Phase singularities, correlation singularities, and conditions for complete destructive interference

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A previously derived condition for the complete destructive interference of partially coherent light emerging from a trio of pinholes in an opaque screen is generalized to the case when the coherence properties of the field are asymmetric. It is shown by example that the interference condition is necessary, but not sufficient, and that the existence of complete destructive interference also depends on the intensity of light emerging from the pinholes and the system geometry; more general conditions for such interference are derived. The phase of the wave field exhibits both phase singularities and correlation singularities, and a number of nonintuitive situations in which complete destructive interference occurs are described and explained.

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1. INTRODUCTION

It is now well appreciated that coherent wave fields possess a nontrivial and singular phase structure in the neighborhood of zeros of intensity, referred to as phase singularities. The zeros typically manifest as lines in three-dimensional space, and the phase has a circulating or helical structure around this line, referred to as an optical vortex. The study of these singularities, and related singularities of power flow and polarization, are now their own subfield of optics, referred to as singular optics [1,2].

The archetypical example of an optical vortex is a Laguerre–Gauss beam ([3], Subsection 18.3.6) of order $n = 0$, $m = 1$, which possesses a zero of intensity along its central axis and a phase that increases continuously as one progresses counterclockwise about this axis—this is the "vortex." The intensity and phase in the waist plane of such a vortex beam is illustrated in Fig. 1. It is to be noted that all contours of equal phase (colors) meet at the central zero, which is therefore a singularity of phase.

In recent years the field of singular optics has been extended to the study of singularities of partially coherent fields. Though it has been shown [4] that zeros of intensity are not typical, or generic, features of a partially coherent wave field; zeros of two-point correlation functions, known as correlation singularities, are common and possess many interesting topological properties. Much research has been done on the behavior of correlation singularities, both theoretical [5,6,7] and experimental [8,9].

When one observation point is fixed, these correlation singularities are mathematically similar to optical vortices, and they are therefore commonly referred to as correlation vortices. It has been demonstrated that there is a strong physical relationship between correlation vortices and their fully coherent counterparts [10], though that relationship is still not completely understood.

Though intensity zeros of a partially coherent field are not generic, it has been demonstrated that they can exist under certain conditions. For instance, a class of partially coherent vortex beams has been theoretically proposed and experimentally demonstrated [11]. Also, it has been shown that zeros can appear in a multiple pinhole interferometer of Young's type when the number of pinholes is greater than 2, even when the degree of coherence between any pair of pinholes is less than unity [12]. These theoretical predictions have been verified both optically [13] and acoustically [14]. The existence of such "pseudocoherent" fields provides an excellent theoretical laboratory to investigate the relationship between correlation vortices and optical vortices, their interactions, and even the meaning of phase for partially coherent fields [15].

A few years ago, it was demonstrated [16] that complete destructive interference in an N-pinhole system can be achieved when the pinholes are symmetrically arranged and the mutual degree of coherence μ_0 between the pinholes satisfies the expression

$$\mathrm{Re}(\mu_0) = -\frac{1}{N-1}. \tag{1}$$

With the use of this condition, the simultaneous existence of phase and correlation singularities was demonstrated, and interactions between different types of singularities was observed.

However, Eq. (1) is only a special case of the more general condition described in [12], and the general condition includes asymmetric values of the degree of coherence between pinholes. In this paper we investigate the behavior of a three-pinhole interferometer with more general correlation properties than previously considered. By varying the source correlation structure, it is shown that the condition on the degree of coherence described in [12] is necessary, but not sufficient, for complete destructive interference and that this interference also depends on the intensity of light emerging from the pinholes and their geometry. The phase of the wave field exhibits both correlation singularities and singularities

