Vortex beam propagation through atmospheric turbulence and topological charge conservation

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Received July 26, 2007; revised October 1, 2007; accepted October 26, 2007; posted November 5, 2007 (Doc. ID 85630); published December 20, 2007

The propagation of vortex beams through weak-to-strong atmospheric turbulence is simulated and analyzed. It is demonstrated that the topological charge of such a beam is a robust quantity that could be used as an information carrier in optical communications. The advantages and limitations of such an approach are discussed. © 2007 Optical Society of America

OCIS codes: 260.6042, 010.1300.

1. INTRODUCTION

In recent years, much work has been done to study the properties of optical beams that possess an intensity null along their propagation axis and hence a singular phase on that axis. The study of such 'vortex beams' (so called because the phase circulates about the central null, much like a fluid circulating a drain) has become an important subfield of the general study of phase singularities in optical fields, known now as singular optics [1].

Vortex beams have been investigated for various applications ranging from being used as information carriers in laser communications [2] to being employed as optical tweezers and spanners [3]. Such beams are of particular interest because they carry orbital angular momentum [4] and can impart this angular momentum to microscopic particles [5]. This orbital angular momentum may help vortex beams propagate through optical turbulence with less distortion than conventional Gaussian beams [2,6], and some theoretical studies have also suggested that they have the ability to self-heal around certain obstacles [7,8]. However, the studies undertaken so far have typically been limited to rather specialized circumstances, namely, single photon communication [2], ultrashort pulse propagation [6], interference-generated vortices [9], and partially coherent vortex beams [10], and no general study of the effectiveness of traditional vortex beams propagating through turbulence seems to have been undertaken.

One of the most important features of a vortex beam is the observation that the topological charge—a measure of the angular momentum of the beam—is a discrete variable and stable under phase perturbations of the field. This suggests that the topological charge might be used as an information carrier in optical communications systems, but as yet no studies have considered the robustness of such a method of communication.

In this paper we study the propagation of vortex beams of various orders through weak-to-strong atmospheric turbulence, using multiple phase screen simulations. It is demonstrated that the topological charge of such a beam is in principle a robust quantity that could be used as an information carrier in optical communications. In Section 2 we briefly review the topological properties of vortex fields. In Section 3 we then study the evolution of the topological properties of vortex beams in turbulence. Section 4 provides concluding remarks.

2. OPTICAL VORTICES AND TOPOLOGICAL CHARGE

A complex-valued, monochromatic scalar wave field $U(\mathbf{r},t)$ of frequency ω can be written in terms of the product of its space-dependent part, $U(\mathbf{r})$, and a complex exponential time dependence, $\exp[-i\omega t]$. The spatially dependent part can be separated further into the product of a real-valued amplitude $A(\mathbf{r})$ and a complex exponential containing the spatial phase $\phi(\mathbf{r})$ of the field, i.e.,

$$U(\mathbf{r}) = A(\mathbf{r})\exp[i\phi(\mathbf{r})]. \tag{1}$$

This factorization is well defined at all spatial points except those at which the amplitude $A(\mathbf{r})=0$. At such points, the definition of the phase is ambiguous, or singular.

The singularity of the phase in regions of zero amplitude has traditionally been treated as a mathematical curiosity of no particular physical significance. However, Nye and Berry [11] demonstrated that the phase around such singular regions has a well-defined structure, analogous to dislocations in crystals. Typically the region of zero amplitude manifests itself as a line, about which the phase takes on a circulating or helical structure, commonly referred to as an optical vortex.

