Can spatial coherence effects produce a local minimum of intensity at focus?

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It is demonstrated that, for high-Fresnel-number focusing systems illuminated by certain classes of partially coherent light, it is possible to produce a local minimum of intensity at the geometrical focus. Such an effect is possible even though the average intensity in the entrance plane of the lens is uniform. An explanation is offered for this effect, and potential applications are considered.

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It is somewhat of an unspoken rule in optical coherence theory that a decrease in spatial coherence generally results in a smearing out of the interference pattern that the light produces; that is, the pattern retains essentially the same structure but its contrast is reduced. For instance, in Young's double-slit experiment, a decrease in the spatial coherence between the two slits results in a lessening of the visibility of the interference fringes (Ref. 1, Sect. 4.3) but the fringe pattern retains its periodic structure.

Also, in the classical Debye theory of the focusing of light (Ref. 2, Sect. 8.8), it is predicted that the maximum intensity will occur at the geometrical focus, provided that the Fresnel number is much greater than unity and the field is monochromatic. It has been shown both for low-Fresnel-number systems[3] and high-Fresnel-number systems[4,5] that the primary effect of decreasing the coherence is a smoothing of the intensity distribution (and, for low-Fresnel-number systems, a shift in the location of the intensity maximum).

However, there is some indication that a decrease in spatial coherence can lead to more drastic changes in the interference pattern. In Ref. 5 it was proved that, in the focusing of light from Gaussian Schell model sources in high-Fresnel-number systems, the maximum spectral intensity occurs at the geometrical focus. It was also noted, though, that such a result requires that the cross-spectral density of the incident field be real and nonnegative, a requirement that is not satisfied by certain correlation functions (such as Bessel correlations). Furthermore, a recent paper by Pu and Nemoto[6] and an early paper by Som and Biswas[7] showed that certain classes of Bessel-correlated fields can show drastic differences in their radiation pattern on diffraction by an aperture.

In this Letter we demonstrate that light of certain states of coherence will produce a local minimum at the geometrical focus on passing through a high-Fresnel-number focusing system. This is purely a consequence of the state of coherence of the incident field; indeed, the intensity of the incident field is uniform across the plane of the lens. This effect may have practical uses in such applications as the production of optical tweezers.

We consider a partially coherent field emerging from a circular aperture with radius a and converging toward a geometrical focus O at distance f from the aperture, as illustrated in Fig. 1. It is assumed that the Fresnel number of the system is much greater than unity, i.e., $N \equiv a^2/\lambda f \gg 1$, where λ is the wavelength of the field. It can be shown[5] that the spectral intensity of the focused field at frequency ω, at a point of observation $P(\mathbf{r})$ in the focal region, is given by the formula

$$S(\mathbf{r}, \omega) = \frac{1}{\lambda^2} \iint_{\mathcal{W}} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega) \times \frac{\exp[-ik(R_1 - R_2)]}{R_1 R_2} \mathrm{d}^2 r_1 \mathrm{d}^2 r_2 , \qquad (1)$$

where $R_i = |\mathbf{r} - \mathbf{r}_i|$ is the distance between the observation point $P(\mathbf{r})$ and a point $Q(\mathbf{r}_i)$ on the reference sphere $\mathcal{W}$ of radius f centered on O, $k = \omega/c$ is the wave number associated with frequency ω, and $W^{(0)}$ is the cross-spectral density of the incident field in the aperture.

For an observation point sufficiently close to the geometrical focus O, we may replace to a good approximation the factors R_1 and R_2 in the denominator of the integrand of Eq. (1) with the focal distance f. Furthermore, we may change the variables of integration to be the transverse coordinates within the aperture. Equation (1) then simplifies to

$$S(\mathbf{r}, \omega) = \frac{1}{(\lambda f)^2} \iint_{A} W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega) \times \exp[-ik(R_1 - R_2)] \mathrm{d}^2 r_1 \mathrm{d}^2 r_2 , \qquad (2)$$

where the domain of integration A is over the area of the aperture.

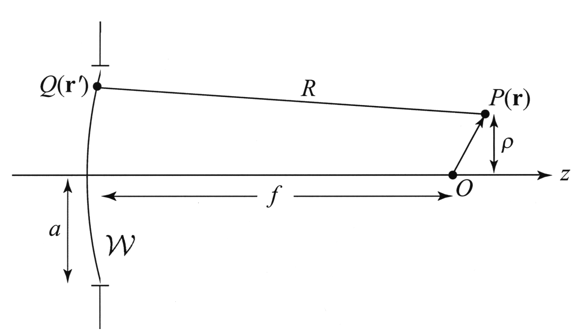

Fig. 1. Illustration of the focusing arrangement.