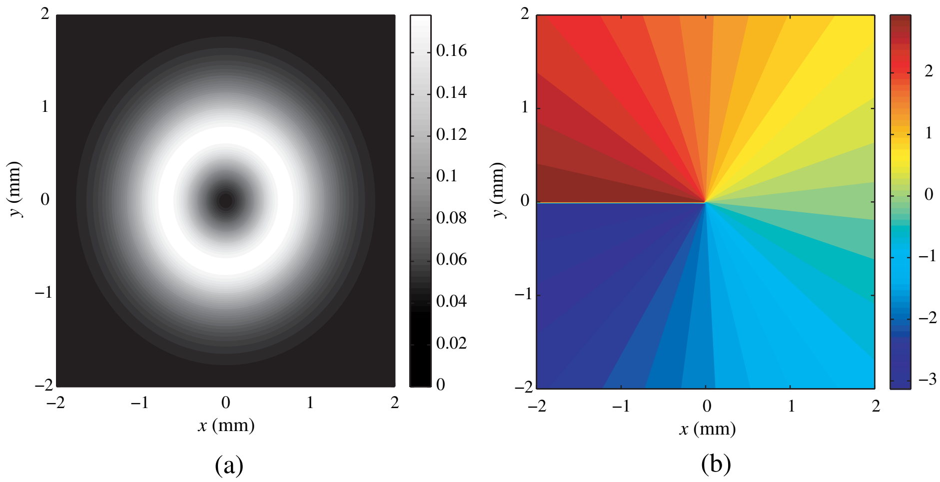


Fig. 1. (Color online) (a) Intensity pattern of an LG_{01} mode in the waist plane. (b) Phase contour of the corresponding wave field in the waist plane. Here the width of the beam is $w_0 = 1$ mm.

associated with zeros of intensity, to be called *field singularities*. A number of nonintuitive situations in which complete destructive interference occurs are described and explained.

2. COHERENCE AND MULTIPLE PINHOLE INTERFEROMETERS

We consider the behavior of partially coherent light in the space–frequency domain; the measurable second-order properties of the wave field at frequency ω are characterized by the cross-spectral density, defined as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) \equiv \langle U^*(\mathbf{r}_1, \omega) U(\mathbf{r}_2, \omega) \rangle, \tag{2}$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the two observation points and $U(\mathbf{r}, \omega)$ is the optical field at position $\mathbf{r}$ and frequency ω. The asterisk denotes complex conjugation, and the brackets indicate averaging over an ensemble of monochromatic realizations of the wave field ([17], Subsection 4.1). Polarization properties are neglected for this study, and only scalar waves are considered.

The cross-spectral density may always be written in the factorized form,

$$\begin{aligned} W(\mathbf{r}_1, \mathbf{r}_2, \omega) &= \sqrt{I(\mathbf{r}_1, \omega)}\sqrt{I(\mathbf{r}_2, \omega)}\mu(\mathbf{r}_1, \mathbf{r}_2, \omega), \\ &= A(\mathbf{r}_1, \omega) A(\mathbf{r}_2, \omega) \mu(\mathbf{r}_1, \mathbf{r}_2, \omega). \end{aligned} \tag{3}$$

In this expression, $I(\mathbf{r}, \omega)$ is the average spectral intensity of the field at position $\mathbf{r}$ and frequency ω, related to the cross-spectral density by

$$I(\mathbf{r}, \omega) = W(\mathbf{r}, \mathbf{r}, \omega), \tag{4}$$

and $\mu(\mathbf{r}_1, \mathbf{r}_2, \omega)$ is the spectral degree of coherence of the field, given by

$$\mu(\mathbf{r}_1, \mathbf{r}_2, \omega) = \frac{W(\mathbf{r}_1, \mathbf{r}_2, \omega)}{\sqrt{I(\mathbf{r}_1, \omega)}\sqrt{I(\mathbf{r}_2, \omega)}}. \tag{5}$$

The quantity $A(\mathbf{r}, \omega) \equiv \sqrt{I(\mathbf{r}, \omega)}$ will be referred to as the average spectral amplitude of the field.

The spectral degree of coherence is a measure of the strength of spatial correlations between the fields at positions $\mathbf{r}_1$ and $\mathbf{r}_2$ and can be shown to be constrained to values $0 \leq |\mu| \leq 1$ ([18], Subsection 2.4.4), 0 representing incoherence and 1 representing complete coherence. It may be also shown to be Hermitian and nonnegative definite, both conditions which follow from the definitions of $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$, Eqs. (2) and (3).

We will focus our attention on the properties of light emanating from an interferometer consisting of three pinholes in an opaque screen, with the pinholes arranged as an equilateral triangle. The geometry of such an interferometer is shown in Fig. 2. The position of the nth pinhole in the screen is labeled Q_n; the position on an observation screen is labeled P. The field that arrives at P from Q_n is of the form

$$U_n(P, \omega) = -i\frac{ka^2}{2\pi} U_0(Q_n, \omega)\frac{e^{ikR_n}}{R_n}, \tag{6}$$

where $k = \omega/c$ is the wavenumber of light, c being the speed of light, a is the radius of an individual pinhole, R_n is the distance from Q_n to P, and $U_0(Q_n, \omega)$ is the field emanating from the nth pinhole. An inclination factor was neglected, assuming only paraxial propagation of light.

