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Phase and coherence singularities generated by the interference of partially coherent fields

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Abstract

A condition for the complete destructive interference of partially coherent fields emerging from pinholes in an opaque screen is derived, with the assumption of symmetry in both their geometric positions and coherence properties. We use this condition to theoretically investigate the simultaneous production of phase singularities of the optical field and of the spectral degree of coherence. We find that in cases where the number of point sources is even, a new type of mixed field/correlation singularity is observed.

1. Introduction

The study of the effects associated with phase singularities in optical wavefields, being a rich subject in itself, has developed into a new branch of physical optics, known as singular optics [1]. Phase singularities of a wavefield exist at points for which the amplitude of the wavefield is zero, so that the phase of the field is singular, and such points that are associated with the optical field itself are commonly referred to as optical vortices (an example is illustrated in Fig. 1). Recent investigations with various kinds of wavefields such as partially coherent light fields [2], bichromatic optical fields [3], vortex waves [4] and focused fields [5] have revealed that field correlation functions may possess phase singularities as well. This extension of singular optics to include field correlation functions has led to the introduction of a new class of singularities, generally called coherence vortices [6,7]. These coherence vortices (pairs of points at which the spectral degree of coherence identically vanishes) are characterized by a vortex structure of the phase of the spectral degree of coherence with respect to one of the observation points (it is to be noted, however, that other types of singularities of correlation functions can exist; see Ref. [8]).

It is noted in Ref. [6] that for pairs of points at which coherence vortices occur, it is not required for the amplitude of the field to vanish. Furthermore, while zeros of the correlation function are quite common in partially coherent fields, zeros of the field amplitude are likely to occur only for fields that are fully coherent. In fact, the study of the phase singularities of the coherence function in Ref. [2] pertains to the case for which phase singularities of the field amplitude do not occur. It seems interesting then to consider a scenario for which phase singularities of both the field amplitude and the correlation function can simultaneously occur, and to study the relationship between the two distinct types of singularities. This will be the subject of study for the present work. We will focus on the phase singularities of fields and correlation functions that arise from the interference of partially coherent, quasi-monochromatic point sources.

The quantity of interest for this study is the cross-spectral density of an optical field, defined as

$$W(\mathbf{r}_1,\mathbf{r}_2,\omega) \equiv \langle U^*(\mathbf{r}_1,\omega)U(\mathbf{r}_2,\omega)\rangle, \tag{1}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the two observation points and $U(\mathbf{r},\omega)$ is the optical field at position $\mathbf{r}$ and frequency ω. The asterisk denotes complex conjugation,

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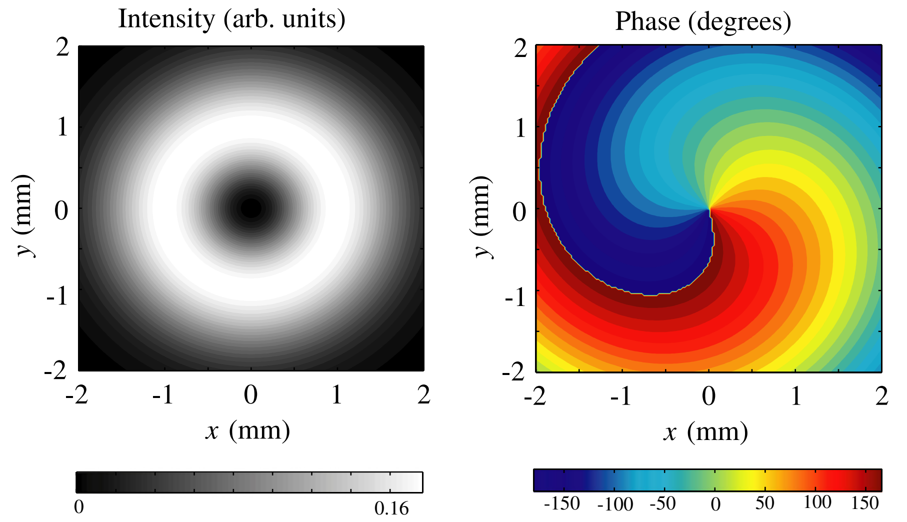