An example of a field containing such a vortex is a Laguerre–Gauss (LG) beam of order $n=1$, $m=1$, the transverse profile of which is illustrated in Fig. 1. The

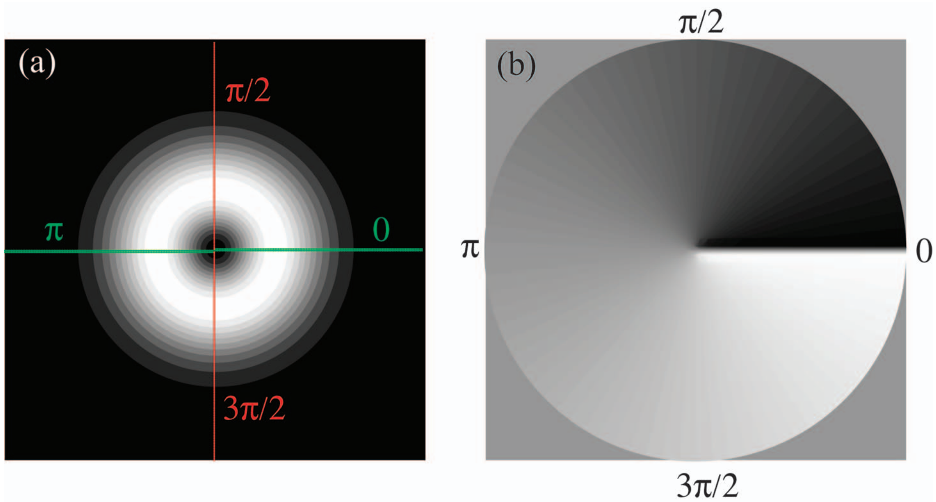

Fig. 1. Simulation of the transverse profile of a Laguerre–Gaussian (LG). beam of order $m=1$, $n=1$. (a) Combined phase-intensity plot. The gray scale indicates the intensity profile of the field, while the red and green lines are lines of constant phase: green represents $\mathrm{Im}\{U_{mn}\}=0$; red represents $\mathrm{Re}\{U_{mn}\}=0$. The intersection of the lines represents a phase singularity. The window size is 10 cm square, and $w_0=2$ cm. (b) Detailed phase plot of the LG beam. The phase increases continuously by 2π as one progresses around the circle.

beam contains an amplitude zero along the length of its central axis, giving the beam a "donutlike" profile. The lines of constant phase form a spokelike pattern, meeting at the zero of amplitude. Following a counterclockwise closed path around the singularity, the phase increases by 2π.

It can be shown ([12], Chapter 5) that the phase along a path enclosing any optical vortex must change by an integer multiple of 2π. This integer multiple is known as the topological charge t and may be defined mathematically by the following path integral:

$$t \equiv \frac{1}{2\pi}\oint_C \nabla \phi(\mathbf{r}) \cdot \mathrm{d}\mathbf{l}, \tag{2}$$

where C represents the contour of integration and $\mathrm{d}\mathbf{l}$ represents an infinitesmal vector path element. It is to be noted that this integral returns the *net* topological charge contained within the contour.

Topological charge is a conserved quantity in a wave field, and optical vortices can be created and annihilated only in pairs of opposing charge. This has been demonstrated experimentally [13] in focusing problems. Furthermore, in pure vortex beams such as LG beams, the topological charge is directly proportional to the quanta of orbital angular momentum the beam possesses [4].

These two properties (stability of topological charge and the presence of orbital angular momentum) suggest that optical vortices might make good carriers of information in optical communication systems. In the next section we consider an analysis of vortex beam behavior in turbulence.

3. VORTEX BEAMS IN TURBULENCE

We have simulated an extended region of atmospheric turbulence by use of a multiple thin phase screen method [14]. The refractive index power spectrum $\phi_n(\kappa)$ of the turbulence was taken to be of the Kolmogorov type,

$$\phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \tag{3}$$

where C_n^2 is the index of refraction structure parameter (strength of turbulence). The number of phase screens used depends on the propagation distance and the strength of the turbulence and is determined by requiring that scattering is weak over the interscreen distance and that the path is sampled enough to represent an extended medium. This results in roughly 3 screens for short distances and weak turbulence and over 20 for longer distances and stronger turbulence. Numerical results for the scintillation of Gaussian beams were compared to longstanding analytical results ([15], Chap. 5)] and were found to be in good quantitative agreement.

