



Topological reactions of optical correlation vortices

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ABSTRACT

It is now well-appreciated that the correlation functions of partially coherent optical wavefields may possess phase singularities with properties similar to those that appear in monochromatic wavefields. Though much work has been done to investigate the generic properties of such correlation vortices, little effort has gone towards studying topological reactions associated with these vortices. In this paper we investigate three such reactions: (1) The break-up of a second-order optical vortex into first-order correlation vortices. (2) Creation and annihilation of correlation vortices. (3) The behavior of correlation vortices on propagation. These results clearly demonstrate the relationship between optical vortices and correlation vortices, and suggest the possible use of correlation vortices as a probe of the statistical properties of a field or a medium.

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1. Introduction

The study of ‘singular’ regions of optical wavefields has developed into a subfield of optics in its own right, now known as *singular optics* [1]. The original studies of singular optics, such as the pioneering work of Nye and Berry [2], investigated regions where the amplitude of coherent wavefields vanish, and hence the phase is undefined. They demonstrated that the zero manifolds of a wavefield are typically lines in three-dimensional space, around which the phase has a circulating or helical structure, now referred to as an ‘optical vortex’. Such vortices have many interesting properties, including a conserved, discrete ‘topological charge’, and a consequent stability on propagation, as well as a relationship to the orbital angular momentum of the wavefield [3].

Other authors have expanded the field of singular optics to include other types of singular behavior. Polarization singularities have been defined [4,5] as locations at which the ellipticity (‘C-points’) or handedness (‘L-lines’) of the polarization ellipse are undefined. Poynting vector singularities, locations at which the direction of Poynting flow is undefined, were first observed some time ago [6] and have been shown to have interesting connections to physical properties in nano-optical systems [7,8].

More recently, authors have demonstrated that spatial correlation functions of partially coherent wavefields may also carry phase singularities [9–14]. These structures, now referred to as *correlation singularities* or coherence singularities, are pairs of points at which the wavefields are spatially uncorrelated. With one of the two observation points fixed, the singularity has a vortex structure referred to as a *correlation vortex*. It has not only been shown that correlation vortices are typical features of a wavefield, but that

they are closely related to the optical vortices of the corresponding spatially coherent field [15,16].

Although much work has been done to study the behavior of generic (first-order) correlation vortices, very little work has been done to study the topological reactions of such vortices. Questions such as the behavior of higher-order correlation vortices, annihilation of correlation vortices, and the propagation of correlation vortices have remained unexplored, except for some recent experimental work. Such studies are essential to gain a complete understanding of correlation singularities and to complete the analogy between them and their fully coherent counterparts.

In this paper we investigate three such topological reactions:

1. The break-up of a second-order optical vortex into first-order correlation vortices.
2. Creation and annihilation of correlation vortices.
3. The behavior of correlation vortices on propagation.

These results clearly demonstrate the relationship between optical vortices and correlation vortices. Beyond this, the strong relationship between the vortex behavior and the correlation length suggest the possible use of correlation vortices as a probe of the statistical properties of a field or a medium.

We begin by reviewing the basic properties of optical vortices and correlation vortices. We then investigate the topological reactions listed above. Finally, concluding remarks are given.

2. Optical vortices and correlation vortices

We begin by considering a monochromatic wavefield $U(\mathbf{r}, t)$ of the form

$$U(\mathbf{r}, t) = U(\mathbf{r}, \omega)e^{-i\omega t}, \quad (1)$$

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where $U(\mathbf{r}, \omega)$ is the spatial dependence of the field. The frequency ω is written as a functional argument in anticipation of the generalization to a partially coherent field.

The space-dependent part may be further factorized into an amplitude $A(\mathbf{r}, \omega)$ and phase $\phi(\mathbf{r}, \omega)$, in the form

$$U(\mathbf{r}, \omega) = A(\mathbf{r}, \omega)e^{i\phi(\mathbf{r}, \omega)}. \quad (2)$$

This factorization is everywhere unique except in regions of space where the field amplitude vanishes; in such regions, the phase is undefined, or *singular*. Though traditionally considered a mathematical curiosity, research over the past few decades has demonstrated that the zeros typically manifest as lines in three-dimensional space, around which the phase has a circulating or helical structure [1]. These optical vortices have associated with them a discrete 'topological charge', which is conserved under perturbations of the field. A familiar family of beams which possess an optical vortex in their center are the Laguerre–Gauss modes, given by [17, Chapter 4]

$$U_{nm}(\rho, \phi, z) = C_{nm} \left(\frac{\sqrt{2}\rho}{W(z)} \right)^m L_n^{(m)} \left(\frac{2\rho^2}{W^2(z)} \right) \times \exp[i(2n + m + 1)\varphi(z)] \times \exp\left(im\phi - \rho^2/W^2(z) - ik\rho^2/2F(z)\right), \quad (3)$$

where (ρ, ϕ, z) are the cylinder coordinates in space, $W(z)$ is the beam width on the transverse plane z , $L_n^{(m)}$ are the Laguerre polynomial of order m and n , $\varphi(z)$ is the longitudinal phase delay on the transverse plane z , $F(z)$ is the radius of curvature of wavefront on the transverse plane z , and C_{nm} is a normalization constant.