Fig. 1. A typical example of an optical vortex formed at the center of a Laguerre-Gaussian beam of radial order $m = 0$, and azimuthal order $n = 1$. Here the beam waist is taken to be 1 mm, and the intensity and phase profiles are taken at $z = z_0$, the Rayleigh range of the beam. The wavenumber of the beam is taken to be $9.9213 \times 10^3\ \text{mm}^{-1}$. Note that because the beam is assumed to be spatially fully coherent, the phase of the field is equal to the phase of the spectral degree of coherence [13].

and the brackets indicate averaging over an ensemble of space-frequency realizations [9, Sec. 4.7]. The cross-spectral density may always be written in the form

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sqrt{I(\mathbf{r}_1, \omega)}\sqrt{I(\mathbf{r}_2, \omega)}\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = A(\mathbf{r}_1, \omega)A(\mathbf{r}_2, \omega)\mu(\mathbf{r}_1, \mathbf{r}_2, \omega), \quad (2)$$

where $I(\mathbf{r},\omega)$ is the average spectral intensity of the field at position $\mathbf{r}$ and frequency ω, $A(\mathbf{r},\omega)$ is the corresponding average spectral amplitude of the field, and $\mu(\mathbf{r}_1,\mathbf{r}_2,\omega)$ is the spectral degree of coherence of the field. The spectral degree of coherence is a measure of the correlation of the field between the two points $\mathbf{r}_1$ and $\mathbf{r}_2$, and can be shown to be constrained by the values $0 \leqslant |\mu| \leqslant 1$. Eq. (2) illustrates that there exist two distinct ways for the cross-spectral density to vanish: the intensity may be equal to zero at a point or the spectral degree of coherence may be equal to zero with respect to a pair of points.

In anticipation of our results, we shall refer to phase singularities associated with zeros of the field amplitude (or, equivalently, the field intensity) as *field singularities*, and those associated with the zeros of the spectral degree of coherence as *coherence singularities*. In addition, we reserve the terms *optical vortex* and *coherence vortex* for a singularity that exhibits a screw dislocation in its phase. To create a situation for which phase singularities of both the amplitude and the spectral degree of coherence are present, we note it has been demonstrated theoretically and experimentally that under certain conditions, complete destructive interference can indeed occur for partially coherent fields [10–12]. Although the results in these references were demonstrated with three partially coherent point sources, we will also consider the superposition of fields emanating from a larger number of point sources (e.g. 4, 5, 6,...) that are partially correlated. While the emphasis in Refs. [10–12] concerned the observation of zeros in the spectral intensity, this work involves an investigation into the phase of the wavefields. As pointed out by Wolf and others [13,14], however, one may associate a well-defined phase function with an optical field only if it is spatially fully coherent, in which case the cross-spectral density may be factorized into the product of the fields evaluated at the pair of observation points. Since the fields are partially coherent in this case, we will study instead the behavior of the phase of the cross-spectral density to gain insight on the singularities of the amplitude and correlation function in the region of superposition.

In Section 2 we briefly review the behaviour of partially coherent light emerging from pinholes. Using a simple argument and some reasonable physical assumptions, we derive a sufficiency condition for complete destructive interference in an N-pinhole system. In Section 3, we theoretically analyze the phase of the cross-spectral density for $N = 3$, 4, 5, and 6 to study the behavior of the singularities which arise, and to determine what relations may exist between the different types of singularities. In Section 4, we summarize the results and offer concluding remarks.