We consider the propagation of standard LG fields that may be written in the source plane ($z=0$) in the form [[16], Section 4.6.2],

$$U_{mn}(r,\theta) = A_{mn}\left(\frac{\sqrt{2}r}{w_0}\right)^m L_n^{(m)}\left(\frac{2r^2}{w_0^2}\right)\exp[im\theta - r^2/w_0^2], \tag{4}$$

where (r,θ) are the polar coordinates representing a position in the source plane, w_0 is the width of the beam in the source plane, $L_n^{(m)}$ are the associated Laguerre functions, and A_{mn} is a normalization constant. In Eq. (4) the integer n represents the radial order of the beam, and m represents the azimuthal order of the beam; $n=0$, $m=0$ represents an ordinary Gaussian beam. The orbital angular momentum of a photon in such a beam is $m\hbar$.

The topological charge of the beam at the detector is calculated by evaluating Eq. (2) around the perimeter of the detector aperture. This determines the total topological charge within the aperture region. A number of experimental techniques could potentially be used to realize such a configuration; a recent paper addressed theoretically the detection of phase singularities with a

Shack–Hartmann sensor [17]. The detector aperture used in these simulations is typically 4 cm, with exceptions noted later.

Quantities of particular interest are the average topological charge, $\bar{t}$, and the standard deviation of the topological charge, Δt, defined as

$$\bar{t} \equiv \frac{1}{N}\sum_{n=1}^{N} t_n, \tag{5}$$

$$\Delta t \equiv \left(\frac{1}{N}\sum_{n=1}^{N} t_n^2 - \bar{t}^2\right)^{1/2}, \tag{6}$$

where t_n is the detected value of the topological charge for the nth realization of the turbulent medium and N is the total number of realizations used to calculate the average, typically 500–1000 realizations.

The average topological charge as a function of propagation distance for some typical LG beams propagating through moderate atmospheric turbulence is shown in Fig. 2. It can be seen in both cases that the average topological charge remains constant, with very little variance, for appreciable distances—roughly 3.5 km for the (1,1) mode and roughly 2 km for the (5,5) mode. Past a critical distance, however, the average topological charge gradually decreases, coinciding with a significant increase in the standard deviation.

This decline can be understood to arise from the finite size of the detector aperture and the spreading of the beam. As the beam propagates and spreads through the atmospheric turbulence, the vortex core "wanders" away from its original position. If it wanders outside the perimeter of the detector aperture, the topological charge will not be measured by the detector. Figure 3 illustrates the transverse intensity and phase profile of a beam with lost topological charge.

This loss of topological charge occurs more or less rapidly for stronger or weaker turbulence, respectively. Figure 4 illustrates the average topological charge as a function of propagation distance for a variety of turbulence strengths. In the regime of weak turbulence ($C_n^2 \sim 10^{-16} - 10^{-17}$ m$^{-2/3}$), the topological charge is transmitted quite well with little variance, while in the regime of moderate turbulence ($C_n^2 \sim 10^{-14}$ m$^{-2/3}$) charge is rapidly lost and fluctuates greatly.

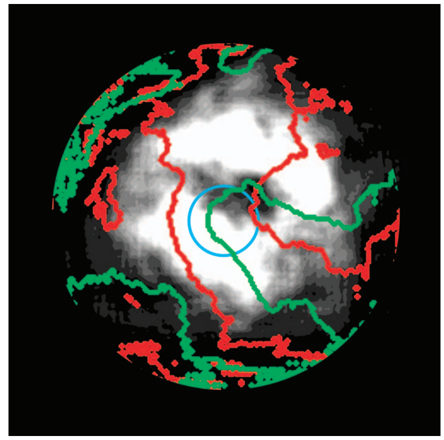

Fig. 3. Simulation of the transverse profile of a LG beam of order $m=1$, $n=1$, at a distance of $d=4$ km from the source. All other beam and turbulence parameters are as in Fig. 2. The image size is 50 cm square. It can be seen that the vortex has wandered outside the detector region (blue circle); no charge is detected.