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We choose as our cross-spectral density the class of Bessel-correlated fields,

$$W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = S^{(0)}(\omega) J_0(\beta|\mathbf{r}_2 - \mathbf{r}_1|), \quad (3)$$

where $S^{(0)}$ is the spectrum of the incident field and J_0 is the Bessel function of zeroth order. The distance over which such a field is correlated (the correlation range) is given approximately by the inverse of β. This choice of correlation function was motivated by the observation from Ref. 5 that unusual effects are possible when $W^{(0)}$ possesses positive and negative values; the particular choice of J_0 was motivated by the fact that such a correlation function possesses an elegant modal expansion[8] that simplifies the integration of Eq. (2). It is to be noted that the spectral intensity of a field with cross-spectral density given by Eq. (3) is uniform across the plane of the aperture.

An example of the intensity on axis ($\rho = 0$), calculated with Eqs. (2) and (3) and the aforementioned modal expansion, is shown in Fig. 2; the intensity is normalized by the intensity of a fully coherent field at the geometrical focus,[5] $S_0 = S^{(0)} k^2 a^4 / 4f^2$. It can be seen that there is a clear minimum at the geometrical focal point $\rho = z = 0$. Figure 3 shows the intensity in the focal plane $z = 0$. Again it can be seen that the intensity takes on a minimum at the geometrical focus. A decrease of the correlation range $(1/\beta)$ of the field results in a broadening of the spot size of the field and an enhancement of the minimum at the geometrical focus, eventually reaching a case in which the light can no longer be considered to be focused at all.

When the correlation range of the field is increased beyond the size of the aperture, the intensity pattern of the field in the region of focus approaches the coherent limit. This is illustrated in Fig. 4, in which $1/\beta = 20$ mm. The curves indicate our numerical results with the cross-spectral density given by Eq. (3), and the circles indicate for comparison analytical results determined long ago [Ref. 2, Sect. 8.8, Eqs. (25) and (26)] for the intensity of a fully coherent converging wave. It can be seen that the two calculations agree well.

The behavior of the cross-spectral density for the two cases mentioned above is illustrated in Fig. 5 as a function of the difference variable $|\mathbf{r}_2 - \mathbf{r}_1|$ and plotted over distances up to the radius of the aperture. It can be seen that the case with a minimum at focus has both positive and negative values of the cross-spectral density within the aperture, as expected.

One can readily understand this somewhat unusual coherence effect by considering a simple model for a random field. Let us assume that the field that is incident on the focusing system is a plane wave, $U(\mathbf{r}) = U_0 \exp(ik\mathbf{s} \cdot \mathbf{r})$, for which the unit vector $\mathbf{s}$, which indicates the direction of incidence, is a slowly varying random function, described by a probability distribution function $f(\mathbf{s})$ such that

$$\int f(\mathbf{s}) \mathrm{d}^2 s_\perp = 1, \quad (4)$$

where $\mathbf{s} = (\mathbf{s}_\perp, s_z)$, $\mathbf{s}_\perp$ is the component of $\mathbf{s}$ perpendicular to the z direction, $s_z = (1 - |\mathbf{s}_\perp|^2)^{1/2}$, and the integration is over all values of $\mathbf{s}_\perp$ such that $|\mathbf{s}_\perp| \leq 1$. From geometrical-optics considerations, one can show that a plane wave that is incident upon the focusing system will produce a bright spot in the focal plane at a distance from the geometrical focus ρ such that $\rho = f \tan\theta$, where θ is the angle that $\mathbf{s}$ makes with the z axis. If the probability density $f(\mathbf{s})$ is centered on and has a maximum at normal incidence, it follows that the maximum intensity of the average field will tend to be at the geometrical focus. If, however, the probability density has a local minimum at normal incidence, it follows that the incident field tends to be focused at points away from the geometrical focus, and the intensity of the average field at the geometrical focus may exhibit a local minimum.

We can determine the cross-spectral density for this plane-wave model by taking the appropriate average of the incident field, i.e.,

$$W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = |U_0|^2 \int f(\mathbf{s}) \exp[ik\mathbf{s} \cdot (\mathbf{r}_2 - \mathbf{r}_1)] \mathrm{d}^2 s_\perp . \quad (5)$$

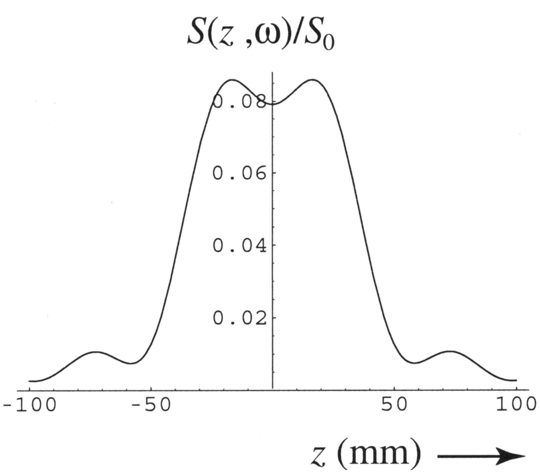

Fig. 2. On-axis intensity distribution for $1/\beta = 3.53$ mm, showing a local minimum at focus, with $a = 10$ mm, $f = 2$ m, and $\lambda = 500$ nm.