Several low-order Laguerre–Gauss beams are illustrated in Fig. 1. The central core of the beam possesses an amplitude zero, and the phase of the field increases by a multiple m of 2π as one progresses around the central core. We refer to m as the *order* of the vortex core. We note that a phase plot consisting of three

equiphase lines is sufficient to describe the location and topological charge of field vortices; we use four equiphase lines in all figures which follow.

When such a vortex beam is randomized, it is expected from previous results that the vortex properties survive, in some sense, in the spatial coherence of the wavefield [11,15]. Traditionally, the coherence properties of a statistically stationary scalar wavefield are characterized by the mutual coherence function,

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau) = \langle U^*(\mathbf{r}_1, t)U(\mathbf{r}_2, t + \tau) \rangle, \quad (4)$$

where the angle brackets indicate a time average or, equivalently, an ensemble average. It is more convenient for our purposes, however, to work with the cross-spectral density of the wavefield [18, Section 4.3], which is the temporal Fourier transform of the mutual coherence function with respect to the time delay τ ,

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau)e^{i\omega\tau} d\tau. \quad (5)$$

The cross-spectral density characterizes the intensity and spatial coherence of the field at frequency ω , and contains the same information at the mutual coherence function.

It has been shown that the cross-spectral density of an arbitrary partially coherent wavefield at frequency ω may always be expressed as the average of an ensemble of monochromatic realizations of the field [19], i.e.

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle U^*(\mathbf{r}_1, \omega)U(\mathbf{r}_2, \omega) \rangle, \quad (6)$$

where $U(\mathbf{r}, \omega)$ is a monochromatic realization of the partially coherent field and the subscript ω denotes averaging with respect to this special ensemble. The advantage of this representation is that it allows one to construct models of partially coherent fields in the frequency domain directly without resorting first to finding the more complicated mutual coherence function. We will use this

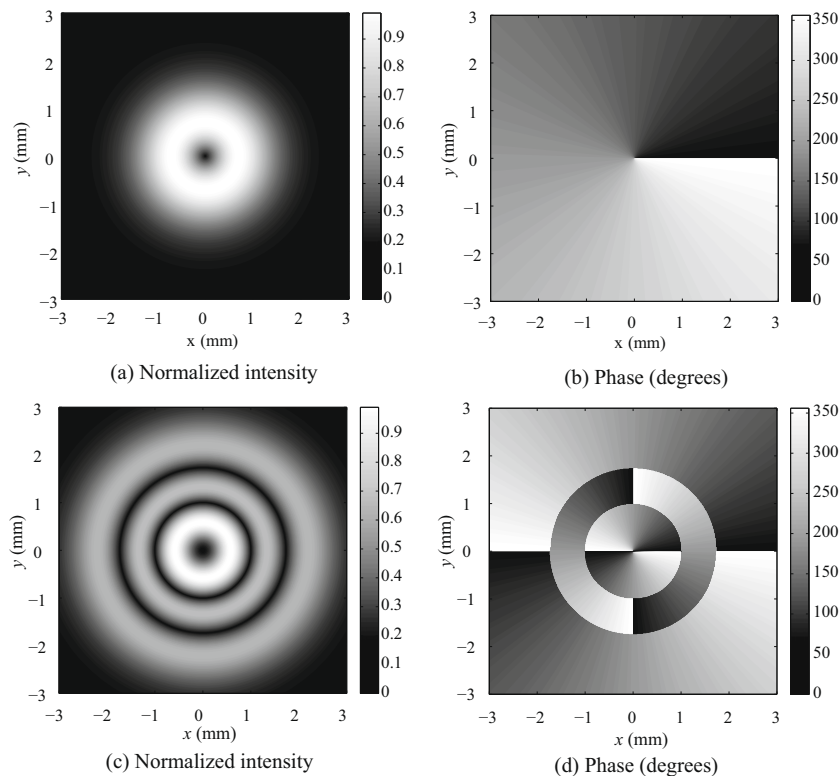


Fig. 1. Illustration of the transverse profile and phase structure of a pair of Laguerre–Gauss beams: $m = 1, n = 0$: (a) intensity, (b) phase, $m = 2, n = 2$, (c) intensity, and (d) phase.

below to construct a model of a quasi-monochromatic, partially coherent field with a ‘built-in’ vortex behavior.

To study the relationship between optical vortices and correlation vortices, we use a ‘beam wander’ model first developed in Ref. [15]. We consider an ensemble of monochromatic fields $U(\boldsymbol{\rho} - \boldsymbol{\rho}_0, \omega)$ in the source plane $z = 0$, with $\mathbf{r} \equiv (\rho, z)$. Each member of the ensemble is centered on a different transverse location $\boldsymbol{\rho}_0$, and this vector is described by a probability distribution $f(\boldsymbol{\rho}_0)$. We may use this ensemble to create a partially coherent field whose coherence properties are dictated by the behavior of the function $f(\boldsymbol{\rho}_0)$. The cross-spectral density is then given by

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \int f(\boldsymbol{\rho}_0) U^*(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_0) U(\boldsymbol{\rho}_2 - \boldsymbol{\rho}_0) d^2 \boldsymbol{\rho}_0, \quad (7)$$

where the integral, representing the ensemble average, is taken in the source plane of the field. This construction may be taken as a simple model of beam wander in atmospheric turbulence [17]; in the Appendix, we show that it can also represent a partially coherent field being propagated through a vortex mask. In Ref. [15], this construction was used to study the behavior of a partially coherent field created with an $n = 0, m = 1$ vortex beam, as the degree of coherence is changed. In Ref. [16], it was demonstrated that the construction produces results consistent with the general behavior of vortices in partially coherent fields.

