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Information Sciences 162 (2004) 3–20

INFORMATION SCIENCES
AN INTERNATIONAL JOURNAL

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The information content of the scattered intensity in diffraction tomography

Greg Gbur [a,*], Emil Wolf [b,1]

[a] *Department of Physics and Astronomy, Free University, De Boelelaan 1081, Amsterdam 1081 HV, The Netherlands*

[b] *Department of Physics and Astronomy and The Institute of Optics, University of Rochester, Rochester, NY 14627, USA*

Accepted 4 January 2003

Abstract

It was recently shown that the structure of a weakly scattering object may be reconstructed from intensity measurements made in a half-space beyond the scatterer. We review these results and show how they may be extended to more complicated situations, and illustrate them by examples.

1. Introduction

The possibility of determining the three-dimensional structure of an object from measurements of the field scattered by the object is of basic importance in many applications, including medical diagnosis, geophysical surveying, and structural testing. The two methods of solving this inverse scattering problem that are most frequently employed are *computed tomography* and *diffraction tomography*.

* Corresponding author. Tel.: +31204447861; fax: +31204447992.
E-mail address: gbur@nat.vu.nl (G. Gbur).
[1] Currently at School of Optics/CREOL, University of Central Florida, Orlando, FL 32816, USA.

0020-0255/$ - see front matter
doi:10.1016/j.ins.2003.01.001

Computed tomography, also referred to as computed axial tomography (see [1, Section 4.11] or [2,3]), which is frequently performed using X-rays, utilizes measurements of the attenuation of the incident field to determine the object structure. The algorithm used to reconstruct the object assumes that the incident field propagates along rays, and is attenuated but not diffracted.

When the wavelength of the probing field is comparable to the scale of spatial variation of the structure of the object, diffraction and scattering effects become significant and computed tomography leads to inaccurate results. Diffraction tomography [1, Section 13.2] or [4], a method which incorporates diffraction effects, can then be used. This method, however, unlike computed tomography, requires measurement not only of the intensity of the scattered field but also its phase. Performing phase measurements at optical frequencies is typically difficult, requiring the use of interferometric techniques and complicated phase unwrapping algorithms. Perhaps because of this, diffraction tomography has not found widespread application as has computed tomography.

To circumvent the difficulties of phase measurements of an optical field, a variety of methods have been proposed and tested. Methods of diffraction tomography that require only intensity measurements have been suggested which use iterative algorithms [5] or are restricted to reconstructions of objects whose refractive index is real-valued [6]. Another method, called power-extinction diffraction tomography [7], requires measurements of the power extinguished from a pair of plane waves incident simultaneously on the scatterer. A method which uses measurements of the intensity of the scattered field at a plane near the scatterer and also in the far zone has been proposed [8]. One paper discusses the validity of performing diffraction tomography without using any phase information at all [9].

It is evidently of great interest to determine how much information about a scattering object is contained in the intensity of the scattered field. A promising method proposed by Teague [10] allows the phase of a paraxial field to be determined by taking intensity measurements on a pair of mutually parallel planes and then solving a two-dimensional Poisson equation. Several authors have discussed the reconstruction of objects using Teague's transport of intensity equation [11,12]. Very recently, however, a new method of diffraction tomography has been developed that allows straightforward determination of the scattering potential using intensity measurements on a pair of planes [13,14]. The method is similar to Teague's but is not restricted to paraxial wavefields and does not require the solution of a partial differential equation to recover phase information.

In this paper we discuss this new "intensity diffraction tomography" method and we examine how much information can be obtained about a scattering object from the intensity of the scattered field. After reviewing the new method and comparing it to ordinary diffraction tomography, we show that the new method can, in principle, be extended to take into account additional information contained in the intensity backscattered from the object. Finally, difficulties that arise when one attempts to reconstruct the low spatial frequency components of the scattering object are discussed, and methods are proposed to overcome them.

2. Ordinary diffraction tomography and intensity diffraction tomography

We consider a monochromatic scalar plane wave $V_i(\mathbf{r}, t) = U_i(\mathbf{r})\mathrm{e}^{-\mathrm{i}\omega t}$ of frequency ω and wave number $k = \omega/c$, with $U_i(\mathbf{r}) = \mathrm{e}^{\mathrm{i}k\mathbf{s}_0\cdot\mathbf{r}}$ and $\mathbf{s}_0^2 = 1$, incident on a scattering object characterized by a complex potential $F(\mathbf{r})$, occupying a volume V. The arrangement is depicted in Fig. 1. Let $\mathbf{r} = x\mathbf{s}_1 + y\mathbf{s}_2 + z\mathbf{s}_0$, where $(\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_0)$ are unit vectors along the axes of a right-handed coordinate system. We denote by $\mathscr{R}^+$ the half-space outside the scatterer into which the incident field is propagating and call it the forward scattering half-space; the region outside the scatterer from which the field is incident is denoted by $\mathscr{R}^-$ and we call it the back scattering region.