A number of options exist for maintaining topological charge over long distances. The most straightforward is to increase the azimuthal order of the beam. Although topological charge decreases steadily over distance, some residual charge remains even at appreciable distances. This occurs because a beam with a central vortex of order m breaks down on propagation into m first-order vortices, which wander the transverse plane quasi-independently. If we define a 1 and 0 bit as the presence or absence, respectively, of any topological charge, we can extend the viable range of a communications system in this manner. The average topological charge for higher-order LG beams in strong turbulence is illustrated in Fig. 5. This technique is limited by the difficulty of experimentally realizing high-order LG beams.

Another option is to increase the size of the detector aperture, which allows the system to detect vortices with greater wander. To test this possibility, we consider a system with an aperture size $r_{ap}(d)$, which is comparable to the size of the diffracted and propagated beam, i.e.,

$$r_{ap}(d) = r_0 - w_0 + w_0\sqrt{1 + 4d^2/(k^2 w_0^4)}, \tag{7}$$

where d is the source-detector distance, r_0 is the aperture size at $d=0$, and $k=2\pi/\lambda$ is the wavenumber of the beam. The simulation results are illustrated in Fig. 6. Though for strong turbulence and low azimuthal order the results

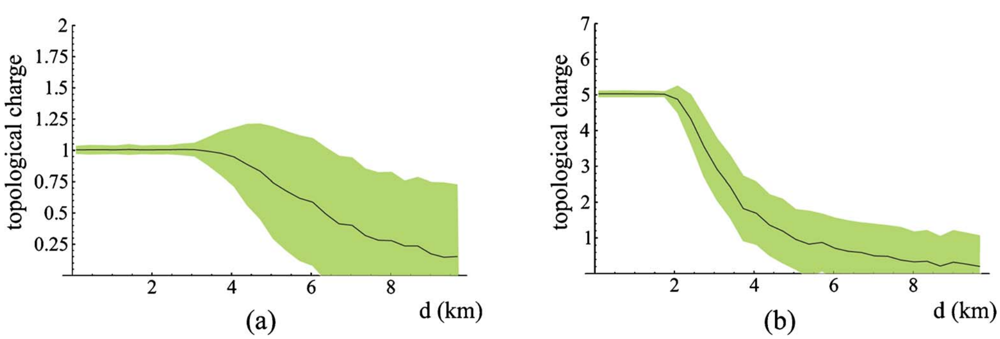

Fig. 2. (Color online) Simulation of the average topological charge for (a) LG beam of order $m=1$, $n=1$, and (b) LG beam of order $m=5$, $n=5$, propagating in moderate $C_n^2 = 10^{-15}$ m$^{-2/3}$ atmospheric turbulence. Here $w_0 = 2$ cm, $\lambda = 1.55$ μm, and the detector radius is 4 cm. The shaded region illustrates the standard deviation of the topological charge.

