Diffraction tomography without phase information

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A modified form of diffraction tomography is presented in which measurements of the phase of the scattered field are replaced with measurements of the intensity on two planes beyond the scatterer. The new method is illustrated by an example. © 2002 Optical Society of America

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Two methods are commonly used for determining the three-dimensional structure of objects from scattering experiments, computed tomography (CT), and diffraction tomography (DT). CT (see, for example, Ref. 1, Sec. 4.11, or Ref. 2), which is typically performed using x rays, utilizes measurements of the attenuation of the incident field to determine the object structure. When diffraction and scattering effects become appreciable, CT generally gives unsatisfactory results, and the use of DT (Ref. 1, Sec. 13.2) is then more appropriate. DT, however, unlike CT, requires knowledge of both the phase and the intensity of the field, and phase measurements at optical (or higher) frequencies present formidable challenges.

In this Letter we introduce a modified form of DT that does not require phase measurements. This method is an extension of our recent work on the relation between CT and DT[3] and is related to a method suggested by Teague[4] for reconstructing the phase of a paraxial field. Our new method is appreciably simpler than Teague's and in general is not limited to the paraxial domain.

We consider a system with the geometry illustrated in Fig. 1. A monochromatic plane wave $U_i(\mathbf{r}) = \exp(ik\mathbf{s}_0\cdot\mathbf{r})$ [time dependence $\exp(-i\omega t)$ is suppressed] is incident on an object occupying a volume V with a (generally) complex index of refraction $n(\mathbf{r})$. Our new tomography method requires knowledge of the field beyond the scatterer on two planes, denoted as 1 and 2, parallel to the wave front of the incident field.

If the scattering is sufficiently weak (as is usually assumed in DT experiments), the field beyond the scatterer may be expressed in the form of the first Rytov approximation (Ref. 1, Sec. 13.5),

$$U(\mathbf{r}) \approx U_i(\mathbf{r})\exp[\psi(\mathbf{r})], \tag{1}$$

where

$$\psi(\mathbf{r}) = \frac{1}{U_i(\mathbf{r})}\int_V F(\mathbf{r}')\,\frac{\exp(ik|\mathbf{r}-\mathbf{r}'|)}{|\mathbf{r}-\mathbf{r}'|}\,U_i(\mathbf{r}')\mathrm{d}^3r', \tag{2}$$

$$F(\mathbf{r}) = \frac{k^2}{4\pi}[n^2(\mathbf{r})-1]. \tag{3}$$

By use of the angular spectrum representation of the free-space Green's function (Ref. 5, Sec. 3.2), ψ may be rewritten in the form

$$\psi(x, y, z) = i(2\pi)^2\iint \frac{1}{w}\tilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w-k)\mathbf{s}_0] \times \exp[i(w-k)z]\exp[i(ux+vy)]\mathrm{d}u\mathrm{d}v, \tag{4}$$

where

$$w = \begin{cases} \sqrt{k^2-u^2-v^2} & \text{when} \quad u^2+v^2 \le k^2, \\ i\sqrt{u^2+v^2-k^2} & \text{when} \quad u^2+v^2 > k^2, \end{cases} \tag{5}$$

$$\tilde{F}(\mathbf{K}) = \frac{1}{(2\pi)^3}\int_V F(\mathbf{r}')\exp(-i\mathbf{K}\cdot\mathbf{r}')\mathrm{d}^3r'. \tag{6}$$

The intensity $I(\mathbf{r})$ of the field is defined by the expression

$$\begin{aligned} I(x, y, z) &= |U(x, y, z)|^2 \\ &= \exp[\psi(x, y, z) + \psi^*(x, y, z)]. \end{aligned} \tag{7}$$

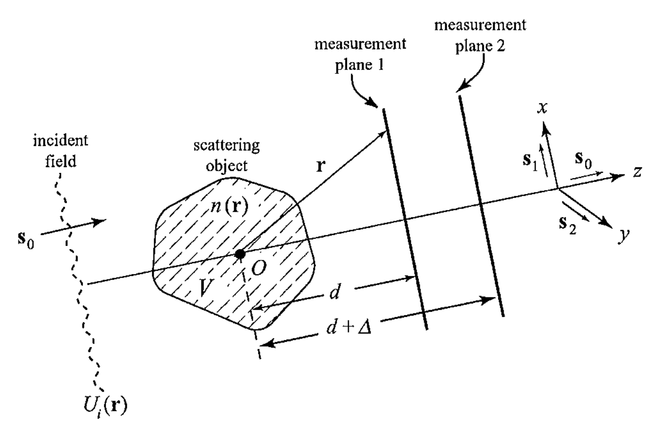

Fig. 1. Illustrating the notation for tomographic reconstruction of a scattering object.