For the remainder of this paper, we will restrict ourselves to the case

$$f(\boldsymbol{\rho}_0) = \frac{1}{\sqrt{\pi} \delta} e^{-\rho_0^2 / \delta^2}, \quad (8)$$

where the quantity δ represents the average beam wander, and also restrict ourselves to a single frequency ω (further expression of the frequency ω will be suppressed). In the limit $\delta \rightarrow 0$, the beam does not wander and is therefore fully coherent. Increasing δ represents a decrease in spatial coherence. We now apply this model to the study of topological reactions in partially coherent vortex beams.

3. Second-order optical vortex into two first-order correlation vortices

In Ref. [20], the propagation of an optical vortex beam through atmospheric turbulence was studied. On propagation, a high-order optical vortex breaks down into a collection of first-order optical vortices, though the total topological charge is conserved. If the field is averaged over times long compared to the fluctuations of the medium, one readily finds that the spatial coherence of a beam decreases on propagation [17]. Consequently, the optical vortices are replaced with correlation vortices. It is natural to ask whether an m th-order optical vortex is replaced with an m th-order correlation vortex when the spatial coherence of the field decreases, or

whether a number of lower-order correlation vortices are produced.

We start with a Laguerre–Gauss beam of order $m = 2$ and $n = 0$, which possesses a second-order optical vortex in its center. Its transverse profile in the source plane is

$$U_{02}(\rho, \phi) = C_{02} \left(\frac{\sqrt{2} \rho}{w_0} \right)^2 \exp(2i\phi) \exp(-\rho^2 / w_0^2), \quad (9)$$

where w_0 is the beam width in the source plane, and $\boldsymbol{\rho} \equiv (\rho, \phi)$. With Eqs. (7)–(9), we can study the topological structure of the corresponding correlation vortex.

Two examples of these correlation vortices are shown in Fig. 2. In both cases we see four first-order correlation vortices. Two of these (the left-most in both figures) are first-order correlation vortices of the same sign which result from the break-up of the second-order optical vortex. Evidently no choice of reference point or state of coherence (other than complete coherence) will maintain a pure second-order vortex, i.e. the second-order optical vortex is unstable under changes in the state of coherence. The other two correlation vortices in each figure (the right-most) are of opposite sign from the first pair and are singularities which are ‘hidden’ at the point at infinity when the field is fully coherent. A similar behavior is seen for a first-order optical vortex, which has been shown to have an opposite ‘hidden’ at infinity [15].

Fig. 3 illustrates the dependence of correlation vortices structure on parameter δ . It can be readily seen that, as $\delta \rightarrow 0$, i.e. as the field becomes fully coherent, the left-most correlation vortices merge and become the second-order optical vortex. It can also be seen that the right-most correlation vortices recede to the point at infinity.

As noted previously, δ characterizes the spatial coherence of the source, and in the context of our simple ‘beam wander’ model is also a rough measure of the strength of turbulence fluctuations. Fig. 3 suggests that the separation of the first-order correlation vortices is directly related to the size of δ . This raises the possibility that this ‘splitting’ could be used as a simple measure of either the spatial coherence properties of a partially coherent field or, indirectly, the strength of a turbulent medium which the vortex beam has passed through. In Fig. 4 the separation of the two vortices (the left-most in Fig. 3a) is plotted as a function of δ . It can be seen that the separation increases monotonically with δ , eventually saturating at a finite value.

This example shows that a second-order optical vortex is unstable under conversion into a correlation vortex, and breaks into a pair of first-order vortices. This result, combined with the well-known general observation that an optical vortex of order greater than 1 is unstable under perturbations of the field, suggests that an m th-order optical vortex will break into a collection of m first-order vortices of equivalent topological charge.

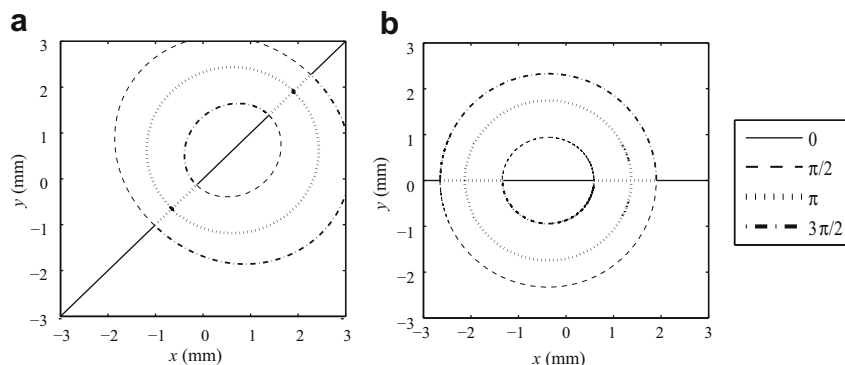


Fig. 2. Equal phase contours of the cross-spectral density, with the reference point at: (a) (0.5 mm, 0.5 mm), and (b) (–0.3 mm, 0). In both plots, $w_0 = 1$ mm and $\delta = 1$ mm.

