INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS A: PURE AND APPLIED OPTICS

J. Opt. A: Pure Appl. Opt. **6** (2004) S277–S280 PII: S1464-4258(04)68462-5

# The diffraction of light by narrow slits in plates of different materials

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Received 3 September 2003, accepted for publication 1 December 2003
Published 19 April 2004
Online at stacks.iop.org/JOptA/6/S277
DOI: 10.1088/1464-4258/6/5/027

**Abstract**
We analyse the diffraction of light incident on a sub-wavelength slit in a thin plate. It is found that plates with different material properties, such as conductivity and thickness, show a fundamentally different behaviour of the field near the slit. Depending on the material properties, the light transmission can either be enhanced or frustrated. A correspondence between the handedness of optical vortices and the transmission behaviour is demonstrated.

**Keywords:** singular optics, phase singularities, optical vortices, diffraction, electromagnetic fields, slits

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

Most theoretical studies that deal with the diffraction of light by a narrow slit in a metal plate assume as simplifying conditions that the plate is vanishingly thin and perfectly conducting (see, for example, [1, 2] and the references therein). Recently, we have developed a rigorous Green tensor formalism that allows us to analyse the diffraction of light at slits in plates of finite thickness and finite conductivity. In [3, 4] we found that *enhanced light transmission* [5] through a sub-wavelength slit in a thin metal plate is accompanied by the annihilation of phase singularities of the Poynting vector in the neighbourhood of the slit. With applications like optical data storage and near-field optics in mind, it is of interest to determine how the transmission process changes for different materials.

In the present paper we examine the diffraction of light and its singular behaviour near a slit in different kinds of materials. It is found that there is a significant difference in the behaviour of the field near metal plates and near those made out of semiconductor material. In the latter case, frustrated transmission may occur—that is, the transmission through a narrow slit in such a plate can be *less* than the transmission through a semiconductor plate *without* a slit. It is shown by numerical simulation that frustrated transmission is accompanied by a handedness of optical vortices that is opposite to that which occurs in the case of metal plates for which enhanced transmission takes place.

Furthermore, we study the dependence of the light transmission on the thickness of the plate. The results found are qualitatively explained by considering Fabry–Pérot resonances of waveguide modes.

## 2. The Green tensor method

A rigorous scattering approach [6] was used to calculate the field in the vicinity of an infinitely long slit in a metal plate. This method can be used to deal with realistic materials of finite conductivity and finite thickness. The illuminating field is taken to be monochromatic with time-dependence $\exp(-i\omega t)$. For the electric field this scattering problem reduces to solving the following integral equation [6]:

$$\mathbf{E} = \mathbf{E}^{(\mathrm{inc})} - i\omega\Delta\varepsilon \int_{\mathrm{slit}} \mathsf{G}\cdot\mathbf{E}\,\mathrm{d}^2 r, \qquad (1)$$

where $\Delta\varepsilon = \varepsilon_0 - \varepsilon_{\mathrm{plate}}$ is the difference in permittivity between the slit (vacuum) and the plate, and $\mathsf{G}$ is the electric Green

 

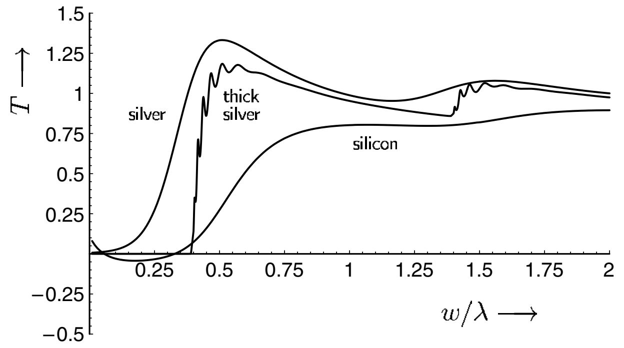


**Figure 1.** The transmission coefficient $T$ of a narrow slit as a function of the slit width $w$, expressed in wavelengths. The lower curve is for a slit in a 100 nm thick silicon plate, the upper curve is for a slit in a 100 nm thick silver plate, whereas the middle curve is for a slit in a 2.5 $\mu$m thick silver plate. The wavelength is $\lambda = 500$ nm, and the refractive indices are taken as $n_{\text{silver}} = 0.05 + \mathrm{i}\,2.87$ and $n_{\text{silicon}} = 4.3 + \mathrm{i}\,0.74$, respectively.