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Let us define an intensity data function as the logarithm of the field intensity, i.e.,

$$D_I(x, y, z) = \log[I(x, y, z)] = \psi(x, y, z) + \psi^*(x, y, z). \tag{8}$$

Next we define the two-dimensional Fourier transform of the data function on a plane $z =$ constant outside the scatterer as

$$\hat{D}_I(u, v; z) = \frac{1}{(2\pi)^2} \iint D_I(x, y, z) \times \exp[-i(ux + vy)]\mathrm{d}x\mathrm{d}y. \tag{9}$$

It can then be shown, after some straightforward calculation, that

$$\hat{D}_I(u, v; z) = i\frac{(2\pi)^2}{|w|^2}(w^*\tilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w - k)\mathbf{s}_0] \times \exp[i(w - k)z] - w\{\tilde{F}[-u\mathbf{s}_1 - v\mathbf{s}_2 + (w - k)\mathbf{s}_0]\}^* \times \exp[-i(w^* - k)z]). \tag{10}$$

We will consider only u, v values such that $u^2 + v^2 \leq k^2$; this amounts to neglecting contributions from evanescent waves. The parameter w is then real. From Eq. (10) it can be seen that a Fourier component (labelled by u, v) of the intensity consists of a superposition of two plane waves traveling in directions $(u, v, w - k)$ and $(-u, -v, w - k)$. If the intensity of the field is measured on a plane beyond the scatterer and then its Fourier transform is taken, it is generally not possible to determine $\tilde{F}(\mathbf{K})$, and subsequently $F(\mathbf{r})$. This difficulty is the reason why DT traditionally requires the measurement of both the phase and the intensity. It is to be noted, however, that a phase difference is generated between the two plane waves (u, v, w) and $(-u, -v, w)$ as they propagate. If the Fourier transform of the intensity is measured on a pair of planes behind the scatterer, the resulting pair of equations may be solved for $\tilde{F}$ and $\tilde{F}^*$. To see this, we consider a new data function $\hat{D}_\Delta$, defined as

$$\hat{D}_\Delta(u, v; d) \equiv \frac{\hat{D}_I(u, v; d) - \hat{D}_I(u, v; d + \Delta)\exp[i(w - k)\Delta]}{\Delta}, \tag{11}$$

which makes use of measurements of the intensity on two planes, $z = d$ and $z = d + \Delta$. On substituting from Eq. (10) into Eq. (11), it can readily be shown that

$$\hat{D}_\Delta(u, v; d) = \frac{(2\pi)^2 i}{w\Delta}\tilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w - k)\mathbf{s}_0] \times \exp[i(w - k)d]\{1 - \exp[2i(w - k)\Delta]\}. \tag{12}$$

Equation (12), which shows that $\hat{D}_\Delta$ is proportional to $\tilde{F}(\mathbf{K})$, is the main result of this Letter; it indicates that it is possible to determine the Fourier components of the scattering potential from measurements of the data function $\hat{D}_\Delta$, i.e., from measurements of the intensity on two planes.

It is clear that this modified tomographic method makes it possible to reconstruct the scattering potential without making any phase measurements; the phase is implicitly determined from $\hat{D}_\Delta$. For comparison, we recall the usual formula of diffraction tomography,

$$\hat{\psi}(u, v; d) = \frac{(2\pi)^2 i}{w}\tilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w - k)\mathbf{s}_0] \times \exp[i(w - k)d], \tag{13}$$

where $\hat{\psi}$ is the two-dimensional Fourier transform of ψ on the plane $z = d$ (ψ can be determined by taking the logarithm of the field in the plane).