2. Behavior of partially coherent light emerging from pinholes

Let us assume partially coherent light that is quasi-monochromatic, with center frequency ω, is incident on a screen with N pinholes. We will employ the space-frequency representation of a partially coherent field [9, Sec. 4.7] to describe its statistical properties. Under the assumption that the angles of incidence and diffraction are small, the field emerging from the pinholes is given by the sum of contributions from the individual pinholes, and the spectral density ('intensity') at an observation point P is given by

$$I(P,\omega) = \left\langle \sum_{n=1}^{N} U_n^*(P,\omega) \sum_{n=1}^{N} U_n(P,\omega) \right\rangle, \quad (3)$$

where

$$U_n(P,\omega) = -\mathrm{i}\frac{ka^2}{2\pi} U_0(Q_n,\omega) \frac{\mathrm{e}^{\mathrm{i}kR_n}}{R_n}, \quad (4)$$

is the field produced by the nth pinhole, $U_0(Q_n,\omega)$ is the value of the incident field at the nth pinhole, R_n is the distance from the nth pinhole to the point P, and $k = \omega/c$ is the wavenumber of the light, c being the speed of light. The geometry for the case of three pinholes is shown in Fig. 2.

To derive the conditions for which $I(P,\omega) = 0$ in Eq. (3), we first review the case $N = 2$. For brevity, the explicit dependence on the frequency ω will be dropped from now on. For 2 pinholes, Eq. (3) reduces to

$$\begin{aligned} I(P) &= \langle [U_1(P) + U_2(P)]^* [U_1(P) + U_2(P)] \rangle \\ &= \langle |U_1(P)|^2 + |U_2(P)|^2 + 2Re[U_1^*(P)U_2(P)] \rangle \\ &= |U_1(P)|^2 + |U_2(P)|^2 \\ &\quad + 2|\mu_{12}||U_1(P)||U_2(P)|\cos(\delta_{12} + \beta_{12}) \end{aligned} \quad (5)$$

where $\mu_{12} = |\mu_{12}|\mathrm{e}^{\mathrm{i}\beta_{12}}$ is the spectral degree of coherence of the light incident at pinholes 1 and 2, β_{12} is the phase of μ_{12}, and $\delta_{12} = k(R_1 - R_2)$ is the phase shift arising from the optical path length difference. If we assume that $|U_1(P)|^2 = |U_2(P)|^2 = |U_0|^2$, then it can be seen from Eq. (5) that complete destructive interference only occurs if $|\mu_{12}| = 1$ and

$$(\delta_{12} + \beta_{12}) = (2m+1)\pi, \quad (6)$$

with m being an integer. We conclude that only a fully coherent illuminating field, with $|\mu_{12}| = 1$, can produce complete destructive interference in the two-pinhole case.

Let us now turn to the $N = 3$ case, and consider the case for which the average amplitudes $U_0(Q_n,\omega)$ of the illuminating fields are all equal. If the pinholes are arranged with geometric symmetry with respect to the origin, and we consider an observation point on the axis of symmetry of the pinholes, then $\delta_{ij} = 0$, $|U_n(P)|^2 = |U_0|^2$ and Eq. (3) for $N = 3$ reduces to the form

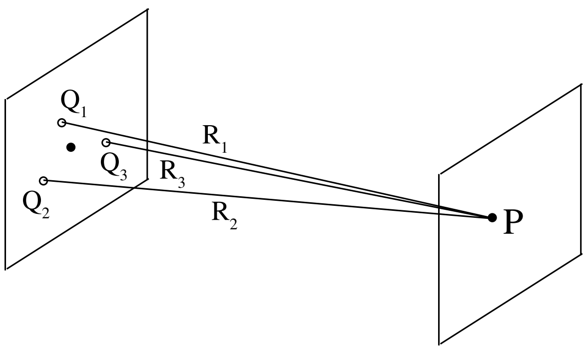

Fig. 2. Illustrating the geometry with $N = 3$ pinholes.

$$\begin{aligned} I(P) &= \langle [U_1(P) + U_2(P) + U_3(P)]^* [U_1(P) + U_2(P) + U_3(P)] \rangle \\ &= |U_1|^2 + |U_2|^2 + |U_3|^2 + 2Re(\mu_{12}U_1^*U_2) \\ &\quad + 2Re(\mu_{13}U_1^*U_3) + 2Re(\mu_{23}U_2^*U_3) \\ &= 3|U_0|^2 + 2|\mu_{12}||U_0|^2\cos(\beta_{12}) + 2|\mu_{13}||U_0|^2\cos(\beta_{13}) \\ &\quad + 2|\mu_{23}||U_0|^2\cos(\beta_{23}), \end{aligned} \quad (7)$$

where $\mu_{ij} = |\mu_{ij}|e^{\mathrm{i}\beta_{ij}}$.