Using Eq. (2), the total intensity at the observation point P may be written as

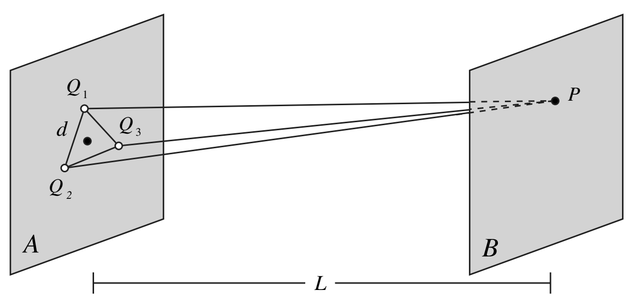


Fig. 2. Illustration of the three-pinhole geometry under consideration. The distance between the pinholes is taken to be $d = 1$ mm.

$$I(P,\omega) = \left\langle \sum_{n=1}^{N} U_n^*(P,\omega) \sum_{m=1}^{N} U_m(P,\omega) \right\rangle. \tag{7}$$

Introducing the cross-spectral density of the field in the pinhole plane,

$$W_0(Q_1, Q_2, \omega) = \langle U_0^*(Q_1,\omega) U_0(Q_2,\omega) \rangle, \tag{8}$$

we may write the spectral intensity at P in a straightforward matrix form. For three pinholes ($N = 3$), this may be expressed as

$$\left(\frac{2\pi}{ka^2}\right)^2 I(P,\omega) = \mathbf{x}^\dagger \mathbf{M} \mathbf{x}, \tag{9}$$

where

$$\mathbf{x} = \begin{bmatrix} A_0(Q_1)e^{ikR_1}/R_1 \\ A_0(Q_2)e^{ikR_2}/R_2 \\ A_0(Q_3)e^{ikR_3}/R_3 \end{bmatrix}, \tag{10}$$

$$\mathbf{M} = \begin{bmatrix} 1 & \mu_{12} & \mu_{13} \\ \mu_{12}^* & 1 & \mu_{23} \\ \mu_{13}^* & \mu_{23}^* & 1 \end{bmatrix}, \tag{11}$$

with $\mu_{ij} = \mu_0(Q_i, Q_j, \omega)$. The matrix $\mathbf{M}$ depends on the spectral degree of coherence of the illuminating field at the pinholes but not the point of observation P nor the amplitude A_0 of the illuminating field. A necessary condition for the intensity of the illuminating wave field to be zero at some point on the observation screen, $I(P) = 0$, is for the determinant of the matrix $\mathbf{M}$ to vanish,

$$1 - |\mu_{12}|^2 - |\mu_{23}|^2 - |\mu_{13}|^2 + \mu_{12}\mu_{23}\mu_{13}^* + \mu_{12}^*\mu_{23}^*\mu_{13} = 0. \tag{12}$$

The condition that $\mathrm{Det}(\mathbf{M}) = 0$ is equivalent to having one or more eigenvalues of the matrix $\mathbf{M}$ be zero. This is only a necessary condition, however, and for the field to vanish at point P, the vector $\mathbf{x}(P)$ must be equal to one of the eigenvectors corresponding to a zero eigenvalue, i.e.,

$$\mathbf{M}\mathbf{x}(P) = 0. \tag{13}$$

Because of the presence of the various R_n in the complex phase factors in $\mathbf{x}$, it is not possible to determine straightforward necessary and sufficient conditions for complete destructive interference. However, we may make a number of physically reasonable assumptions that lead to a simple set of conditions. For an observation point sufficiently far from the pinhole screen, $R_1 \approx R_2 \approx R_3$, and the denominators of the components of $\mathbf{x}$ may all be considered approximately equal. Also, due to the shortness of the wavelength of visible light, the phase terms will vary rapidly and semi-independently with a change in P, and one expects that there will be multiple points on the observation screen at which the phases are equal. We therefore expect to find one or more zeros of intensity when the following equation is satisfied:

$$\mathbf{M}\mathbf{y} = \mathbf{0}, \tag{14}$$

where

$$\mathbf{y} = \begin{bmatrix} A_0(Q_1) \\ A_0(Q_2) \\ A_0(Q_3) \end{bmatrix}. \tag{15}$$

It is to be noted that, along the axis passing through the center of the pinholes, $\mathbf{x}$ is proportional to $\mathbf{y}$; we therefore automatically expect to find a zero along this central axis when Eq. (14) is satisfied. This is still not the most general condition for destructive interference but does provide a nontrivial sufficiency condition.