The time-independent part $U(\mathbf{r})$ of the total field satisfies the differential equation

$$[\nabla^2 + k^2]U(\mathbf{r}) = -4\pi F(\mathbf{r})U(\mathbf{r}), \tag{1}$$

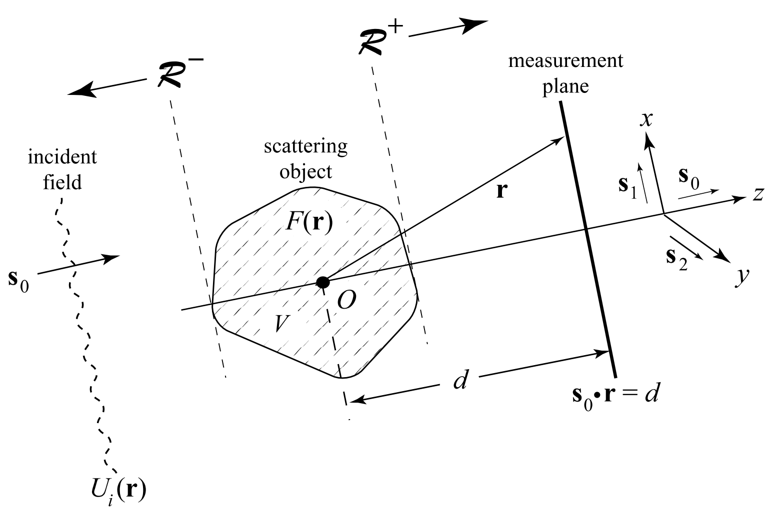

Fig. 1. Illustrating the notation relating to tomographic reconstruction of scattering objects.

where the scattering potential $F(\mathbf{r})$ is defined by the formula

$$F(\mathbf{r}) = \frac{k^2}{4\pi}[n^2(\mathbf{r}) - 1], \tag{2}$$

$n(\mathbf{r})$ being the (generally complex) index of refraction of the scattering object.

If the scattering potential is sufficiently weak [$n(\mathbf{r}) \approx 1$], the total field is well-represented by the lowest-order term in a perturbation expansion of its complex phase, i.e. as

$$U(\mathbf{r}) \approx U_i(\mathbf{r})\mathrm{e}^{\psi(\mathbf{r})}, \tag{3}$$

where

$$\psi(\mathbf{r}) = \frac{1}{U_i(\mathbf{r})}\int_V F(\mathbf{r}')\frac{\mathrm{e}^{\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}U_i(\mathbf{r}')\mathrm{d}^3r'. \tag{4}$$

This approximation to the total field is known as the first Rytov approximation (see [1, Section 13.5]).

In many tomographic experiments, the total field is measured on a plane $z = d$, taken to be perpendicular to the direction of incidence $\mathbf{s}_0$, an arrangement often referred to as the classical measurement configuration. Expression (4) may be expressed in a form particularly suitable to this configuration by use of the Weyl representation of a diverging spherical wave (see [15, Section 3.2]), viz.,

$$\frac{\mathrm{e}^{\mathrm{i}k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} = \frac{\mathrm{i}}{2\pi}\iint \frac{1}{w}\mathrm{e}^{\mathrm{i}[u\mathbf{s}_1+v\mathbf{s}_2\pm w\mathbf{s}_0]\cdot(\mathbf{r}-\mathbf{r}')}\,\mathrm{d}u\,\mathrm{d}v, \tag{5}$$

where the integration extends over the entire u, v plane,

$$w = \begin{cases} \sqrt{k^2-u^2-v^2} & \text{when } u^2+v^2 \leqslant k^2, \\ \mathrm{i}\sqrt{u^2+v^2-k^2} & \text{when } u^2+v^2 > k^2, \end{cases} \tag{6}$$

and the sign of w in Eq. (5) is taken to be positive or negative according as z is greater than or less than z', respectively. On substituting from Eq. (5) into Eq. (4), the complex phase $\psi(\mathbf{r})$ on the plane $z = d$ may be expressed in the form

$$\psi(x,y;d) = (2\pi)^2\mathrm{i}\iint \frac{1}{w}\widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w-k)\mathbf{s}_0]\mathrm{e}^{\mathrm{i}(w-k)d}\mathrm{e}^{\mathrm{i}(ux+vy)}\mathrm{d}u\,\mathrm{d}v, \tag{7}$$

where

$$\widetilde{F}(\mathbf{K}) = \frac{1}{(2\pi)^3}\int_V F(\mathbf{r}')\mathrm{e}^{-\mathrm{i}\mathbf{K}\cdot\mathbf{r}'}\mathrm{d}^3r' \tag{8}$$

is the three-dimensional Fourier transform of the scattering potential.

From the previous expressions it can be shown that structural information about the scattering potential may be determined from a careful processing of the measured field data. It is to be noted that the complex phase $\psi(\mathbf{r})$ may be determined from the field data by the use of the relation

$$\psi(x,y;d) = \log\left[\frac{U(x,y;d)}{U_i(x,y;d)}\right]. \tag{9}$$

If we take the two-dimensional Fourier transform

$$\hat{\psi}(u,v;d) = \frac{1}{(2\pi)^2}\iint \psi(x,y;d)\mathrm{e}^{-\mathrm{i}(ux+vy)}\mathrm{d}x\mathrm{d}y \tag{10}$$

of the complex phase in the plane $z = d$, it is straightforward to show that

$$\hat{\psi}(u,v;d) = \frac{(2\pi)^2\mathrm{i}}{w}\widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w-k)\mathbf{s}_0]\mathrm{e}^{\mathrm{i}(w-k)d}. \tag{11}$$