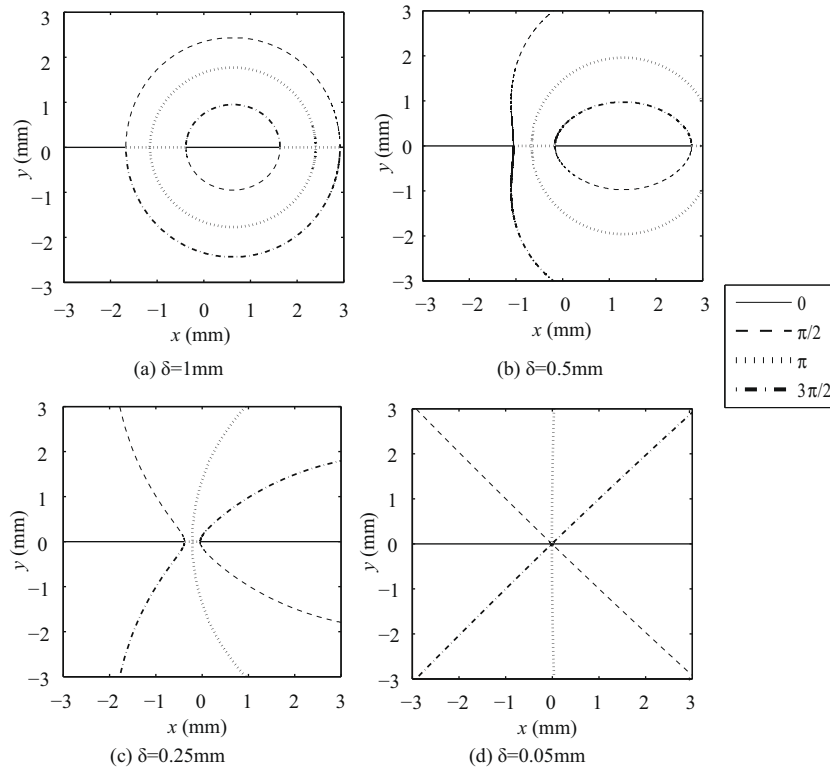


Fig. 3. Illustration of the relationship between a second-order optical vortex and a pair of first-order correlation vortices. The reference point is at (0.5 mm, 0) and $w_0 = 1$ mm.

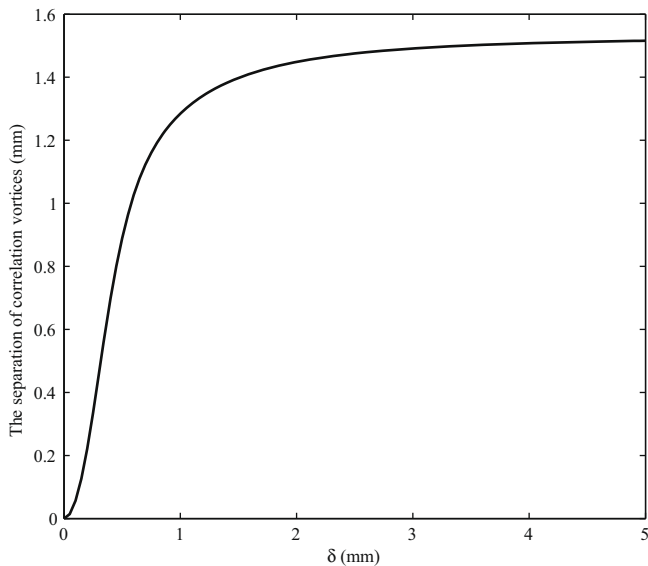


Fig. 4. Illustration of the dependence of the correlation vortices separation on the average beam wander parameter δ . The reference point is at (0.5 mm, 0) and $w_0 = 1$ mm.

4. Creation/annihilation of correlation vortices

Annihilation of two singularities with opposite topological charge is another typical topological reaction. For example, a creation-annihilation process of phase singularities in the focal region of a lens happens when the field illuminating the lens is gradually changed from Gaussian to uniform illumination [21].