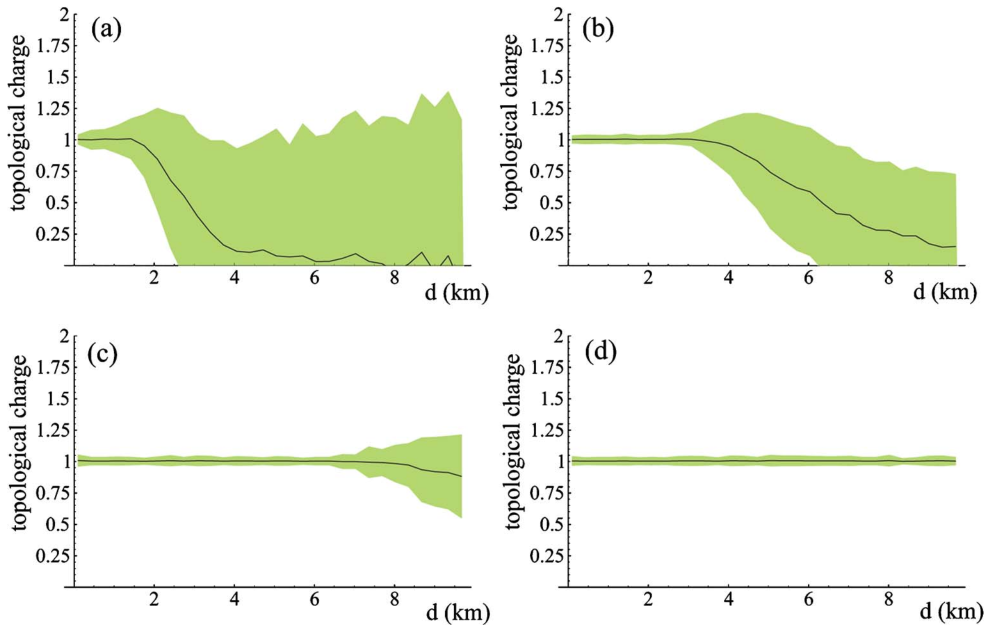

Fig. 4. (Color online) Simulation of the average toplogical charge for a LG beam of order $m=1$, $n=1$, for various turbulence strengths. (a) $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$, (b) $C_n^2=10^{-15}\ \mathrm{m}^{-2/3}$, (c) $C_n^2=10^{-16}\ \mathrm{m}^{-2/3}$, (d) $C_n^2=10^{-17}\ \mathrm{m}^{-2/3}$. All other parameters are as in Fig. 2.

are modest (average charge decreases slowly, but standard deviation increases dramatically), for moderate turbulence the results are dramatic—the average detected topological charge remains close to the original value even out beyond a propagation distance of 10 km. The combination of increasing aperture and higher topological charge, shown in Fig. 6(c), improves on the fixed aperture results of Fig. 5(c).

It is to be noted in Figs. 6(b) and 6(d) that even though the average topological charge remains essentially constant at large distances, the standard deviation increases dramatically. This cannot be the result of charge "wander" alone, which would also result in a decrease in the average. Instead, this increase in variance is the result of two factors: errors in the charge-counting algorithm due to increasing complexity of the phase profile of the field and the creation of new optical vortices through pair production.

As mentioned previously, topological charge in a wave field is a conserved quantity, and singularities can be created only in pairs of opposite topological charge. At large propagation distances and/or high turbulence strength, speckle and focusing effects become important (the caustic region) and pair production becomes common. Most often, these pairs produce no net change in the topological charge of the system [see Fig. 7(a)], but occasionally one member of the pair will drift outside the detector region, resulting in an increase or decrease in the detected charge [see Fig. 7(b)].

Additional simulations were undertaken to analyze the effect of source size w_0 on the topological properties of the wave field. It was found that even for appreciable variations in source size ($w_0=2$ cm to $w_0=4$ cm), the results presented in Figs. 2–7 were essentially the same.

Similar results, and similar solutions, exist when a vortex beam is propagated in strong ($C_n^2 \sim 10^{-13}\ \mathrm{m}^{-2/3}$) turbulence. Topological charge is lost very rapidly on propagation, making the use of low-order beams impractical. Figure 8 shows the results for a high-order beam, with $n=1$, $m=10$. It is to be noted that the distance scale of these plots is smaller than the previous ones; one does not expect the field to remain beamlike after propagating a