It can be seen from Eq. (11) that the two-dimensional Fourier transform of the complex phase ψ on the plane $z = d$ is directly related to the three-dimensional Fourier transform of the scattering potential $F(\mathbf{r})$. In particular, knowledge of the components of $\hat{\psi}$ for $u^2 + v^2 \leqslant k^2$ is equivalent to the knowledge of $\widetilde{F}$ on a half-spherical surface in **K**-space displaced from the origin (see Fig. 2(a)). If the direction of propagation of the incident field $\mathbf{s}_0$ is moved through all possible directions, and measurements of the transmitted field are made for each direction using the standard measurement configuration, one can determine all those Fourier components of $\widetilde{F}(\mathbf{K})$ for which $|\mathbf{K}| \leqslant \sqrt{2}k$ (Fig. 2(b)).

Eq. (11) forms the theoretical basis of diffraction tomography. Algorithms have been described in the literature that may be used to efficiently reconstruct $F(\mathbf{r})$ [16,17]. It is to be noted that reconstruction methods based on Eq. (11) assume that both the amplitude and the phase of the scattered field are accessible to measurement; however, in many circumstances, especially at optical wavelengths, the phase is not easily measurable and often such measurements require the use of complicated phase unwrapping techniques.

Because it is often difficult to obtain phase information , it is of interest to determine how much information intensity measurements can provide about the structure of the scattering object. Now the intensity of the field on the plane $z = d$ may be defined as

$$I(x,y;d) \equiv |U(x,y;d)|^2 = \mathrm{e}^{\psi(x,y;d)+\psi^*(x,y;d)}, \tag{12}$$

where the asterisk denotes the complex conjugate. We next introduce an *intensity data function*

$$D_I(x,y;d) \equiv \log[I(x,y;d)] = \psi(x,y;d) + \psi^*(x,y;d). \tag{13}$$

The two-dimensional Fourier transform of this data function on the plane $z = d$ is given by the expression

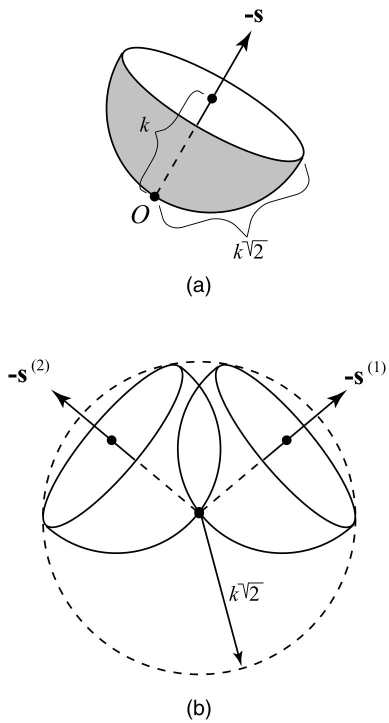

Fig. 2. Accessible Fourier components of the scattering potential from measurements of the field in the forward scattering region in the so called classical measurement configuration. (a) Components of $\widetilde{F}(\mathbf{K})$ accessible with one direction of incidence. (b) Components accessible with several directions of incidence.

$$\widehat{D}_I(u,v;d) = \frac{1}{(2\pi)^2}\iint D_I(x,y;d)\mathrm{e}^{-\mathrm{i}(ux+vy)}\,\mathrm{d}x\,\mathrm{d}y. \tag{14}$$

On substituting from Eqs. (13) and (7) into Eq. (14), one readily finds that

$$\begin{aligned}\widehat{D}_I(u,v;d) = \mathrm{i}\frac{(2\pi)^2}{|w|^2}\Big\{&w^*\widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w-k)\mathbf{s}_0]\mathrm{e}^{\mathrm{i}(w-k)d}\\ &- w\Big[\widetilde{F}[-u\mathbf{s}_1 - v\mathbf{s}_2 + (w-k)\mathbf{s}_0]\Big]^*\mathrm{e}^{-\mathrm{i}(w^*-k)d}\Big\}.\end{aligned} \tag{15}$$

Let us consider u, v values such that $u^2+v^2 \leqslant k^2$; values outside this range are associated with evanescent waves, from which it is appreciably more difficult to extract structural information (see, for example, [18,19]). The quantity w is then real, and it can be seen from Eq. (15) that the (u,v)th Fourier component of D_I consists of a superposition of two plane waves travelling in the directions $(u,v,w-k)$ and $(-u,-v,w-k)$, illustrated in Fig. 3. Because this Fourier

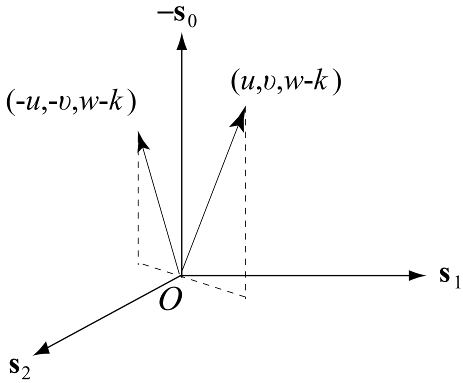

Fig. 3. Vectors $(u, v, w - k)$ and $(-u, -v, w - k)$.

transform consists of a weighted sum of $\widetilde{F}$ and $\widetilde{F}^*$ which is typically complex, it follows that it is not, in general, possible to determine $\widetilde{F}$ from measurements of the intensity on a single plane $z = d$. What is not immediately obvious, however, is that it is possible to determine $\widetilde{F}$ by measuring the intensity on two or more planes perpendicular to $\mathbf{s}_0$, as we will now show.