The right-hand sides of Eqs. (12) and (13) differ in that Eq. (12) possesses an additional factor, contained in the braces. This factor vanishes for values of u, v such that

$$2[k - \sqrt{k^2 - u^2 - v^2}]\Delta = 2n\pi, \tag{14}$$

where n is the integer. For such values of u and v the function $\tilde{F}$ cannot be determined; this includes the value of the function at the origin $u = v = 0$, but that value can be estimated by extrapolation from neighboring values of the function $\tilde{F}$. One is therefore restricted to reconstructing only those components of u, v such that the left-hand side of Eq. (14) does not exceed 2π, i.e., values such that

$$u^2 + v^2 \leq 2\pi k/\Delta. \tag{15}$$

In inequality (15) we assume that $u^2 + v^2 \ll k^2$. This inequality constrains how closely spaced the two measurement planes must be to determine a given Fourier component of $\tilde{F}$.

To illustrate the new method, let us consider scattering from a two-layer spherical object of inner radius $ka = 20$ and outer radius $kb = 40$, with the refractive index of the inner sphere taken to be $n_a = 1.001 + 0.001i$ and that of the outer shell taken to be $n_b = 1.0005 + 0.001i$. The scattered field was determined with a partial wave expansion, and the measurement planes were taken to be at distances $kd = 60$ and $k(d + \Delta) = 62$, measured from the center of the sphere. In performing the inversion, we made use of the spherical symmetry of the object. The field intensity was calculated in the measurement planes, its Fourier transform taken as in Eq. (9), and combined to form the data function $\hat{D}_\Delta$. Equation (12) was then used to determine $\tilde{F}$ from $\hat{D}_\Delta$; low spatial frequencies were determined by fitting the higher frequency data to a polynomial. Finally, $\tilde{F}$ was inverted to determine the scattering potential.

The assumed scattering potential and the reconstructed version are shown in Fig. 2. It can be seen that there is good agreement and that the reconstruction has matched both the general shape of the potential and its absolute magnitude. It is to be noted

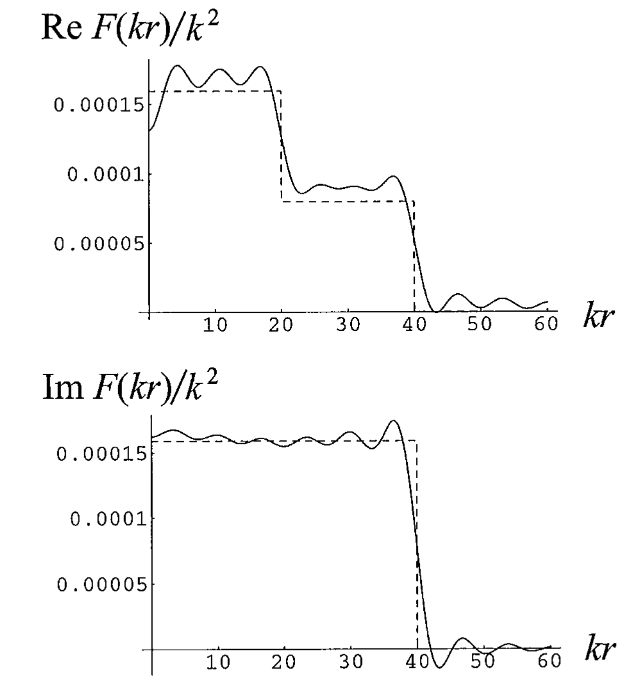

Fig. 2. Reconstruction of a complex scattering potential by use of the new tomographic method. The dashed curve indicates the actual scattering potential. It can be seen that there is good agreement between the actual and the reconstructed potentials.

that both the real and the imaginary parts of the scattering potential were successfully reconstructed. The differences between the assumed potential and the reconstructed one can be attributed to our use of only the homogeneous parts of the scattered field (resulting in a Gibbs phenomenon) and the presence of noise from multiple scattering.

In conclusion, we have introduced a new method of determining a scattering potential by diffraction tomography without the knowledge of the phase of the scattered field. Measurements of the intensity on two planes beyond the scatterer replace the usual amplitude and phase measurements of DT. It is to be noted that this method, unlike Teague's phase reconstruction technique,[4] is not limited to paraxial fields. However, it can be seen from inequality (15) that determining the Fourier components of plane waves propagating in a direction that differs appreciably from $\mathbf{s}_0$ ($u^2 + v^2 \approx k^2$) requires the measurement planes to be spaced at distances smaller than a wavelength. At optical wavelengths ($\lambda \sim 10^{-6}$ m) this is impractical, and it should therefore be expected that the method can be used to determine only the low-frequency components of the scattering potential. This new tomographic method will be described in more detail in a forthcoming publication.[6]

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