Fig. 5 is a simple illustration of the annihilation of optical vortices. The wavefield in the source plane is defined as

$$U(\rho, \phi) = \frac{U_0}{w_0} \left(\rho^2 a + \frac{1}{2} \rho e^{i\phi} - \frac{1}{2} \rho e^{-i\phi} + ia - 1 \right) \exp(-\rho^2/w_0^2), \quad (10)$$

where quantity a is a dimensionless real-valued parameter characterizing the behavior of the optical vortices. From Eq. (10) and Fig. 5, it can be seen that the field possesses two optical vortices of opposite charge for $|a| \leq 1$, and when $|a| > 1$, they annihilate. It is to be noted that this annihilation event is not the most general, or generic, annihilation event possible. It is, however, physically realizable, as the appropriately-weighted coherent superposition of Laguerre–Gauss beams of order $(n, m) = (0, 0)$, $(0, 1)$, $(0, -1)$, and $(1, 0)$, evaluated in a common waist plane. We have found that the general behavior described here is stable under variations of the relative amplitudes of the Laguerre–Gauss modes.

It is of interest to ask whether the corresponding correlation vortices of a partially coherent field annihilate in a similar manner. By substituting from Eqs. (8) and (10) into Eq. (7), the topological behavior of the correlation vortices as a function of parameter a can be studied.

As shown in Fig. 6, the correlation vortices undergo a non-trivial creation-annihilation process as a is increased. In Fig. 6a, when $a = 0$, there are four singularities and two saddles (crossed equiphase lines of same value) along the x -axis. The topological charges for the left two singularities (A and B) and the right two singularities (C and D) are -1 and $+1$, respectively. Two of the vortices, as in the previous section, have come from the point at infinity. When we increase a , singularities A and D move toward each other as well as singularities B and C though at different speed (Fig. 6b and c). During this process, two saddles (E and F in Fig. 6a) have disappeared. When $a = 1$, only singularities B and C remain while singularities A and D in Fig. 6c have annihilated (Fig. 6d).

From Fig. 6d, it shows that correlation vortices still exist when the corresponding optical vortices have already annihilated. It can be anticipated that the remaining two correlation vortices will

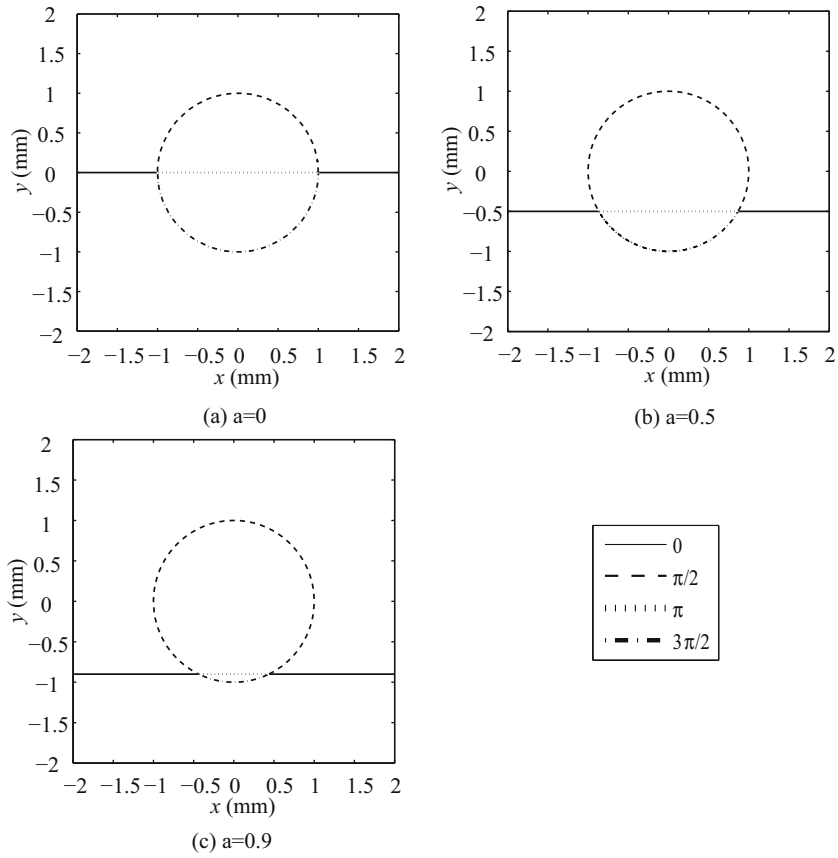


Fig. 5. Illustration of the annihilation of optical vortices, $w_0 = 1$ mm.