We consider a new data function $\widehat{D}_\Delta$, defined as

$$\widehat{D}_\Delta(u, v; d) \equiv \frac{\widehat{D}_I(u, v; d) - \widehat{D}_I(u, v; d + \Delta)\mathrm{e}^{\mathrm{i}(w-k)\Delta}}{\Delta}. \tag{16}$$

This new data function is a linear combination of the Fourier transforms of the intensity data function on two planes, situated at distances $z = d$ and $z = d + \Delta$ (see Fig. 4). On substituting from Eq. (15) into (16), it immediately follows that

$$\widehat{D}_\Delta(u, v; d) = \frac{(2\pi)^2\mathrm{i}}{w}\widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 + (w - k)\mathbf{s}_0]\mathrm{e}^{\mathrm{i}(w-k)d}\Gamma[w, \Delta], \tag{17}$$

where

$$\Gamma[w, \Delta] \equiv \frac{\left[1 - \mathrm{e}^{2\mathrm{i}(w-k)\Delta}\right]}{\Delta}. \tag{18}$$

Eq. (17), which was first derived in Refs. [13,14], demonstrates that $\widehat{D}_\Delta$ is proportional to $\widetilde{F}$. It should be compared to Eq. (11), which forms the basis of diffraction tomography. The two equations differ significantly only in that Eq. (17) possesses the additional factor $\Gamma[w, \Delta]$. This factor vanishes for values of u, v such that

$$2\left[k - \sqrt{k^2 - u^2 - v^2}\right]\Delta = 2n\pi, \tag{19}$$

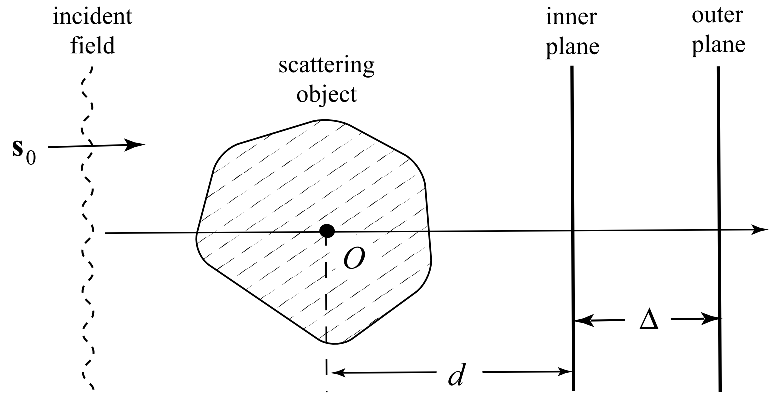

Fig. 4. A two-plane measurement scheme for performing diffraction tomography with only intensity measurements.

where n is an integer. For such values of u and v the function $\widetilde{F}$ cannot be determined. This includes the value of the function at the origin in Fourier space, $u = v = 0$, which we will discuss in Section 4. The simplest reconstruction schemes will require that only values of u, v such that the left-hand side of Eq. (19) does not exceed 2π are used, i.e. values such that

$$u^2 + v^2 \leqslant \frac{2\pi k}{\Delta}. \tag{20}$$

In deriving inequality (20) it was assumed for simplicity that $u^2 + v^2 \ll k^2$. This inequality places a constraint on how closely spaced the measurement planes must be in order to determine a given Fourier component of $\widetilde{F}$; it can be seen that larger values of u and v require a closer spacing of the measurement planes.

That there exists such a tomographic method is perhaps not surprising when one considers that the propagation of a field depends on both its intensity and phase. One might therefore expect that the *change* in the intensity of the field as it propagates contains some information about its phase. We have seen that at any plane the spatial Fourier transform of the intensity may be considered to have a contribution from a pair of plane waves. By measuring the same Fourier components of the intensity further away from the scatterer, we obtain a pair of equations with two unknowns ($\widetilde{F}$, $\widetilde{F}^*$), which may be solved for the two components of $\widetilde{F}$. It should be noted from Eq. (15) that, for given values of u, v, the Fourier transform of the intensity is periodic in the $\mathbf{s}_0$ direction; the zeros defined by Eq. (19) correspond to distances at which the measurements at the two planes give redundant information.

Our tomographic method bears a striking resemblance to the method developed by Teague [10] for reconstructing the phase of a paraxial field. In both methods, the phase is determined from intensity measurements on a pair of measurement planes. Our method differs from Teague's in that it applies even when the scattered field is not paraxial. However, it can be seen from inequality (20) that determining the Fourier components of plane waves propagating in a direction appreciably different from $\mathbf{s}_0$ requires the measurement planes to be spaced at distances smaller than a wavelength. At optical wavelengths ($\lambda \sim 10^{-6}$ m) such measurements cannot be readily made and it should be expected in such cases that the method can be used to determine only the lower frequency components of the scattering object.

