}\frac{\sigma_I^2}{F_2}}. \tag{44}$$

This expression will be valid when z_T is much smaller than the Rayleigh range, i.e., when

$$\sqrt[3]{\frac{\sigma_J^3}{\sigma_I F_2}} \ll 1. \tag{45}$$

We note that the parameters in inequalities (41) and (45) are the same. It is convenient to introduce a turbulence parameter R_T as

$$R_T \equiv \frac{\sigma_J^3}{\sigma_I F_2}. \tag{46}$$

As we have noted, this parameter characterizes the spreading behavior of the beam in a turbulent medium. To see this more clearly, consider the mean squared width of the beam at the Rayleigh range, $z = z_R$. The second term on the right-hand side of Eq. (32) will then be equal to unity. The third term, which represents the influence of turbulence on the spreading of the beam, can be determined by substitution from Eq. (31). It follows that

$$\frac{F_2 z_R^3}{\sigma_I^2} = \frac{F_2 \sigma_I}{\sigma_J^3} = \frac{1}{R_T}. \tag{47}$$

If $R_T \gg 1$, the turbulence will not affect the spreading of the beam until $z \gg z_R$; if $R_T \ll 1$, then the turbulence will dominate the spreading of the beam well before the Rayleigh distance is reached.

When neither of the conditions expressed by inequalities (41) and (45) is satisfied, the full cubic equation [defined by Eq. (33)] must be solved for z_T; in many cases, though, either inequality (37) or inequality (44) may be used to characterize the distance over which a partially coherent beam propagates in a random medium without appreciable distortion due to turbulence.

5. EXAMPLES

We will illustrate our main results by considering beams generated by Gaussian Schell-model sources [see Ref. 11 or Ref. 10, Sec. 5.6.4], i.e., sources with

$$I_0(\boldsymbol{\rho}) = A \exp[-2\rho^2/w_0^2], \tag{48}$$

$$\mu_0(\boldsymbol{\rho}') = \exp[-\rho'^2/2\sigma_\mu^2]. \tag{49}$$

The limiting case $\sigma_\mu \to \infty$ represents a spatially fully coherent source. The other limit $\sigma_\mu \to 0$ represents a spatially incoherent source.

For Gaussian Schell-model sources, it is not difficult to show that

$$\sigma_I^2 = \frac{w_0^2}{2}, \tag{50}$$

$$\sigma_J^2 = \frac{2}{k^2}\left(\frac{1}{w_0^2} + \frac{1}{\sigma_\mu^2}\right), \tag{51}$$

$$z_R = \frac{k w_0}{2}\left(\frac{1}{w_0^2} + \frac{1}{\sigma_\mu^2}\right)^{-1/2}. \tag{52}$$

To model the turbulence, we will use the so-called Tatarskii spectrum [Ref. 3, Sec. 3.3.2],

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp[-\kappa^2/\kappa_m^2], \tag{53}$$

where C_n^2 is known as the structure parameter of the index of refraction, and $\kappa_m = 5.92/l_0$, where l_0 is the inner scale of turbulence. On substituting from Eq. (53) into Eq. (29), one can numerically evaluate the resulting integral, and one finds that

$$F_2 = 1.095 C_n^2 l_0^{-1/3}. \tag{54}$$

Typical values for C_n^2 and l_0 are $C_n^2 = 10^{-14}\ \text{m}^{-2/3}$ and $l_0 = 0.01$ m (see Ref. 2).

We first consider an incident beam of wave number $k = 10^7\ \text{m}^{-1}$ ($\lambda = 628$ nm) and waist radius $w_0 = 0.01$ m. Figure 3 shows the spreading of Gaussian Schell-model beams in free space and in a turbulent medium for different values of the width σ_μ of the spectral degree of coherence at the waist plane, calculated from Eq. (30). It can be seen by direct substitution that inequality (41) is satisfied for each value of σ_μ, and therefore z_T is given by approximation (37). The dashed lines indicate the value of the propagation distance z_T. It is to be noted that each beam spreads more rapidly in the turbulent medium than in free space. It can also be seen that z_T is larger for smaller values of σ_μ, though it should be noted that the beams of lower coherence have larger σ_J and hence greater angular divergence.

Next we consider an incident beam of the same wave number, $k = 10^7\ \text{m}^{-1}$ ($\lambda = 628$ nm), but with a greater waist radius $w_0 = 0.1$ m. Figure 4 shows the spreading of two such beams in free space and in a turbulent medium, with different values of σ_μ. Inequality (45) is satisfied for these cases, and therefore z_T is given by approximation (44). In this regime, z_T is independent of the coherence properties of the incident beam, and this is reflected in the nearly identical spreading properties of the beams of different states of coherence. However, it should be noted that z_T indirectly depends on the state of coherence of the incident field, as inequality (45) depends on σ_J, which in turn depends on the spectral degree of coherence at the beam waist.