We briefly review the special case for which $\mu_{12} = \mu_{23} = \mu_{13} = \mu_0$, with μ_0 real. Then Eq. (12) takes on the simple cubic form

$$1 - 3\mu_0^2 + 2\mu_0^3 = 0.$$

Two of the roots of this equation are $\mu_0 = 1$, i.e., full coherence, but the third root is $\mu_0 = -1/2$, in agreement with Eq. (1). One can readily show that, for $\mu_0 = -1/2$, the appropriate eigenvector is

$$\mathbf{y} = A_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; \tag{16}$$

i.e., all of the amplitudes are equal. This case was dealt with in detail in [16].

The existence and behavior of correlation singularities may be determined by generalizing the formalism given here. The field that arrives at an observation point P_j from a pinhole Q_n is given by

$$U_n(P_j,\omega) = -i\frac{ka^2}{2\pi} U_0(Q_n,\omega) \frac{e^{ikR_{nj}}}{R_{nj}}, \tag{17}$$

where R_j is the distance from Q_n to P_j. The cross-spectral density with respect to a pair of points P_1 and P_2 is therefore

$$W(P_1, P_2, \omega) = \left\langle \sum_{n=1}^{N} U_n^*(P_1,\omega) \sum_{m=1}^{N} U_m(P_2,\omega) \right\rangle. \tag{18}$$

In matrix form, this may be written as

$$W(P_1, P_2, \omega) = \left(\frac{2\pi}{ka^2}\right)^2, \qquad I(P,\omega) = \mathbf{x}^\dagger(P_1)\mathbf{M}\mathbf{x}(P_2), \tag{19}$$

where $\mathbf{M}$ is still given by Eq. (11) and $\mathbf{x}(P_j)$ may be written as

$$\mathbf{x}(P_j) = \begin{bmatrix} A_0(Q_1)e^{ikR_{1j}}/R_{1j} \\ A_0(Q_2)e^{ikR_{2j}}/R_{2j} \\ A_0(Q_3)e^{ikR_{3j}}/R_{3j} \end{bmatrix}. \tag{20}$$

Field singularities are still present in this equation and are those points P_j for which $\mathbf{M}\mathbf{x}(P_j) = 0$. We also now have the possibility of correlation singularities, defined as pairs of points P_1 and P_2 for which the vector $\mathbf{x}(P_1)$ is orthogonal to the vector $\mathbf{M}\mathbf{x}(P_2)$. It is to be noted that correlation singularities do not require $\mathrm{Det}(\mathbf{M}) = 0$ for existence.

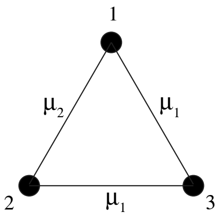


Fig. 3. Relation of the correlations between the three pinholes in terms of μ_1 and μ_2.

3. ASYMMETRIC COHERENCE PROPERTIES OF THE THREE-PINHOLE INTERFEROMETER: TWO VALUES EQUAL

We now consider what happens when the condition that all μ_{ij} are equal is relaxed. We take the pinholes to be arranged in an equilateral triangle and take two of the correlations between pinholes to be equal to μ_1, with the third equal to μ_2; this arrangement is illustrated in Fig. 3. The existence of a zero eigenvalue follows from Eq. (12), which for the special values considered becomes

$$1 - \mu_1^2 - \mu_2[\mu_2 - \mu_1^2] + \mu_1^2[\mu_2 - 1] = 0. \tag{21}$$

Given values of μ_1 and μ_2 satisfying this equation, the pinhole amplitudes required for zeros of intensity can be determined from Eq. (14).

Equation (21) is a quadratic equation for μ_2, which can be readily solved. The two solutions are

$$\mu_2 = 1, \qquad \mu_2 = 2\mu_1^2 - 1. \tag{22}$$

As noted earlier, real-valued μ_1 is itself bounded to $-1 \le \mu_1 \le 1$, which automatically constrains μ_2 to the same range.

We consider the eigenvector for the solution $\mu_2 = 1$ first. Introducing $a_2 = A_2/A_1$ and $a_3 = A_3/A_1$, we find, provided $\mu_1 \neq 1$, that $a_2 = -1$, $a_3 = 0$. The condition $a_3 = 0$ implies that no light is emerging from the third pinhole at all and that the system is a fully coherent Young's two-pinhole interferometer. Since $a_3 = 0$, the quantity μ_1 may take on any value. The condition $a_2 = -1$ is equivalent to having $a_2 = 1$ and $\mu_2 = -1$ and implies that the fields of the pinholes 1 and 2 must be anticorrelated to have a zero of intensity along the central axis.