We observe from Eq. (7) that there are, in principle, many combinations of values of μ_{ij} that can satisfy $I(P) = 0$. We restrict ourselves to the special case for which all off-diagonal μ_{ij}'s are equal, i.e., $\mu_{ij} = \mu_0 = |\mu_0|\mathrm{e}^{\mathrm{i}\beta_0}$ $(i \neq j)$. This reduces Eq. (7) to the form

$$I(P) = 3|U_0|^2 + 6|\mu_0||U_0|^2\cos\beta_0. \quad (8)$$

For complete destructive interference in the case of three pinholes, we thus require that $\mu_0 = -\frac{1}{2}$, in agreement with the results of Ref. [10]. Extending our analysis to N pinholes, it can be easily seen by extending the derivation of Eqs. (5) and (7) that

$$I(P) = N|U_0|^2 + \sum_{i<j}^{N}(N-1)|U_0|^2|\mu_{ij}|\cos\beta_{ij}. \quad (9)$$

Again restricting ourselves to the case $\mu_{ij} = \mu_0$ (and $\beta_{ij} = \beta_0$), one arrives at the following condition for complete destructive interference, viz.

$$\begin{aligned} 0 &= N|U_0|^2 + N(N-1)|U_0|^2|\mu_0|\cos\beta_0 \\ N(N-1)|\mu_0|\cos\beta_0 &= -N \\ |\mu_0|\cos\beta_0 &= -\frac{1}{N-1} \\ Re(\mu_0) &= -\frac{1}{N-1}. \end{aligned} \quad (10)$$

It is to be noted that condition (10) presents a solution for the special case of geometric symmetry in the position of the pinholes and equal values of μ_{ij}. We have therefore found a sufficiency condition for the complete destructive interference of partially correlated fields emanating from N pinholes, though it is important to note that it is not a necessary condition. The general condition for complete destructive interference, $\det(M) = 0$, where det denotes determinant, and M is the $N \times N$ matrix containing the μ_{ij}'s, has been derived in Ref. [10, Appendix A].

With the condition (10), we can readily simulate the simultaneous occurrences of phase singularities of both the optical field and correlation function through the interference of fields from multiple pinholes. For a pair of points P_1 and P_2 in the region of superposition, the coherence vortices are related to the zeros of the spectral degree of coherence $\mu(P_1,P_2)$, which may be derived from the cross-spectral density (2) by the relation

$$\mu(P_1,P_2) = \frac{W(P_1,P_2)}{\sqrt{I(P_1)I(P_2)}}. \quad (11)$$

Eq. (11) is well-defined as long as $I(P_1),I(P_2)\neq 0$. If condition (10) is satisfied and the pinholes are arranged in a symmetric geometry, a field singularity is expected along the axis of symmetry. By taking P_2 as a fixed point not at the origin, the phase of $\mu(P_1,P_2)$ as a function of P_1 can be readily computed from Eq. (11).

It is worth considering briefly the construction of a source which satisfies Eq. (10). In Ref. [10] it was demonstrated that for 3 pinholes this correlation can be achieved with a pair of counter-rotating Laguerre-Gauss beams, but the construction for N pinholes requires a bit more subtlety. We consider a system of X independent (uncorrelated) lasers, each of which produces a field U_i, where $i = 1, \ldots ,X$, i.e.

$$\langle U_i^* U_j \rangle = \begin{cases} 0, & i \neq j, \\ |U_0|^2, & i = j. \end{cases} \tag{12}$$