It is clear from Eq. (13) that the intensity data function of the field is singular at points where the intensity of the field vanishes. Under such circumstances our method cannot be used. This fact is a consequence of the fact that the phase reconstruction of a field is not unique when the field contains optical vortices [20,21]. However, a requirement for the validity of the first Rytov approximation is that the incident field is only weakly perturbed by the scattering object. Conversely, as long as the first Rytov approximation is valid, the solution to the phase problem will be unique. Evidently the phase reconstruction problem is well suited for weakly scattered fields.

We have seen that a single Fourier component of $\widetilde{F}$ may be determined by the use of a data function $\widehat{D}_{\varDelta}$. This data function employs Fourier components of the intensity data function on a pair of measurement planes. However, an additional Fourier component of $\widetilde{F}$ may be determined from the same intensity information by using a slightly modified data function, as we now show.

Let us define a modified data function $\widehat{D}^m_{\varDelta}$ by the relation

$$\widehat{D}^m_{\varDelta}(u, v; d) \equiv \frac{\widehat{D}_I(u, v; d) - \widehat{D}_I(u, v; d + \varDelta)\mathrm{e}^{-\mathrm{i}(w-k)\varDelta}}{\varDelta}. \tag{21}$$

The only difference between this equation and Eq. (16) is the sign of the exponent. On substituting the appropriate intensity data functions into this equation, one finds that

$$\widehat{D}^m_{\varDelta}(u, v; d) = -\frac{(2\pi)^2\mathrm{i}}{w}\left[\widetilde{F}[-u\mathbf{s}_1 + -v\mathbf{s}_2 + (w-k)\mathbf{s}_0]\right]^* \mathrm{e}^{-\mathrm{i}(w-k)d}\varGamma^m[w, \varDelta], \tag{22}$$

where

$$\varGamma^m[w, \varDelta] \equiv \frac{\left[1 - \mathrm{e}^{-2\mathrm{i}(w-k)\varDelta}\right]}{\varDelta}. \tag{23}$$

By taking the complex conjugate of Eq. (22), we may determine the quantity $\widetilde{F}[-u\mathbf{s}_1 - v\mathbf{s}_2 + (w-k)\mathbf{s}_0]$, a component of $\widetilde{F}$ distinct from that determined by the use of $\widehat{D}_{\varDelta}$. The values of two Fourier components of $\widetilde{F}$ may therefore be

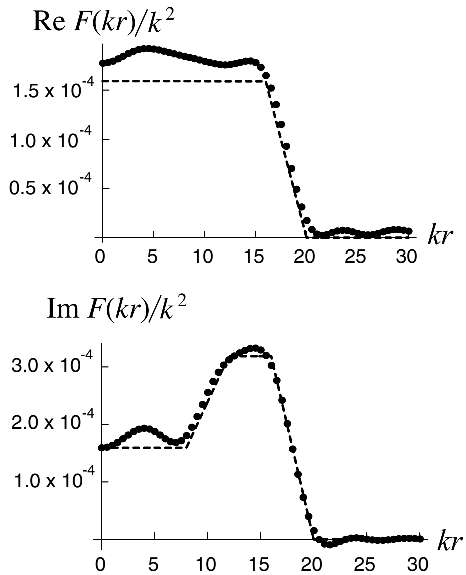

Fig. 5. Reconstructions of a layered, spherically symmetric scattering potential from forward scattering data. The dashed line indicates the actual scattering potential.

deduced from a single u, v Fourier component of the intensity data function, measured on the two measurement planes.

As an example of the new tomographic method, we consider scattering from a spherically symmetric object with a scattering potential defined by

$$F(\mathbf{r}) = \begin{cases} f_a & \text{when } r \leqslant a \\ \dfrac{f_a - f_b}{a - a_1}(r - a) + f_a & \text{when } a < r \leqslant a_1 \\ f_b & \text{when } a_1 < r \leqslant b_1 \\ f_b \dfrac{r - b}{b_1 - b} & \text{when } b_1 < r \leqslant b \\ 0 & \text{when } r > b, \end{cases} \tag{24}$$

with $n_a = 1.001 + 0.001\mathrm{i}$, $n_b = 1.001 + 0.002\mathrm{i}$, and $f_i = (n_i^2 - 1)/4\pi$, where $i = a, b$. The radii were chosen to have the (scaled) values $ka = 8$, $ka_1 = 12$, $kb_1 = 16$, and $kb = 20$. The scattered field was calculated using the first Born approximation [1, Section 13.1.2].

The measurement planes were taken to be at distances $kd = 70$ and $k(d + \Delta) = 71.4$; the distance Δ was chosen so that no zeros of $\Gamma[w, \Delta]$ are present in the forward or back scattering arrangements. The intensity was sampled in-plane at radial intervals $k\rho = 1$, to a maximum radius of $k\rho_{\max} = 250$. In performing the reconstruction of the scattering potential, the spherical symmetry of the object was assumed as prior knowledge. To demonstrate the stability of the method, Gaussian noise was added to the intensity data on the two planes with a standard deviation equal to 6% of the average value of the scattered intensity on a sphere of radius equal to the distance of the measurement plane from the origin.