tensor pertaining to the plate without the slit. The incident field $\mathbf{E}^{(\text{inc})}$ is the field that would occur in the absence of the slit in the plate. Here it is taken to be a plane wave propagating perpendicular to the plate, with the electric field polarized along the slit. (The other polarization, i.e., with the electric field perpendicular to the slit, has a guided mode without cut-off width and can give rise to surface plasmons. This case will be discussed elsewhere.) It is to be noted that the incident field also consists of a reflected part and a part that is transmitted by the plate. The second term on the right-hand side of (1) is the scattered field due to the presence of the slit. For points within the slit, (1) is a Fredholm equation of the second kind for $\mathbf{E}$, which is solved numerically by the collocation method with piecewise-constant basis functions [7].

The definition of the transmission coefficient $T$ of the slit consists of two parts. The first part is the component of the actual (time-averaged) Poynting vector $\mathbf{S}$ normal to the plate, integrated over the area of the slit at the dark side of the plate. The second part consists of the difference of the normal components of the actual Poynting vector and that of the Poynting vector in the absence of the slit, $\mathbf{S}^{\text{inc}}$, integrated over the dark side of the plate (excluding the slit). The result is normalized by the normal component of $\mathbf{S}^{(0)}$, the Poynting vector of the field emitted by the source and impinging on the slit, i.e.

$$T \equiv \frac{\int_{\text{slit}} S_z \, \mathrm{d}^2x + \int_{\text{plate}} \left(S_z - S_z^{\text{inc}}\right) \mathrm{d}^2x}{\int_{\text{slit}} S_z^{(0)} \, \mathrm{d}^2x}. \tag{2}$$

The subtraction in the second integral in the numerator corrects for the small part of the incident field that may flow through the plate itself. It is to be noted that the normalization factor that appears as the denominator is a geometrical optics term. In other words, according to geometrical optics the transmission coefficient will always be unity.

## 3. Transmission through silver and silicon plates

In figure 1 the transmission coefficient is shown as a function of the slit width for a thin silver plate and for a thin silicon plate (the middle curve will be discussed in the next section). For the silver plate we observe a damped oscillating behaviour of the transmission coefficient. This is explained in detail in [3, 4]. There the increase of the transmission coefficient at certain widths (e.g., $w \approx 0.4\lambda$, $1.4\lambda$) was found to be due to the occurrence of a cut-off width of a guided mode. A good qualitative insight into the behaviour of the transmission is obtained by studying the field of power flow near the slit. In particular, the role of phase singularities is found to be significant. For example, near these cut-off widths annihilations of several phase singularities in the field of power flow occur. In figure 2 the enhanced transmission (i.e., $T > 1$) is found to coincide with a handedness of optical vortices within the plate that gives rise to a 'funnel effect'.

The behaviour of the transmission coefficient of a slit in a silicon plate is seen from figure 1 to be quite different: for small slit widths the transmission is frustrated (i.e., $T < 0$). From (2) it follows that in this case the power flow through a plate *with* a slit is less than that through a plate *without* a slit. Also, in contrast to what we found for a slit in a silver plate, the transmission coefficient now never exceeds unity.