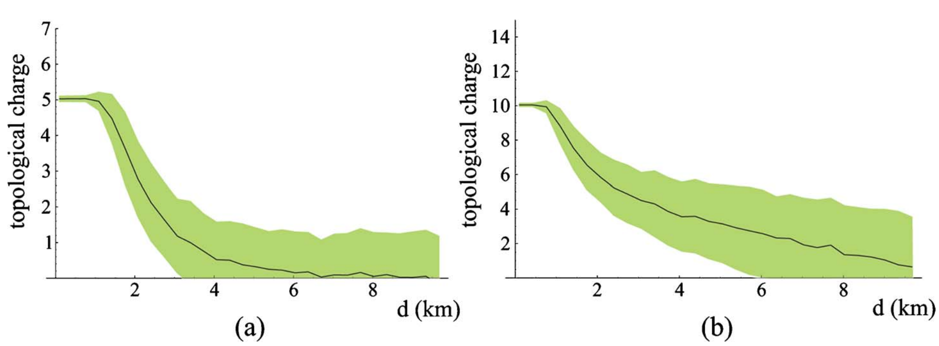

Fig. 5. (Color online) Simulation of the average toplogical charge for LG beams of order (a) $m=5$, $n=5$, and (b) $m=10$, $n=1$, in strong turbulence $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$. All other parameters are as in Fig. 2.

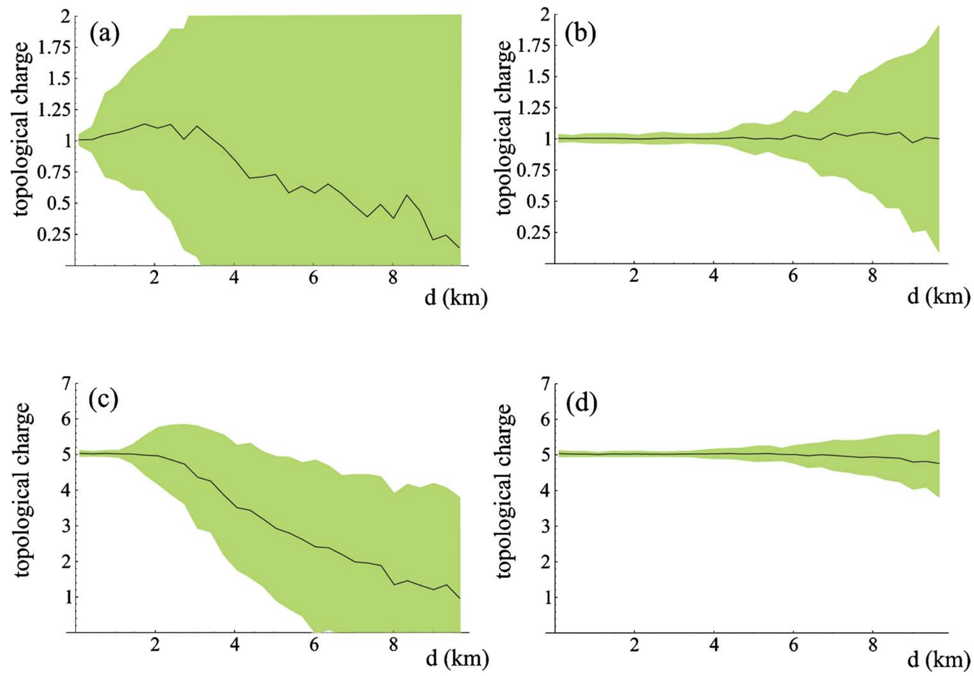

Fig. 6. (Color online) Simulation of the average topological charge for LG beams with variable aperture detectors. (a) $m=1$, $n=1$, $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$; (b) $m=1$, $n=1$, $C_n^2=10^{-15}\ \mathrm{m}^{-2/3}$; (c) $m=5$, $n=5$, $C_n^2=10^{-14}\ \mathrm{m}^{-2/3}$; (d) $m=5$, $n=5$, $C_n^2=10^{-15}\ \mathrm{m}^{-2/3}$. The quantity $r_0=4$ cm; all other parameters are as in Fig. 2.