Because the laser fields are uncorrelated, their cross-correlations vanish. By the use of fiber splitters and delay lines, we may add these fields together in any combination with any phase delay we choose. At each pinhole, we combine equal contributions from $N - 1$ independent lasers, which in Eq. (11) will produce a denominator of $(N - 1)\ |U_0|^2$. However, we require that each pinhole only have one contribution which is present at any other pinhole; this will result in a numerator for Eq. (11) of $|U_0|^2$. At the first pinhole, we therefore add together the contributions of $N - 1$ independent lasers, at the second pinhole, we introduce an additional $N - 2$ independent lasers, at the third, an additional $N - 3$, and so on. We therefore require a total number of independent lasers for an N-pinhole system equal to the result

$$X \equiv \sum_{n=1}^{N-1} n = \frac{N(N-1)}{2}. \tag{13}$$

Because each independent laser only makes contributions to two pinholes, we can select one of these contributions to be π-phase delayed from the other; this results in the negative sign in the numerator of Eq. (11).

This rather tricky construction is illustrated in Fig. 3 for $N = 4$. Using the fields in this figure, one can readily show that the spectral degree of coherence between pinholes A and D, for instance, is given by

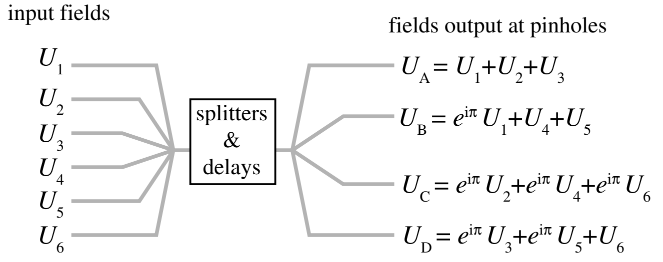

Fig. 3. Illustrating the technique for experimentally realizing the condition (10) for $N = 4$. The fields U_i $(i = 1, \ldots , 6)$ from six independent lasers are mixed using fiber splitters and delay lines into the combinations U_α $(\alpha = A, \ldots ,D)$. These combinations are fed into the pinholes.

$$\langle U_A^* U_D \rangle = \frac{\langle (U_1 + U_2 + U_3)^* (-U_3 - U_5 + U_6) \rangle}{\sqrt{\langle |U_1 + U_2 + U_3|^2 \rangle \langle | - U_3 - U_5 + U_6|^2 \rangle}} = \frac{-|U_0|^2}{3|U_0|^2} = -\frac{1}{3}. \tag{14}$$

This construction demonstrates that the correlations of Eq. (10) can be experimentally generated, at least in principle.

3. Phase singularities of the cross-spectral density for *N*-pinhole system

We have studied the phase of the cross-spectral density in the region of superposition of fields from multiple pinholes for $N = 3$, 4, 5, and 6. The condition (10) was applied to ensure that, in addition to the coherence singularities, a field singularity will occur at least at the point $x = y = 0$ in the region of superposition. We have made the choice in all cases that $\mu_0 = -1/(N - 1)$.

The behaviour of the phase of $W(P_1, P_2)$ at a plane for which $z = 2000$ mm is shown in Fig. 4. The reference point P_2 has been taken to lie at $x = y = 1$ mm. In general, the phase structure is quite complicated, and numerous singularities appear in every figure. We examine the $N = 3$ and $N = 4$ (diamond) cases in detail in Fig. 5.

For $N = 3$, we find, as expected, optical vortices at locations *a*, *b*, and *c*, characterized by their evident intensity null and vortex-like phase structure (recall Fig. 1). However, we also see numerous vortices at locations for which the intensity is non-zero, marked as *d*, *e* and *f*. These are coherence vortices, and one is very closely associated with each optical vortex. This behavior is similar to what has been observed in other studies of coherence vortices [6,15,16].

The $N = 4$ (diamond) case, however, illustrates previously unobserved structures. There is an intensity null at the center of the pattern, at point *a*, but there is not an optical vortex in the phase at this point. Instead, we see that a line of phase discontinuity (a π phase jump) intersects the intensity null. This line discontinuity is not associated with any intensity null, and therefore must be a coherence singularity. Though line discontinuities have been predicted previously in the context of Young's double slit experiment [2], no such mixed field/coherence singularity has yet been predicted or observed.