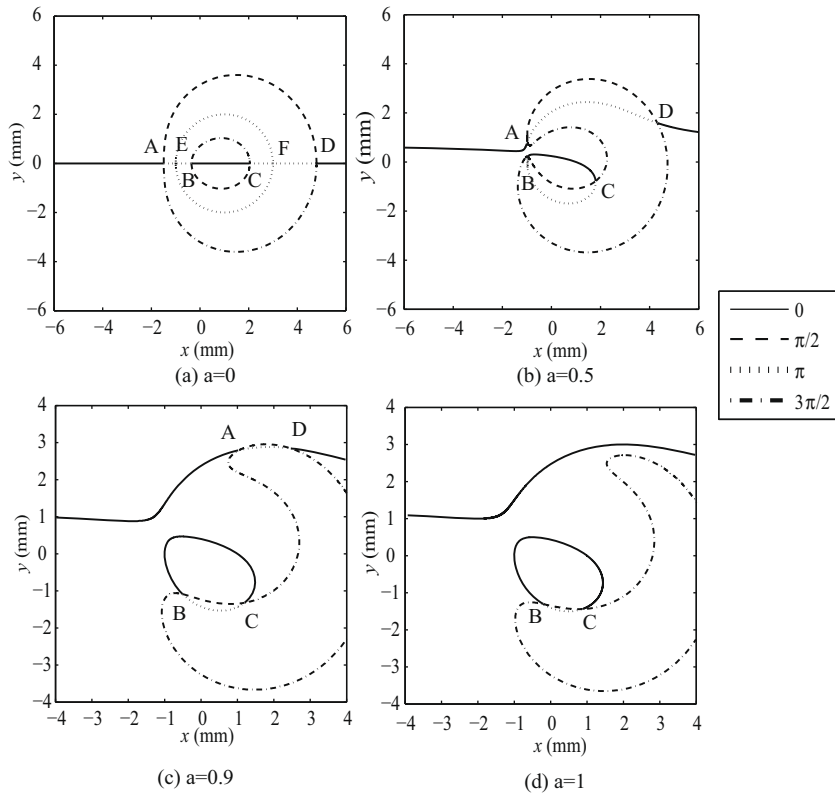


Fig. 6. Illustration of the annihilation of correlation vortices. The reference point is at (1 mm, 0), $\delta = 1$ mm and $w_0 = 1$ mm.

annihilate finally for some critical value of a which is larger than 1. It means that correlation vortices annihilate later than their counterparts, optical vortices. For the example in Fig. 6, this value is between 1.06 and 1.07.

This phenomenon can be roughly explained in terms of the fluctuations of the field. Reconsidering our discussion of vortex beams in turbulence at the beginning of Section 3, a movie of the temporal evolution of a partially coherent field would show the optical vortices randomly moving about the source plane. When the initial field possesses a pair of vortices of opposite sign, some of these wanderings will result in an annihilation of the vortex pair; also, other fluctuations may result in the creation of vortices in the field. Depending on the phase structure of the field and its statistical properties, these fluctuations may make it more likely (correlation vortices annihilate earlier than optical ones) or less likely (correlation vortices annihilate later than optical ones) for the original vortex pair to exist.

It is well-known [22,23] that phase saddles are associated with the creation/annihilation of ordinary optical vortices. One would certainly expect correlation saddles to be generally involved with correlation vortices: this follows immediately from the observation that, with observation point \mathbf{r}_1 fixed, the cross-spectral density behaves as a monochromatic field. In Fig. 6a and b one can see saddles present in the field, and associated with the correlation vortex interactions.

5. Propagation of a correlation vortex

In the previous two sections, the topological reactions of correlation vortices are restricted to the source plane $z = 0$. Now we consider the propagation of a correlation vortex beam, and the behavior of correlation vortices on propagation. At first glance, this would seem to be a trivial problem: from the definition (6), with \mathbf{r}_1 fixed, the cross-spectral density acts exactly as a monochromatic

field with respect to the variable \mathbf{r}_2 . The correlation vortices should satisfy all the propagation characteristics of an optical vortex.

Rewriting $\mathbf{r}_1 = (\rho_1, z_1)$, $\mathbf{r}_2 = (\rho_2, z_2)$, however, it is clear that most experiments will involve measurements of the correlation properties in a single transverse plane $z_1 = z_2$, and only the transverse coordinate ρ_1 will be fixed. On propagation, then, z_1 takes the role of a continuously varying parameter of the system, and the behavior of the correlation singularities under such circumstances is unclear, though some work has been done to elucidate the behavior of the cross-correlation function of the field on propagation [24].

The beam wander model treats a partially coherent field as an incoherent collection of monochromatic wavefields, each of which propagates independently. The statistical properties of the total field at any propagation distance can therefore be found by propagating these modes of the field individually and adding them with the appropriate weights, represented by $f(\rho_0)$. This is done by substituting Eqs. (3) and (8) into Eq. (7). We can study the vortex structure of correlation vortices at any transverse plane along the propagation direction. For simplicity, the simplest vortex Laguerre–Gauss beam of order $m = 1$ and $n = 0$ is used. Fig. 7 shows an example of the propagation traces of correlation vortices and their corresponding projections in the transverse plane, with reference point at (0.7 mm, 0) and $w_0 = 1$ mm. For this case, the two singularities first approach each other but quickly separate. Finally, the first correlation vortex arrives at a fixed position while the second one moves to infinity.