To circumvent the difficulties of the data function near the origin, the value of $\widetilde{F}(\mathbf{K})$ for $|\mathbf{K}|$ below a certain spatial frequency was extrapolated as described in Section 4.

The reconstructions are shown in Fig. 5. The dashed curve represents the actual scattering potential, while the solid curve represents the reconstruction. It can be seen that there is excellent agreement between them. It is to be noted that both the real and imaginary parts of the scattering potential can be determined, despite the fact that only intensity data was used in the reconstruction.

3. Intensity diffraction tomography in backscattering

The tomographic method described in Section 2 showed how information in the intensity of a field scattered by an object into the half-space $\mathscr{R}^+$ may be used to determine some of the Fourier components of the scattering potential. One might expect that it might be possible to determine structural information also from the backscattered field. The backscattered field may be shown to contain information about higher spatial frequencies of the object and, as suggested in the previous section, will therefore require the spacing between measurement planes to be smaller than a wavelength, which would be impractical at optical wavelengths. However, a method which can retrieve information from backscattered fields might be useful for acoustical or other experiments, and we will therefore consider such a method here.

Returning to Eq. (4) and the Weyl expansion, Eq. (5), it follows that the complex phase in the region $\mathscr{R}^-$ is given by

$$\psi(x,y;d) = (2\pi)^2 \mathrm{i} \iint \frac{1}{w} \widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 - (w+k)\mathbf{s}_0] \mathrm{e}^{-\mathrm{i}(w+k)d} \mathrm{e}^{\mathrm{i}(ux+vy)} \,\mathrm{d}u\,\mathrm{d}v. \tag{25}$$

In a manner strictly similar to that used to derive Eq. (15), it can be shown that

$$\begin{aligned}\widehat{D}_I(u,v;d) = \mathrm{i}\frac{(2\pi)^2}{|w|^2}\Big\{ & w^* \widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 - (w+k)\mathbf{s}_0] \mathrm{e}^{-\mathrm{i}(w+k)d} \\ & - w\Big[\widetilde{F}[-u\mathbf{s}_1 - v\mathbf{s}_2 - (w+k)\mathbf{s}_0]\Big]^* \mathrm{e}^{\mathrm{i}(w+k)d}\Big\},\end{aligned} \tag{26}$$

where we are again considering only values of u and v such that $u^2 + v^2 \leqslant k^2$.

We next define a data function $\widehat{D}_\delta$ for the back scattering region $\mathscr{R}^-$ as

$$\widehat{D}_\delta(u,v;d) \equiv \frac{\widehat{D}_I(u,v;d) - \widehat{D}_I(u,v;d+\delta)\mathrm{e}^{-\mathrm{i}(w+k)\delta}}{\delta}, \tag{27}$$

with $\delta < 0$. On substitution from Eq. (26) into Eq. (27), we find that

$$\widehat{D}_\delta(u,v;d) = \frac{(2\pi)^2\mathrm{i}}{w\delta}\widetilde{F}[u\mathbf{s}_1 + v\mathbf{s}_2 - (w+k)\mathbf{s}_0]\mathrm{e}^{-\mathrm{i}(w+k)d}\left\{1 - \mathrm{e}^{-2\mathrm{i}(w+k)\delta}\right\}. \tag{28}$$

It can be seen that, as in the forward scattering region, the Fourier components of the scattering potential are directly related to the Fourier transform of the scattered intensity on a pair of measurement planes. The Fourier components available for reconstruction are shown in Fig. 6. These Fourier components are represented by points on a hemisphere that is the complement of the hemisphere of available Fourier components in the forward scattering region. Because the hemisphere shown in Fig. 6 no longer intersects the origin in Fourier space, there is no difficulty in determining the $u = v = 0$ component. However, in order to avoid other zeros of the term in the curly brackets in Eq. (28), it is necessary that

$$(w+k)2|\delta| < 2\pi \tag{29}$$

for all possible values of w. Since the maximum value of w is k, Eq. (29) suggests that

$$|\delta| < \frac{\pi}{2k} = \frac{\lambda}{4}. \tag{30}$$

It is therefore seen that, as noted at the beginning of this section, to take advantage of the information in the backscattered field the measurement planes must be spaced at a distance appreciably smaller than a wavelength. This would be impractical at optical wavelengths, but for electromagnetic radiation

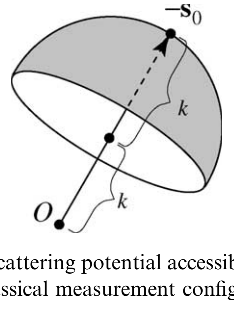

Fig. 6. Fourier components of the scattering potential accessible from measurements of the field in the back scattering region in the classical measurement configuration.