As noted in the introduction, a coherence singularity is a singularity of a two-point correlation function, and therefore its behavior should depend on the choice of reference point. In Fig. 6 we evaluate the behavior of the phase of the cross-spectral density as the location of the reference point is changed. It can be readily seen that the location of the line singularities depend strongly on the location of the reference point; however, they are always 'pinned' to the field singularities whose locations are independent of the choice of reference.

To gain further insight on the relationship between the field and coherence singularities, we study the behavior of

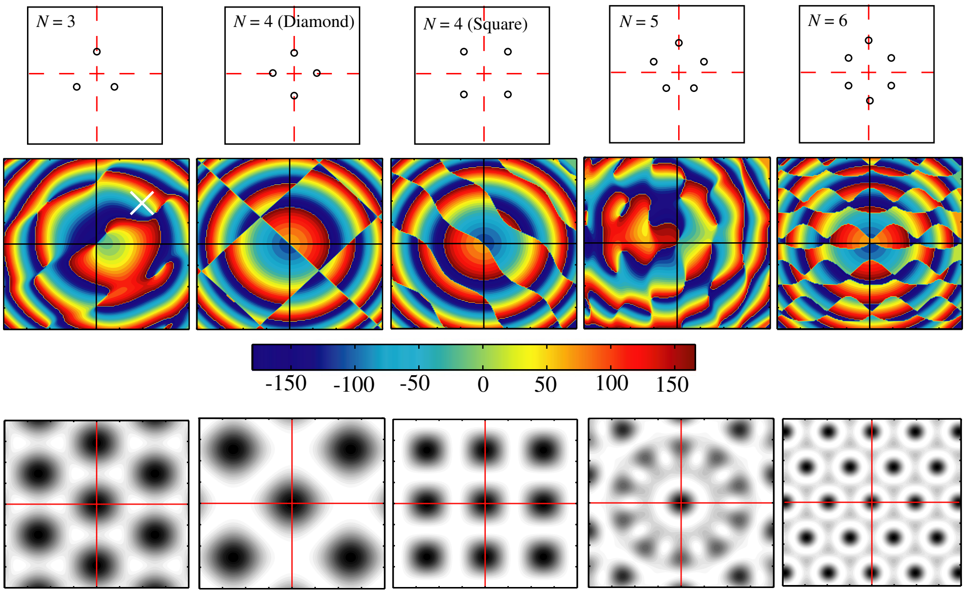

Fig. 4. Phase plots of the cross-spectral density, and field intensity plots (bottom row) when condition (10) is satisfied. From left to right: $N = 3$, 4 (diamond), 4 (square), 5, and 6. For all cases, the plots extend from the region bounded by $x = -2$ to 2 mm and $y = -2$ to 2 mm, $z = 2000$ mm, and the cross on the leftmost figure corresponds to the location of the reference point, which is taken to be $x = y = 1$ mm. The grayscale intensity plots are plotted using arbitrary units, hence the scale of the respective plots have been omitted.