The results are quantitatively different for a different choice of reference point. Fig. 8 shows the case when the reference point is at $(x, y) = (0.3 \text{ mm}, 0)$. Two correlation vortices almost immediately annihilate and then are recreated. Therefore there is a certain distance along propagation for which no vortices exist. This annihilation/recreation happens for a range of observation point positions, as illustrated in Fig. 9. When the observation point is taken

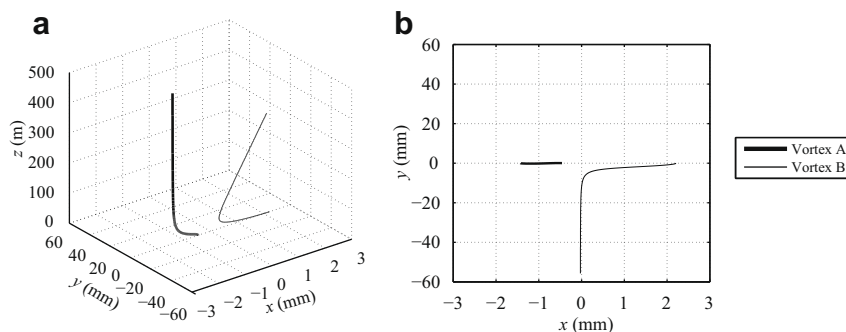


Fig. 7. Propagation trace of correlation vortices and the corresponding projection on the xy -plane. The reference point is at (0.7 mm, 0), $\lambda = 500$ nm, $\delta = 1$ mm and $w_0 = 1$ mm.

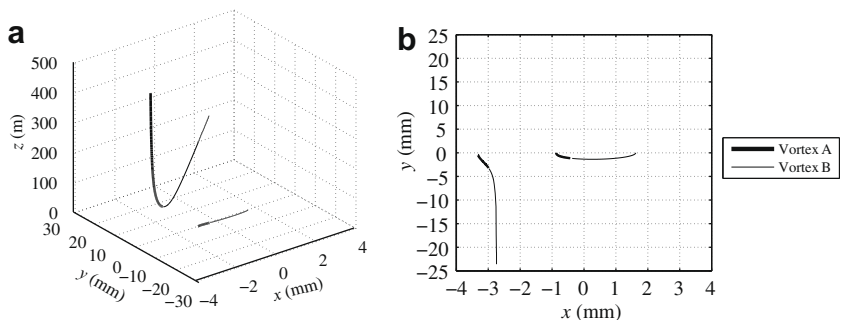


Fig. 8. Propagation trace (left) of correlation vortices and the corresponding projection on the xy -plane. The reference point is at (0.3 mm, 0), $\lambda = 500$ nm, $\delta = 1$ mm and $w_0 = 1$ mm.

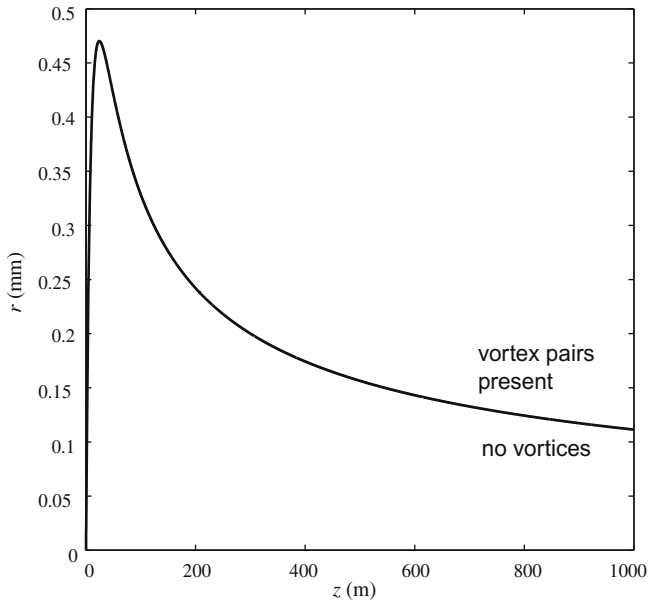


Fig. 9. Presence/absence of correlation vortices as a function of propagation distance and observation point. For this example, $\delta = 1$ mm, $\lambda = 500$ nm and $w_0 = 1$ mm.

to be greater than roughly 0.47 mm from the origin, however, we find that the vortices never annihilate.

As noted, this non-conservation of topological features on propagation is not surprising: field propagation for the configuration considered is coupled with the variation of an effective system parameter, namely z_1 .

6. Conclusions

We have studied the nature of several topological reactions of correlation vortices. In particular, the break-up of a second-order optical vortex into first-order correlation vortices, creation/annihilation of correlation vortices and the behavior of correlation vortices on propagation have been investigated. These results clearly demonstrate the relationship between optical vortices and correlation vortices. Curiously, however, there is not necessarily a direct correspondence between the behavior of the correlation vortices and the corresponding optical vortices: on propagation, correlation vortices may disappear entirely for a finite propagation distance, and in the creation/annihilation example the correlation vortices disappear under different conditions than the optical vortices.

In all three cases, however, there is a direct connection between the behavior of the correlation vortices and the coherence properties of the optical field. In the second-order case and the creation/annihilation case, the separation of the correlation vortices is directly related to the spatial coherence of the field. This suggests that correlation vortices could in principle be used to measure the statistical properties of a field or of a medium. To measure the statistical properties of a medium, a coherent vortex field could be passed through the random medium and the location of the correlation vortices can then be related to the medium's effect on the field's spatial coherence. To measure the statistical properties of a partially coherent field, the partially coherent field could be transmitted through a vortex mask (as described in the Appendix), and the position of the resulting correlation vortices is a measure of the field's spatial coherence. Such techniques may serve as a simpler alternative to traditional interferometric methods of measuring spatial coherence.