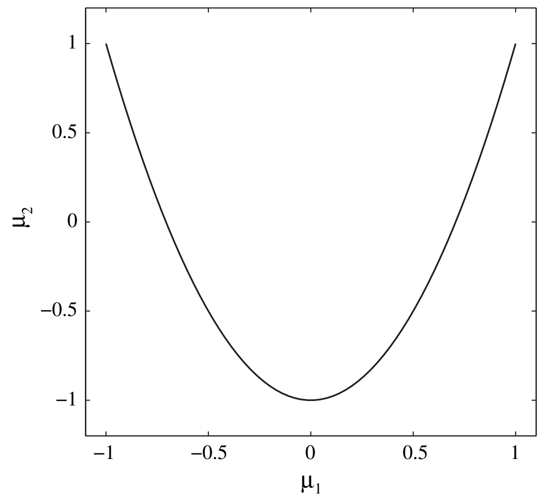


Fig. 4. Illustration of μ_2 as a function of μ_1.

However, in the special case $\mu_1 = 1$, the three equations derived from the eigenvector formula are degenerate, and A_1, A_2, and A_3 need only satisfy the condition

$$A_1 + A_2 + A_3 = 0. \tag{23}$$

This suggests that at least one of the pinholes must be out of phase with the other two to give complete destructive interference on the central axis.

The other solution for μ_2 in Eq. (22) is plotted in Fig. 4. The corresponding eigenvector is

$$\mathbf{y} = A_0 \begin{bmatrix} 1 \\ 1 \\ -2\mu_1 \end{bmatrix}. \tag{24}$$

For $\mu_1 = 1$, we get $\mu_2 = 1$, and this case reduces to a special case of that described by Eq. (23). Other cases are not so trivial, and we find that there is a range of values of $|\mu_1|$ and $|\mu_2|$ less than unity for which there is complete destructive interference on axis and nonzero intensities at the three pinholes.

A typical example is shown in Fig. 5, for $\mu_1 = -1/3$, $\mu_2 = -7/9$. Here $A_1 = 1$, $A_2 = 1$, and $A_3 = 2/3$. The intensity

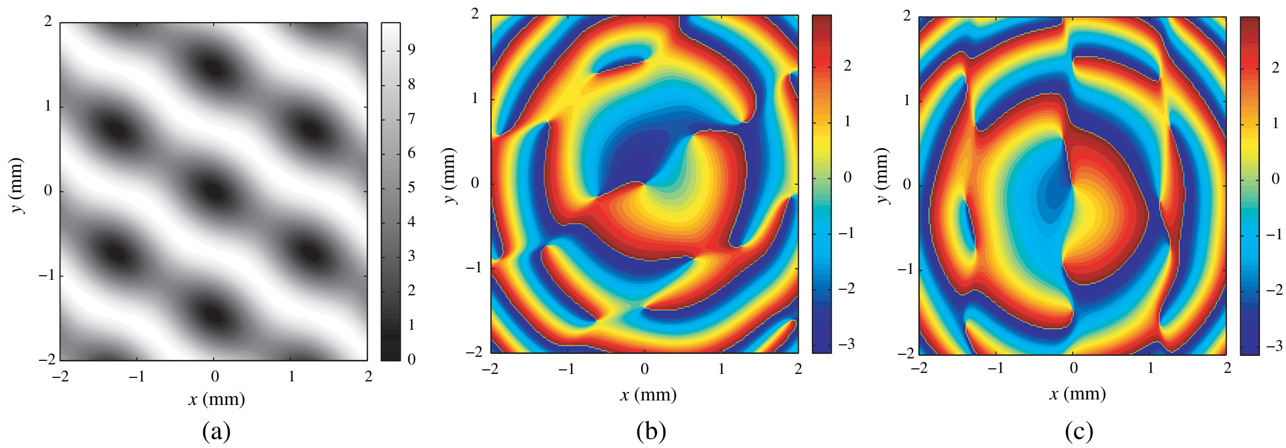


Fig. 5. (Color online) (a) Intensity pattern on the observation plane ($z = 2$ m), for $\mu_1 = -1/3$. (b) Phase contour of the cross-spectral density on the observation plane when the reference point is at $(x_1, y_1) = (1\ \text{mm}, 1\ \text{mm})$. (c) Phase contour of the cross-spectral density on the observation plane when the reference point is at $(x_1, y_1) = (1\ \text{mm}, 0.8\ \text{mm})$. For all cases, $\mu_1 = -1/3$, and $\mu_2 = -7/9$.