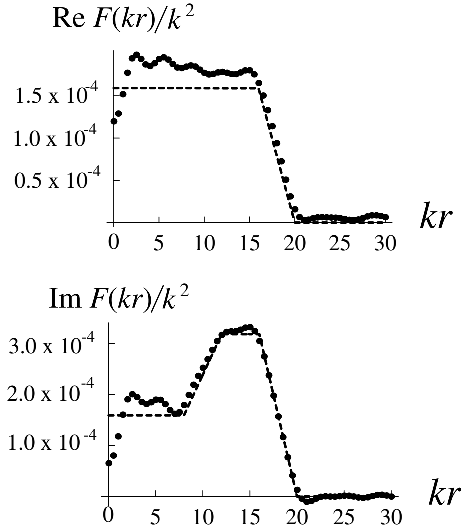

Fig. 7. Reconstructions of a layered, spherically symmetric scattering potential using both the forward and backward scattering data. The dashed line indicates the actual scattering potential.

of longer wavelength or for acoustical waves such measurements could be performed.

There is a difficulty in the analysis just described which is not immediately obvious. It is to be noted that the field described by Eqs. (3) and (4) represents the *total* field, the sum of incident and scattered fields. In the region $\mathscr{R}^+$, where the incident and scattered fields are both propagating into the same half-space, it is natural to assume that it is the total field $U(\mathbf{r})$ that will be measured by a detector, i.e. that one can measure the data function $D_I(\mathbf{r})$ directly. However, in the region $\mathscr{R}^-$, the incident field and the scattered field are propagating into different half-spaces, and a detector set up in the region $\mathscr{R}^-$ will generally measure only the scattered field U_s, where

$$U_s(\mathbf{r}) = U(\mathbf{r}) - U_i(\mathbf{r}). \tag{31}$$

This scattered field will generally have a small amplitude and will possess zeros of intensity, i.e. phase singularities, a difficulty we have already mentioned. To measure the quantities considered in this section will require that a plane wave with a phase equal to the phase of the incident plane wave be made to interfere with the scattered field. Such a procedure could be carried out with an appropriate interferometric technique. One might expect that to perform such a procedure would require sensitivity in measuring position much less than a wavelength; however, we have also seen that such sensitivity is assumed already in the positioning of the pair of measurement planes.

As an example, we again consider the spherically symmetric object with the scattering potential given by Eq. (24). We take all parameters to be the same as in the previous example, except that now a pair of additional planes is used in the backscattering direction at distances $kd = -70$ and $k(d + \delta) = -71.4$, and the higher spatial frequency Fourier data are reconstructed from these planes. The reconstruction is shown in Fig. 7. There seems to be little improvement in the reconstructions, save perhaps that the flat surfaces of the scattering potential are more accurately reconstructed.

4. Reconstruction of low spatial-frequency components of scattering object

A helpful property of the new tomographic method is that it does not require the processing of more data than does traditional diffraction tomography. Measurements of the intensity and phase of the field on a plane are replaced by measurements of the intensity on a pair of planes. One complication that arises in the use of the new method is the vanishing of $\widehat{D}_\Delta$ when $u = v = 0$. Consequently $\widetilde{F}$ at the origin in Fourier space cannot be determined directly from the data function. Furthermore, the reconstruction of components of $\widetilde{F}$ near the origin will be highly susceptible to noise because these components are determined by dividing the data function $\widehat{D}_\Delta$ by the function $\Gamma[w, \Delta]$, which has a very small value for small values of u, v. One must therefore use indirect methods to find the behavior of $\widetilde{F}$ near the origin. We will briefly consider some possibilities of doing so.

The reason for this complication may be understood by returning to the arguments which follow Eq. (15). At a given pair of values (u, v), the Fourier transform of the field intensity in a plane contains contributions from a pair of plane waves travelling in different directions. By measuring the intensity in two planes, it is possible to isolate the contribution of each of these plane waves and so determine $\widetilde{F}$. The values of (u, v) determine the angular spread of the pair of waves. For smaller values of (u, v), these plane waves propagate farther along the z-direction before an appreciable phase difference is built up between them. When $u = v = 0$, $\widehat{D}_I$ is constant with respect to position, and the wave number of each plane wave has zero value.

Several options exist for overcoming this difficulty. To begin with, it is to be noted that since the scattering object is finite, $F(\mathbf{r})$ is nonzero only within a finite domain. It follows from a theorem concerning functions of several complex variables (Ref. [22, p. 352]) that $\widetilde{F}(u, v, w)$ is the boundary value on the real axes of an analytic function in three complex variables. It is therefore, in principle, possible to use analytic continuation methods to determine unknown values of the function $\widetilde{F}$ from the known values.

The measurement of values of $\widetilde{F}$ near the origin may be directly improved simply by using additional measurement planes spaced at greater distances

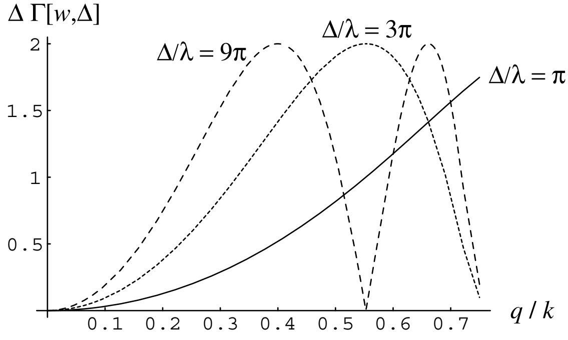

Fig. 8. Showing the form of the function $\Gamma[w, \Delta]$ for a variety of values of the (normalized) parameter spacing Δ. The parameter $q = \sqrt{u^2 + v^2}$.

apart. Fig. 8 shows the value of the function $\Gamma(w, \Delta)$ for several values of spacing Δ between the planes. It can be seen that as the spacing of the planes is increased, the low values of the function become more compressed about the origin. This suggests that a three-plane measurement system might be useful in some circumstances: two planes spaced as constrained by inequality (20), and a third plane placed at a distance far from the first two.