To better understand the behaviour of the transmission coefficient, we have plotted in figure 3 the effective index of the first guided mode for both silicon and silver waveguides as a function of the slit width. For the first guided mode in silver (discussed in [3]), it is found that there is a cut-off width at $w \approx 0.4\lambda$, i.e. for a width smaller than this critical width

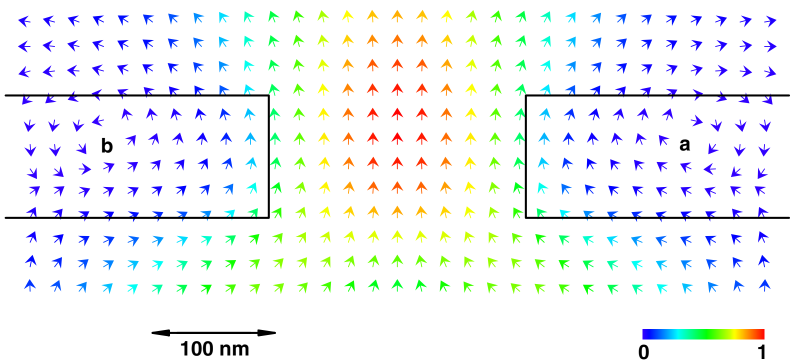


**Figure 2.** The behaviour of the time-averaged Poynting vector near a 200 nm wide slit in a 100 nm thick silver plate. The incident light (coming from below) has a wavelength $\lambda = 500$ nm. The shading/colour coding indicates the modulus of the (normalized) Poynting vector (see legend). Two vortices ($a$ and $b$) can be observed.

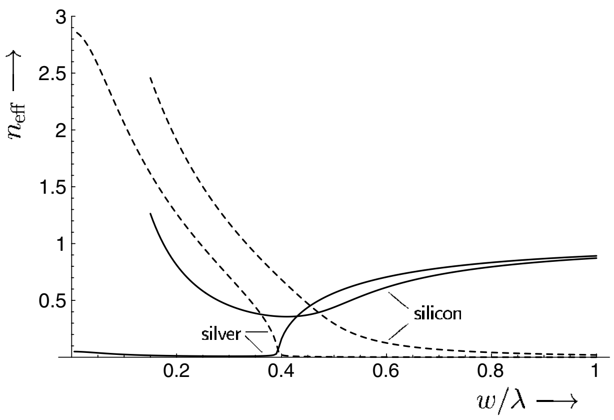


**Figure 3.** The effective indices $n_{\text{eff}}$ of the first waveguide mode inside a narrow slit as a function of the slit width $w$, expressed in wavelengths. The full curves denote the real part of the effective index, the dashed curves denote the imaginary part.

the mode is evanescent, whereas for a width larger than the cut-off width the mode is propagating. The silicon waveguide has a completely different behaviour: for small slit widths ($w < 0.15\lambda$) the first guided mode is not present (i.e., only radiation modes [8] exist), whereas for a somewhat larger slit width (i.e., $w \approx 0.15\lambda$–$0.4\lambda$) the first guided mode is more damped than a plane wave through the bulk material. The extraordinarily high damping of this guided mode is caused by its extension into the lossy cladding. It should be noted that at these widths the transmission coefficient is smaller than that of a silicon plate without a slit (see figure 1). For slit widths $w > 0.4\lambda$, the guided mode has a propagating character, which corresponds with a positive transmission coefficient.

In figure 4, the time-averaged Poynting vector is plotted around a 100 nm wide slit in a silicon plate. In the case of a silver plate (figure 2), two vortices are visible, which correspond with a funnel-like power flow into the slit [3, 4]. In contrast, the power flow near a slit in a silicon plate exhibits two vortices and two saddle points located inside the slit, coinciding with a power flow into the silicon plate rather than into the slit. It should be noted that this coincides with a different handedness of the vortices: vortex *b* in figure 2 is left-handed, whereas vortex *b* in figure 4 is right-handed. In figure 5 a detail of figure 4 is shown. There it can be observed that in this region a sink and a saddle point are present. Due to conservation of energy, a sink is only possible inside a lossy material. If the width of the slit is decreased, the sink and the saddle point annihilate each other, a process in which the topological charge is conserved [4].