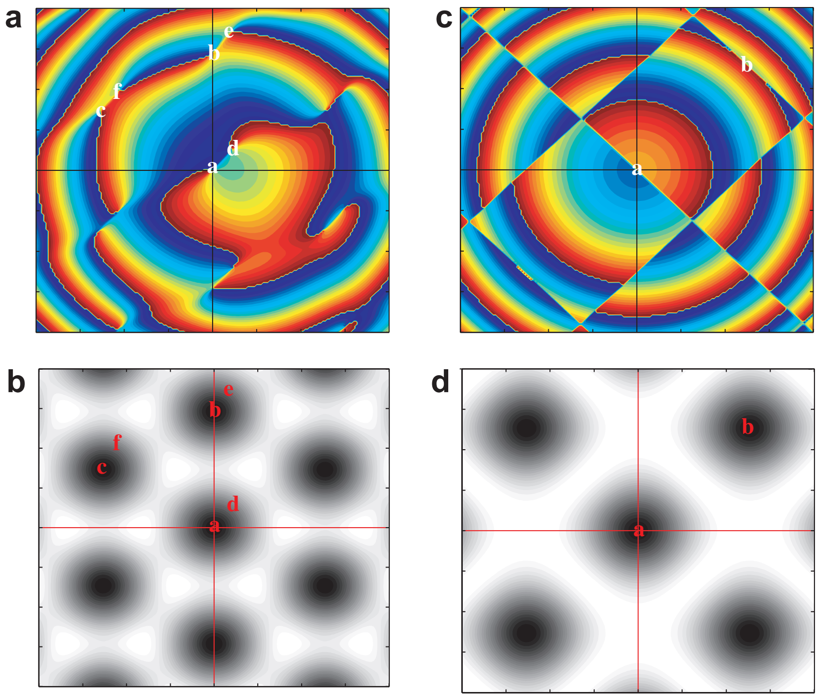

Fig. 5. Detail plots of the phase structure of the cross-spectral density (top row) and intensity structure of the field (bottom row). (a) and (b) represent the $N = 3$ case, while (c) and (d) represent the $N = 4$ (diamond) case. All other parameters are as in Fig. 4.

$W(P_1,P_2)$ when condition (10) is not satisfied. We note that there are numerous ways in which condition (10) can be modified so that the optical vortex in each case ceases to exist: (a) change the value of μ_0 for all the fields emanating from the pinholes; (b) change the value of μ_0 for all but one of the pinhole fields; (c) change the position of one of the pinholes while keeping μ_0 unchanged. Topological reactions in the vortices and singularities were observed in the phase plots of $W(P_1,P_2)$ with each of the three methods mentioned above.

In Fig. 7, we consider the effect of changing the value of μ_0 equally for all pairs of pinholes. As expected, in all cases

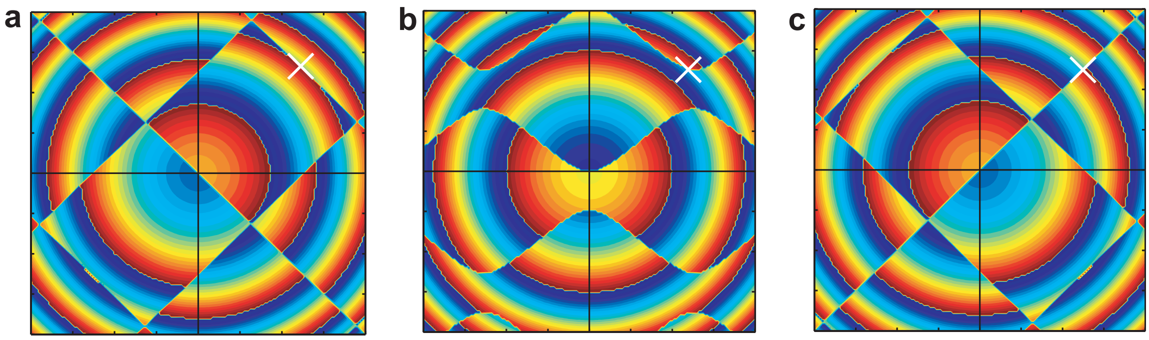

Fig. 6. Behavior of the cross-spectral density in the $N = 4$ (diamond) case when the reference point is changed. The reference point is taken to be (a) $(x, y) = (1,1)$, (b) $(x, y) = (0,1)$, and (c) $(x, y) = (-1,1)$. The cross indicates the position of an off-axis zero of intensity.