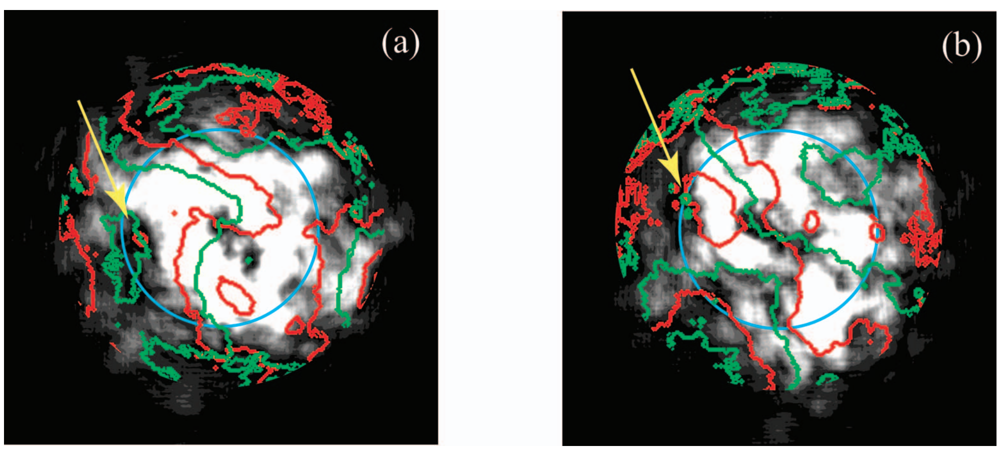

Fig. 7. Simulation of vortex pair production (intersection of red and green circles) in a LG beam of order $m=1$, $n=1$, with a variable aperture detector at a propagation distance of 6.5 km. All other parameters are as in Fig. 2. (a) Pair production (indicated by yellow arrow) results in no net change in topological charge. (b) Pair production (indicated by yellow arrow) results in one member of the pair lying outside the aperture; the detected topological charge is 2. The image size is 80 cm square.

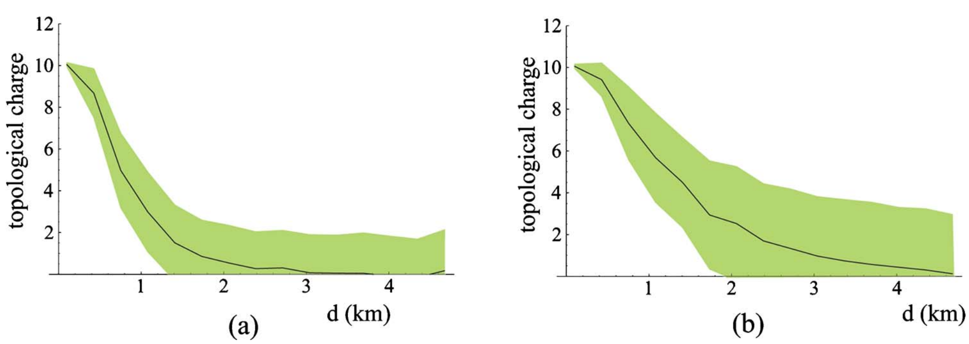

Fig. 8. (Color online) Simulation of the average topological charge for a LG beam of order $m=10$, $n=1$, in strong turbulence, $C_n^2=10^{-13}\ \mathrm{m}^{-2/3}$: (a) with a fixed detector radius 4 cm and (b) with a variable detector radius. All other parameters are as in Fig. 2.

distance of 10 km in strong turbulence. With a fixed aperture size [Fig. 8(a)], we find that the beam maintains a charge distinct from $m=0$ to a distance of about 1 km. When the aperture size is allowed to vary [Fig. 8(b)], this distinction is pushed to about 1.5 km. The variation in this case was taken to be faster than in Eq. (7) by a factor of $\sqrt{2}$. Larger apertures and/or higher beam orders can in principle push the useable range farther. Additional discrimination could be achieved by using a positive charge as the 1 bit and a negative charge as the 0 bit. The plots for negative topological charge have been computationally shown to be the mirror image about the horizontal axis of the positive charge plots.