In figure 6 another detail of figure 4 is presented. If the width of the slit is slightly increased, the saddle points (*a* and *d*) move together to form a *monkey saddle singularity* [9]. This is shown in figure 7. A monkey saddle is similar to a saddle point, but possesses three attracting and three repulsing directions, instead of two attracting and two repulsing directions. The monkey saddle is unstable, as it exists for one value of the slit width only; for a larger width it decays into two saddle points (see figure 8). With the aid of symmetry considerations, one can show that the singularity in figure 7 is indeed a monkey saddle, and not two closely spaced saddle points.

## 4. Transmission through thick silver plates

Next we discuss the influence of the plate thickness on the light transmission. In figure 1 the transmission coefficient is plotted as a function of the slit width for a thick (i.e., $5\lambda$) silver plate. Several differences with the thin silver plate can be observed. Below the cut-off width at $w = 0.4\lambda$ there is hardly any transmission through the thick silver plate. At the cut-off width a steep rise of the transmission as a function of the width is seen. Furthermore we note that there are some fast oscillations of the transmission near the cut-off widths.

The negligible transmission below $w \approx 0.4\lambda$ can be explained by noting that all guided modes are evanescent in that region, i.e., the transmission will decrease exponentially as a function of the thickness of the plate. The explanation for the fast oscillations near the cut-off widths is more subtle: the guided mode travelling in one direction can be reflected into a guided mode travelling into the opposite direction at the end of the slit. Therefore a Fabry–Pérot type resonance exists, i.e., the transmission as a function of the thickness has an oscillating behaviour, with a period determined by the real part of the effective index. Because the effective index as a function of the width changes rapidly near the cut-off width (see figure 3), the interference of the guided modes will change repeatedly

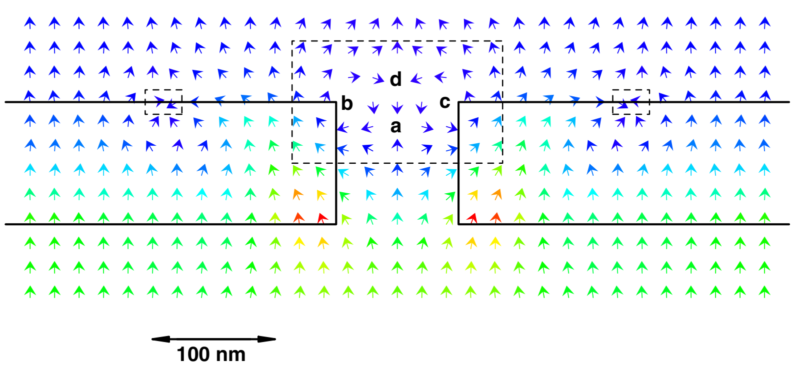


**Figure 4.** The behaviour of the time-averaged Poynting vector near a 100 nm wide slit in a 100 nm thick silicon plate. The incident light (coming from below) has a wavelength $\lambda = 500$ nm. Two vortices (b and c) and two saddle points (a and d) are present in the middle region. In the dashed regions on the right and the left a saddle point and a sink are present (see figure 5). The dashed box in the middle denotes the region depicted in figure 6.