6. CONCLUSIONS

In this paper we have studied the spreading of partially coherent beams in random media, with the aim of determining what types of beams are least affected by the medium.

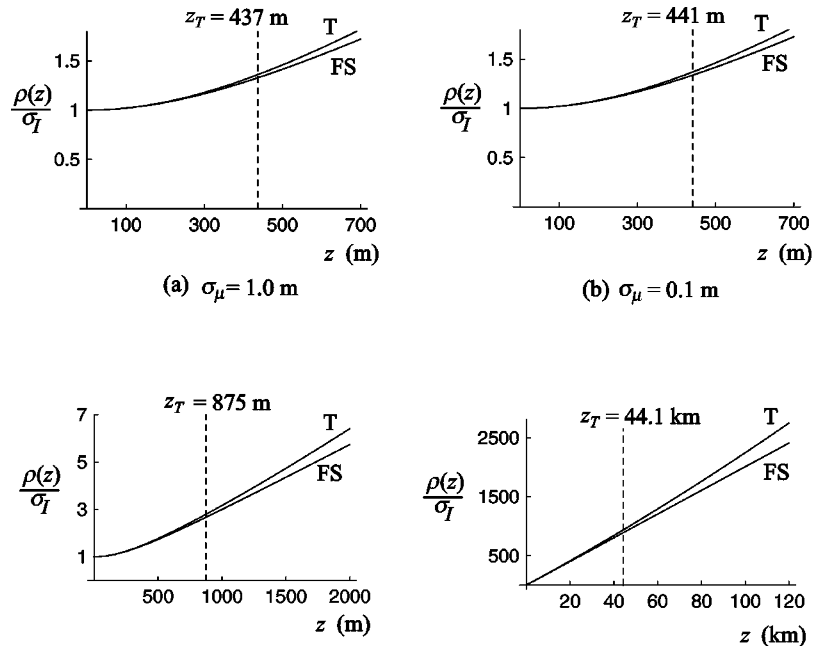

Fig. 3. Spreading of Gaussian Schell-model beams in free space (FS) and in turbulent media (T) for various values of the width of the spectral degree of coherence in the waist plane, σ_μ. The turbulence distance z_T is indicated by a dashed line. Case (a) represents a beam produced by a spatially fully coherent source, while case (d) represents a beam produced by a spatially incoherent source. In all cases $w_0 = 0.01$ m and $R_T \gg 1$.

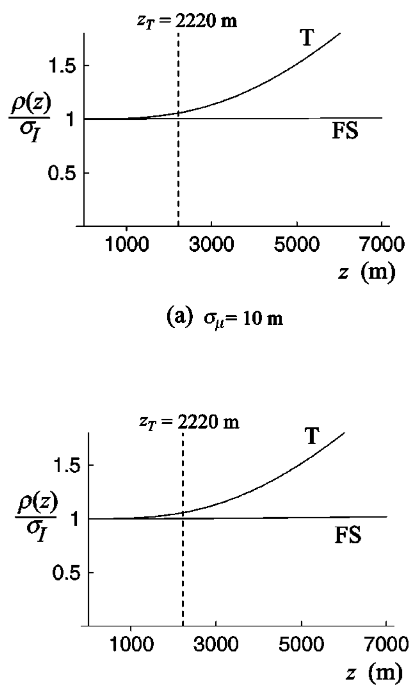

Fig. 4. Spreading of Gaussian Schell-model beams in free space (FS) and in turbulent media (T) for two values of the width of the spectral degree of coherence, σ_μ. In both cases $z_T = 2220$ m, $w_0 = 0.1$ m, and $R_T \ll 1$.

We have noted two relevant parameters. The first one, z_T, is the propagation distance at which the area of the beam is 10% larger in a turbulent medium than it would be in free space. We showed that z_T [defined by Eq. (33)] is a reasonable measure of the "resistance" of a beam to turbulence. The second parameter, R_T [defined by Eq. (46)], may be used to classify beams roughly as one of two varieties: those beams for which the turbulence spreading becomes appreciable before diffractive free-space spreading occurs, and those for which the turbulence affects the beam only after appreciable diffractive free-space spreading has occurred.

Our results indicate, in general agreement with the results of Refs. 5 and 6, that partially coherent beams are generally more resistant to turbulence; that is, for a partially coherent beam the parameter z_T has a value equal to or greater than that for a fully coherent beam with the same initial intensity profile. However, a partially coherent beam will also have a larger angular spread in free space; it might be said that the effects of the turbulence are masked by the larger free-space spreading. The usefulness of partially coherent beams that have greater turbulence resistance than coherent beams will evidently depend on the particular application for which they are to be used. The optimal beam will necessarily involve a trade-off between turbulence resistance and free-space spreading.

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