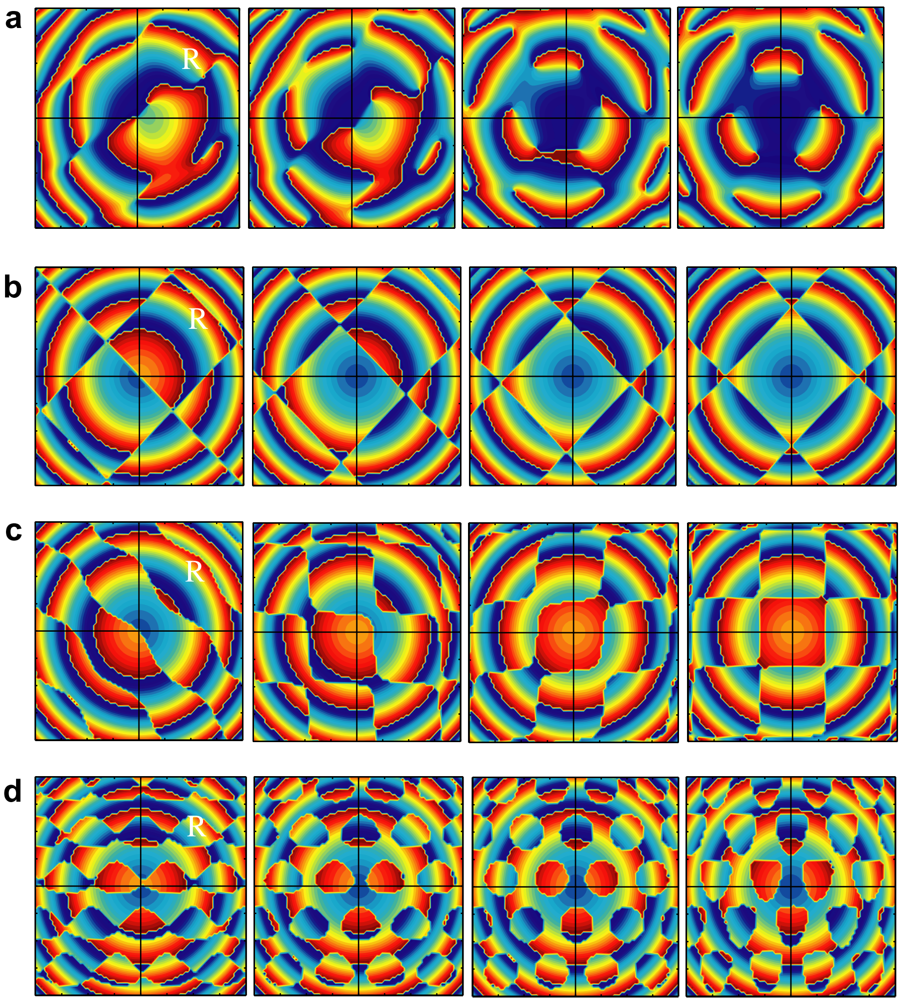

Fig. 7. Phase plots as the value of μ_0 changes so that condition (10) ceases to be satisfied. From top to bottom, left to right: (a) $N = 3$, $\mu_0 = -0.5, -0.3, 0.2, 0.5$; (b) $N = 4$ (diamond), $\mu_0 = -0.33, -0.05, 0.23, 0.93$; (c) $N = 4$ (square), $\mu_0 = -0.33, -0.05, 0.51, 0.93$; (d) $N = 6$, $\mu_0 = -0.2, -0.12, 0, 0.2$. R indicates the location of the reference point.

once the condition (10) is not satisfied, the intensity nulls in the field are no longer present. For the $N = 3$ case, this corresponds to the field singularity becoming a coherence singularity, and moving down and to the right as μ_0 is changed. For $N = 4$ and 6, the mixed field/coherence singularities become pure coherence singularities. For $N = 4$, for instance, the line singularity which intersects the axis of symmetry moves up and to the right.

4. Conclusion

A condition for complete destructive interference of partially coherent fields emerging from pinholes that have symmetric geometric and coherence properties has been derived. This condition was used to generate phase singularities of both the optical field and correlation function simultaneously in the region of superposition of partially coherent fields. In addition to the expected field singularities and coherence singularities, a new mixed form of phase singularity was observed for the $N = 4$ and $N = 6$ cases. By breaking the requirement for complete destructive interference, the field and mixed singularities were demonstrated to convert directly into pure coherence singularities.

The observation that field singularities can convert to coherence singularities, along with the observation of mixed singularities, supports the somewhat philosophical contention in Ref. [16] that traditional field singularities may be considered a special case of the broader class of coherence singularities. The newfound existence of mixed singularities suggests that more remains to be learned about the properties of phase singularities of two-point correlation functions.

Acknowledgement

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