It is also to be noted that one additional piece of information is available about $\widetilde{F}$ at the origin. Returning to Eq. (15), it can be seen that, for $u = v = 0$, the equation reduces to

$$\widehat{D}_I(0, 0; d) = -\frac{2(2\pi)^2}{k} \mathrm{Im}\left\{\widetilde{F}(0, 0, 0)\right\}. \tag{32}$$

The imaginary part of $\widetilde{F}(0, 0, 0)$ can therefore readily be determined from the integral of the intensity over a single measurement plane. This result is analogous to the well-known optical theorem, according to which the power extinguished from a plane wave incident on a scattering object is proportional to the imaginary part of the forward scattering amplitude [1, Section 13.3]. The information in Eq. (32) can be used as a check on the validity of any scheme used to determine the low-frequency components of $\widetilde{F}$.

Because $\widetilde{F}(\mathbf{K})$ is the boundary value of an entire analytic function, it may be expanded in a Taylor series about the origin,

$$\widetilde{F}[\mathbf{K}] = \widetilde{F}[0] + \mathbf{K} \cdot \nabla_{K'} \widetilde{F}[\mathbf{K}']|_{\mathbf{K}'=0} + \frac{1}{2} (\mathbf{K} \cdot \nabla_{K'})^2 \widetilde{F}[\mathbf{K}']|_{\mathbf{K}'=0} + \mathrm{O}(K^3), \tag{33}$$

where $\mathbf{K} = K_1\mathbf{s}_1 + K_2\mathbf{s}_2 + K_0\mathbf{s}_0$ and $\nabla_{K'} = \partial/\partial K_1'\mathbf{s}_1 + \partial/\partial K_2'\mathbf{s}_2 + \partial/\partial K_0'\mathbf{s}_0$. This equation contains 10 complex parameters: one complex number for the zeroth order term, three complex numbers for the linear term, and six for the quadratic term. One can therefore assume that, near the origin, $\widetilde{F}$ is well-approximated by an equation of the form

$$\widetilde{F}[\mathbf{u}] \approx A + \mathbf{u}\cdot\mathbf{B} + (\mathbf{u}\cdot\mathbf{C}\cdot\mathbf{u}), \tag{34}$$

where A is a complex constant, $\mathbf{B}$ is a complex vector, and $\mathbf{C}$ is a complex symmetric matrix. The parameters A, $\mathbf{B}$, and $\mathbf{C}$ can be found most readily by matching Eq. (34) to reconstructed values of $\widetilde{F}$ near the origin. In the most general case, this results in 10 complex equations which can be solved to determine the coefficients of the Taylor expansion; if more data is available, it can be used to improve the quality of the reconstruction.

In the examples described in the previous two sections, this method was used to determine the low spatial frequency components of the scattering potential. For a spherically symmetric scattering potential, the linear term of Eq. (33) vanishes and the quadratic term simplifies, leaving only two undefined complex parameters (four real parameters). Near the origin, the potential was therefore assumed to have the form

$$\widetilde{F}(K) = A_1 + A_2K^2, \tag{35}$$

where A_1 and A_2 are complex constants. This approximation was used for K-values below a chosen cutoff spatial frequency K_{cutoff}; above this spatial frequency, Eq. (17) was used to reconstruct the Fourier components of $\widetilde{F}(\mathbf{K})$. The constants A_1 and A_2 were then determined by minimizing a function of the form

$$M(A_1, A_2) = \sum_{n=1}^{N} \left|A_1 + A_2K_n^2 - \widetilde{F}[K_n]\right|^2 + \left(\mathrm{Im}[A_1] + \frac{1}{2(2\pi)^2}\widehat{D}_I(0,0;d)\right)^2, \tag{36}$$

where the sum is over N selected values of the reconstructed data whose spatial frequencies K_n were above K_{cutoff}. The use of Eq. (36) represents a simplified method of curve fitting. In the examples of the previous two sections, the parameters were chosen to be $K_{\mathrm{cutoff}}/k = 0.1$ and $N = 20$. It can be seen that this method and this choice of parameters resulted in good reconstructions of the scattering potential.

5. Conclusions

In this paper we have reviewed and extended a new method of performing diffraction tomography without the use of phase information; it has been shown that intensity data is sufficient for reconstructing the scattering object. Although there exists a problem in the reconstruction of the low frequency components of the scattering potential as discussed in Section 4, practical methods exist for overcoming this problem. It seems that this new tomographic method could be used in circumstances when phase measurements are not feasible, or could be used to simplify existing diffraction tomography techniques by eliminating the need for a phase measuring apparatus.

Acknowledgements

This research was supported by the US Air Force Office of Scientific Research under grant F49260-03-1-0138, by the Engineering Research Program of the Office of Basic Energy Science at the US Department of Energy under grant DE-FG02-2ER 45992, and by the Defense Advanced Research Project Agency under grant MDA9720110043.

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