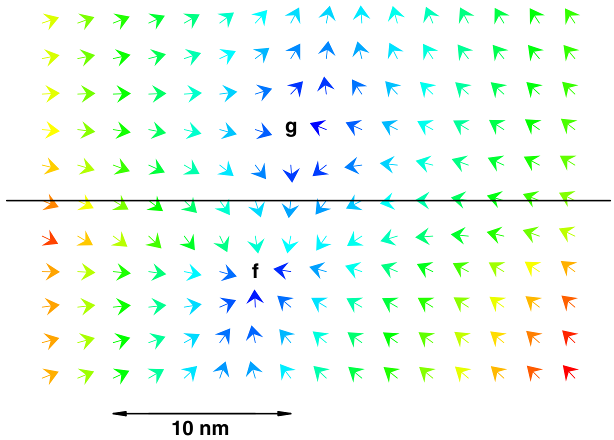


**Figure 5.** Detail (from the right-hand side) of figure 4. A sink (f) and a saddle point (g) are present.

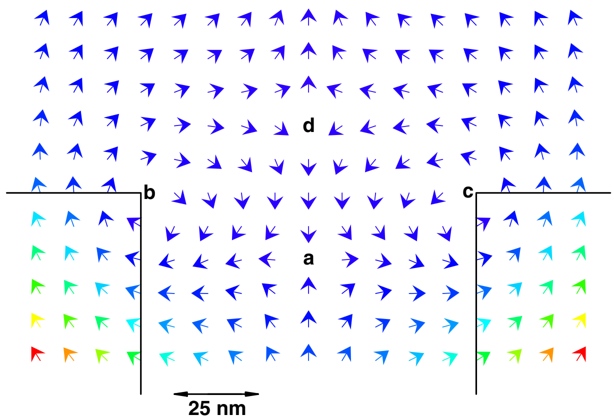


**Figure 6.** Detail of the time-averaged Poynting vector field near a 100 nm wide slit in a 100 nm thick silicon plate. The depicted region is indicated in figure 4.

from constructive to destructive and back. This results in the fast oscillations observed in figure 1. Similar Fabry–Pérot-like resonances inside slits were reported in [10, 11] for the case where the incident light is TM-polarized, rather than TE-polarized, as discussed in the present paper.

In conclusion, we have shown that the process of light transmission through a narrow slit in a plate strongly depends on the material properties of the plate. In particular, a slit in a silver plate can give rise to enhanced transmission, whereas a slit in a comparable silicon plate exhibits frustrated transmission. This was found to coincide with a change in handedness of certain optical vortices. Our results suggest that the material properties of the plate and its thickness are as important as the width of the slit in the application of extraordinary light transmission in any practical optical system.

## Acknowledgments

This research is supported by the Dutch Technology Foundation STW and the European Union, within the framework of the Future and Emerging Technologies program (SLAM).

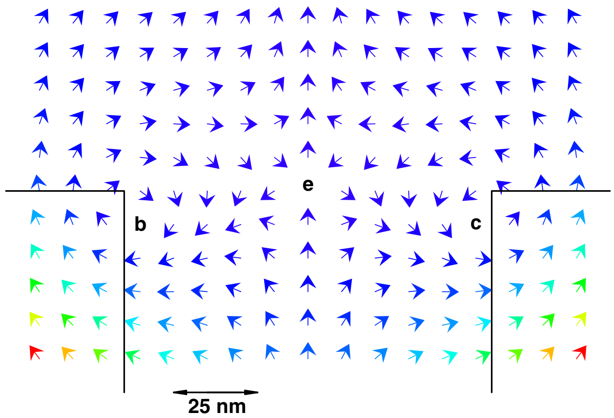


**Figure 7.** The time-averaged Poynting vector near a 109.5 nm wide slit in a 100 nm thick silicon plate. A monkey saddle (e) with topological charge −2 can be seen.

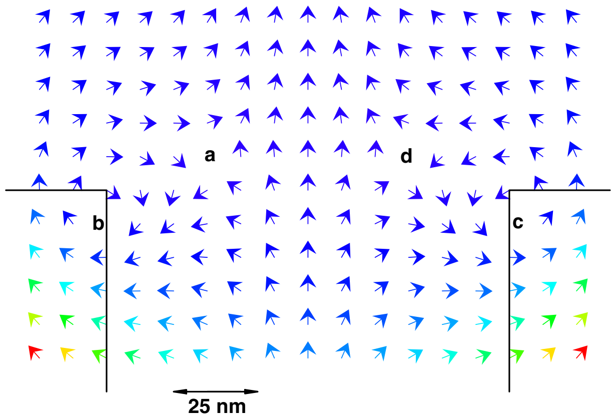


**Figure 8.** The time-averaged Poynting vector near a 120 nm wide slit in a 100 nm thick silicon plate. The monkey saddle (visible in figure 7) has decayed into two saddle points (a and d).

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