In this paper we have tried to illustrate some of the non-trivial topological reactions which can occur with correlation vortices. It is worth noting, however, that we have not explicitly proven that these behaviors are generic, i.e. that these reactions are the 'typical' topological reactions occurring in partially coherent wavefields. Questions of generic behavior in partially coherent wavefields have not, to our knowledge, been resolved, mainly due to the abundance of parameters in the system. In an annihilation event, for instance, the field behavior depends not only on the parameter a , but also on the position of the observation point \mathbf{r}_1 and the correlation parameter δ .

It is expected, though, that these results are illustrative of the general behavior for two reasons. First, it was shown in Ref. [16] that the 'beam wander' model produces first-order correlation vortex behavior in agreement with the general behavior of such vortices in linear optical systems. Second, Appendix A illustrates that the 'beam wander' model produces fields identical to those produced by a quite different system, a partially coherent beam passing through a phase mask.

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Appendix A. Beam wander vs. vortex mask

In this Appendix we demonstrate that we can arrive at Eq. (7) not only by assuming a 'beam wander' model for a partially coherent field, but also by transmitting a partially coherent field through a vortex mask and then focusing it. The geometry is illustrated in Fig. 10. We consider a Gaussian Schell-model field [18, Section 5.2.2] in the plane $z = 0$ of the form

$$W_0(\rho_1, \rho_2) = A^*(\rho_1)A(\rho_2)\mu(\rho_2 - \rho_1), \tag{A1}$$

where $A(\rho)$ is the average field amplitude and $\mu(\rho)$ is the spectral degree of coherence. This field is incident upon a vortex phase mask of order 1, which imparts a phase $\exp[i\phi]$ onto the field, so that immediately beyond the mask the cross-spectral density takes on the form

$$W(\rho_1, \rho_2) = W_0(\rho_1, \rho_2)e^{-i\phi_1} e^{i\phi_2} = A^*(\rho_1)A(\rho_2)\mu(\rho_2 - \rho_1)e^{-i\phi_1} e^{i\phi_2}. \tag{A2}$$

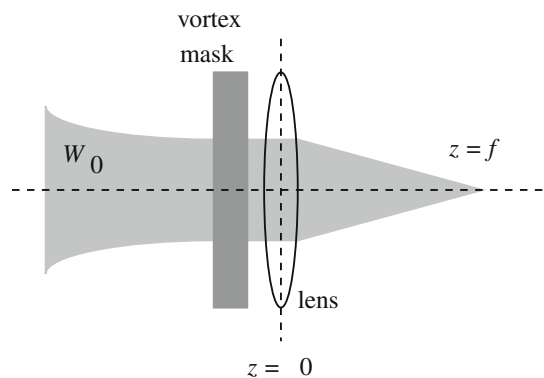


Fig. 10. Illustration of a focusing system which can produce correlation vortices in a partially coherent field.

We may define a new field amplitude $B(\rho)$ as

$$B(\rho) \equiv A(\rho)e^{i\phi}. \quad (\text{A3})$$

This field is then immediately focused by a thin lens of focal length f . As is well-known, the field in the focal plane is simply related to the Fourier transform of the field at the lens input [25, Section 5.2.1]. For a partially coherent field, this becomes a Fourier transform with respect to both ρ_1' and ρ_2' , i.e.

$$W(\rho_1, \rho_2) \propto \int \int B^*(\rho_1')B(\rho_2')\mu(\rho_2' - \rho_1')e^{i\alpha(\rho_1'\rho_1 - \rho_2'\rho_2)}d^2\rho_1'd^2\rho_2', \quad (\text{A4})$$

where $\alpha \equiv k/f$. This formula may be written in a more suggestive form by using the following Fourier expansions:

$$\mu(\rho) = \int \tilde{\mu}(\mathbf{K})e^{i\mathbf{K}\cdot\rho}d^2K, \quad (\text{A5})$$

$$B(\rho_j) = \int \tilde{B}(\mathbf{K}_j)e^{i\mathbf{K}_j\cdot\rho_j}d^2K_j, \quad (\text{A6})$$

where $j = (1, 2)$. On substitution, and applying some standard Fourier transform techniques, one finds that

$$W(\rho_1, \rho_2) \propto \int \tilde{\mu}(\mathbf{K})\tilde{B}^*(\alpha\rho_1 - \mathbf{K})\tilde{B}(\alpha\rho_2 - \mathbf{K})d^2K. \quad (\text{A7})$$

This formula can be directly related to Eq. (7),

$$W(\rho_1, \rho_2, \omega) = \int f(\rho_0)U^*(\rho_1 - \rho_0)U(\rho_2 - \rho_0)d^2\rho_0. \quad (\text{A8})$$

The function $\tilde{\mu}$ plays the role of $f(\rho_0)$, while \tilde{B} plays the role of U . The function $B(\rho)$ is assumed to possess a helical phase structure, which results in its Fourier transform \tilde{B} possessing the same helical phase.

We readily see that the focusing of a partially coherent field through a vortex phase mask results in a field with the same functional form as given by the ‘beam wander’ model.

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