4. CONCLUSIONS

In this paper we have studied the topological charge properties of vortex beams propagating through atmospheric turbulence. It was demonstrated that the topological charge is a robust quantity that can be transmitted over significant distances without loss. At a critical distance that depends on the strength of turbulence, the vortex "wanders" from the detector region, resulting in a loss of detected topological charge. This difficulty can be mitigated by two strategies: increasing the azimuthal order of the beam, to increase the amount of initial charge in the beam, and increasing the aperture size, to detect more wandering charge. At greater distances, an increase of topological charge fluctuation occurs because of detector limitations and charge pair production.

These results suggest that the use of vortex topological charge as an information carrier in optical communications is physically plausible. In this paper we have investigated theoretically the physical behavior of vortex beams in turbulence; much more work will need to be done to evaluate the experimental feasibility of the method and to optimize vortex beam communication strategies.

ACKNOWLEDGMENTS

We acknowledge support from L3-Communications Brashear, Pittsburgh, Pennsylvania, and the Air Force Office of Scientific Research under grant FA9550-05-1-0288.

REFERENCES

1. M. S. Soskin and M. V. Vasnetsov, "Singular optics," in *Progress in Optics, Vol. XLII*, E. Wolf, ed. (Elsevier, North-Holland, 2001), pp. 219–276.
2. C. Paterson, "Atmospheric turbulence and orbital angular momentum of single photons for optical communication," Phys. Rev. Lett. **94**, 153901 (2005).
3. N. B. Simpson, K. Dholakia, L. Allen, and M. J. Padgett, "Mechanical equivalence of spin and orbital angular momentum of light: an optical spanner," Opt. Lett. **22**, 52–54 (1997).
4. L. Allen, S. M. Barnett, and M. J. Padgett, eds., *Optical Angular Momentum* (IOP, 2004).
5. W. M. Lee, X.-C. Yuan, and W. C. Cheong, "Optical vortex beam shaping by use of highly efficient irregular spiral phase plates for optical micromanipulation," Opt. Lett. **29**, 1796–1798 (2004).
6. A. Vinotte and L. Berg, "Femtosecond optical vortices in air," Phys. Rev. Lett. **95**, 193901 (2005).
7. Z. Bouchal, "Resistance of nondiffracting vortex beam against amplitude and phase perturbations," Opt. Commun. **210**, 155–164 (2002).
8. M. V. Vasnetsov, I. G. Marienko, and M. S. Soskin, "Self-reconstruction of an optical vortex," JETP Lett. **71**, 130–133 (2000).
9. Y. Zhang and C. Tao, "Wavefront dislocations of Gaussian beams nesting optical vortices in a turbulent atmosphere," Chin. Opt. Lett. **2**, 559–561 (2004).
10. Y. Zhang, M. Tang, and C. Tao, "Partially coherent vortex beam propagation in a turbulent atmosphere," Chin. Opt. Lett. **3**, 559–561 (2005).
11. J. F. Nye and M. V. Berry, "Dislocations in wave trains," Proc. R. Soc. London, Ser. A **336**, 165–190 (1974).
12. J. F. Nye, *Natural Focusing and the Fine Structure of Light* (IOP, 1999).
13. G. P. Karman, M. W. Beijersbergen, A. van Duijl, and J. P. Woerdman, "Creation and annihilation of phase singularities in a focal field," Opt. Lett. **22**, 1503–1505 (1997).
14. J. M. Martin and S. M. Flatté, "Intensity images and statistics from numerical simulation of wave propagation in 3-D random media," Appl. Opt. **27**, 2111–2126 (1988).
15. L. C. Andrews, R. L. Phillips, and C. Y. Hopen, *Laser Beam Scintillation with Applications* (SPIE, 2001).
16. L. C. Andrews and R. L. Phillips, *Laser Beam Propagation through Random Media* (SPIE, 1998).
17. M. Chen, F. S. Roux, and J. C. Olivier, "Detection of phase singularities with a Shack–Hartman wavefront sensor," J. Opt. Soc. Am. A **24**, 1994–2002 (2007).