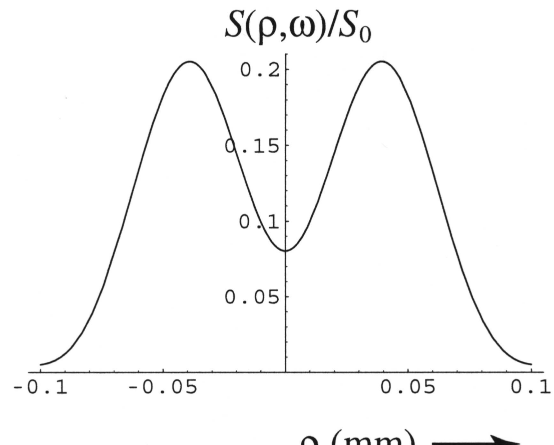

Fig. 3. Intensity in the focal plane for $1/\beta = 3.53$ mm, showing a local minimum at focus. All other parameters are as in Fig. 2.

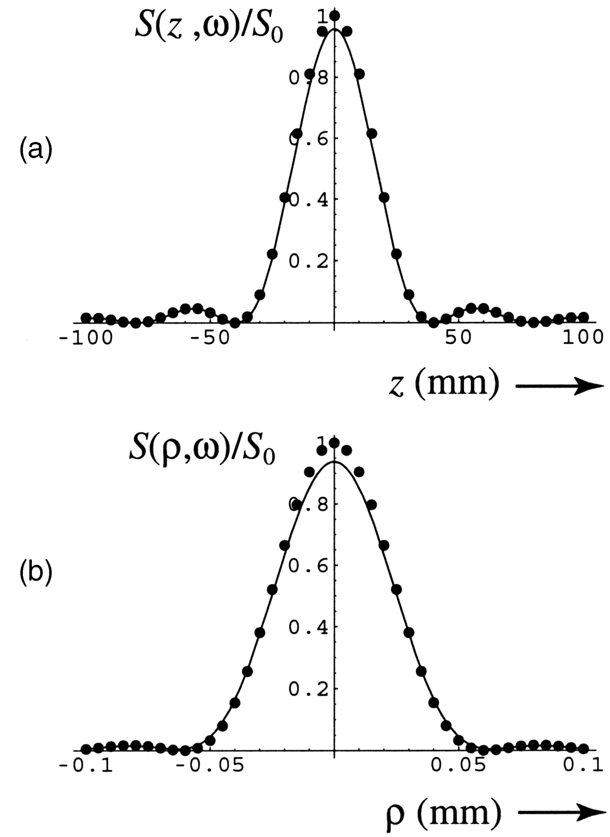

Fig. 4. (a) On-axis and (b) focal plane intensities for $1/\beta = 20$ mm. The curves indicate our numerical calculations, and the circles indicate the classical results for fully coherent fields (from Ref. 2, Sect. 8.8). All other parameters are as in Fig. 2.

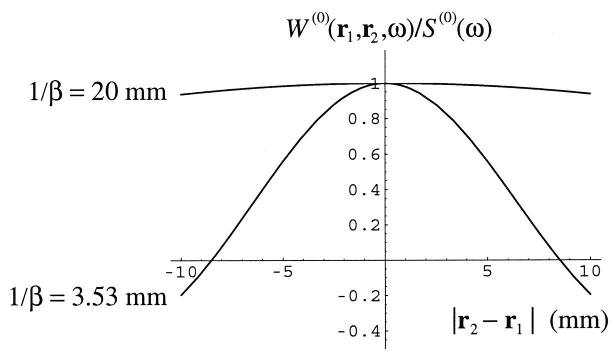

Fig. 5. The normalized form of the correlation function $W^{(0)}(\mathbf{r}_1, \mathbf{r}_2, \omega)$ for the two different values of $1/\beta$, as discussed in the text.

Let us choose as a probability distribution

$$f(\mathbf{s}) = \frac{k^2}{2\pi\beta}\,\delta(\beta - k|\mathbf{s}_\perp|)\,, \tag{6}$$

which constrains the direction of the incident plane wave to lie on a cone defined by $|\mathbf{s}_\perp| = \beta/k$. It follows from direct substitution of Eq. (6) into Eq. (5) and from a well-known integral representation of J_0 [see Ref. 9, Eq. (11.30c)] that $W^{(0)}$ is given by Eq. (3), with $S^{(0)} = |U_0|^2$.

Fields with J_0 correlations can be synthesized by placement of a thin, annular incoherent source in the first focal plane of a converging lens; the output of the lens is the J_0 field.[8]

These results could have practical use in the production of optical tweezers, in which focused beams are used to trap neutral dielectric particles.[10] The ability to shape the intensity pattern of the focused field might allow manipulation of trapped particles on an extremely fine scale.

In closing, it is worth mentioning that J_0 correlations are just one possibility for producing unusual intensity distributions in the focal region. As mentioned above, any correlation functions with positive and negative values have the potential to produce unusual effects. For instance, correlation functions of the form $J_n(x)/x^n$ are also good candidates.[11,12] A detailed study of the behavior of such correlations may lead to a general ability to shape the intensity pattern of the focused field.

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