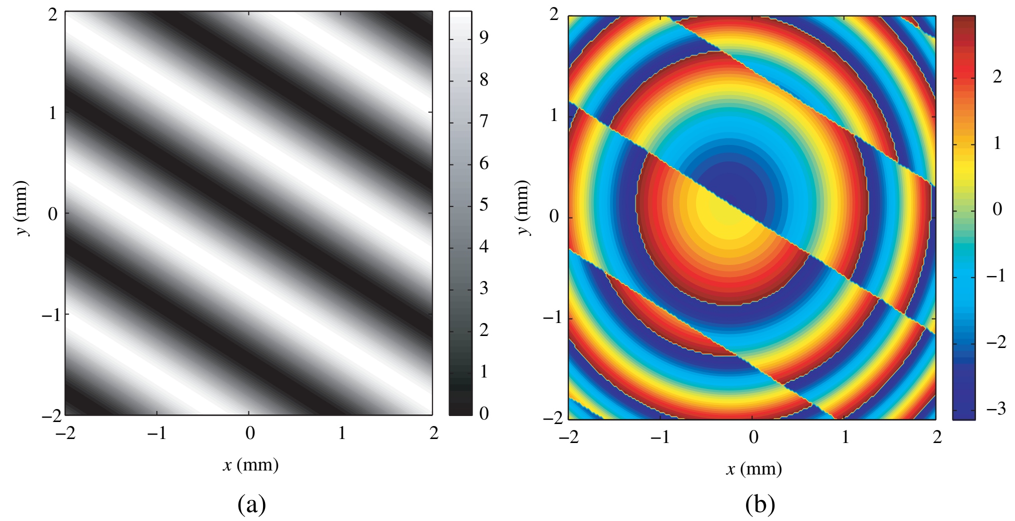


Fig. 6. (Color online) (a) Intensity pattern on the observation plane ($z = 2$ m). (b) Phase contour of the cross-spectral density on the observation plane. Here $\mu_1 = 0$, $\mu_2 = -1$, and the reference point is at $(x_1, y_1) = (1\ \mathrm{mm}, 1\ \mathrm{mm})$.

of light on the observation screen, shown in Fig. 5(a), exhibits clear dark spots both at the center of the pattern and in a hexagonal arrangement around it. Figure 5(b) shows the phase of $W(\mathbf{r}_1, \mathbf{r}_2, \omega)$ for a fixed observation point $(x_1, y_1) = (1\ \mathrm{mm}, 1\ \mathrm{mm})$; it can be seen that there are phase singularities associated with each dark spot, indicating that they are true field singularities.

Furthermore, there also exist phase singularities that are located at points where the intensity is nonzero. These are singularities of the two-point cross-spectral density function, i.e., correlation singularities. An examination of Fig. 5(b) suggests that, for every field singularity, there exists a correlation singularity of opposite handedness; this can be seen by following the deep red phase contour from any field singularity to another. This does not imply that any pair of singularities are uniquely "linked" to each other, however, as equiphase contours of different colors exist that can connect different singularities.

The existence of two types of singularities, correlation and field, raises the question of how we can uniquely distinguish them. It is to be noted that correlation singularities are defined by $W(\mathbf{r}_1, \mathbf{r}_2, \omega) = 0$, and their location depends on the choice of both $\mathbf{r}_1$ and $\mathbf{r}_2$, while field singularities are defined by $W(\mathbf{r}_2, \mathbf{r}_2, \omega) = 0$ and are independent of the choice of $\mathbf{r}_1$. In Fig. 5(c), the reference point is moved to $(x_1, y_1) = (1\ \mathrm{mm}, 0.8\ \mathrm{mm})$; it can be seen that the field singularities remain "pinned" to the zeros of intensity, while the position of the correlation singularities change with the choice of reference. This test can be used to determine the nature of an observed singularity and has been used in all the examples to follow.

A number of special values of μ_1 are worth special discussion.

A. $\mu_1 = 0$, $\mu_2 = -1$

This is a case equivalent to the $\mu_2 = 1$ solution described above, with $a_3 = 0$. The intensity of the field on the observation screen and the phase of the cross-spectral density are shown in Fig. 6, and it can be seen that we have a traditional Young's two-pinhole result. The phase of the cross-spectral

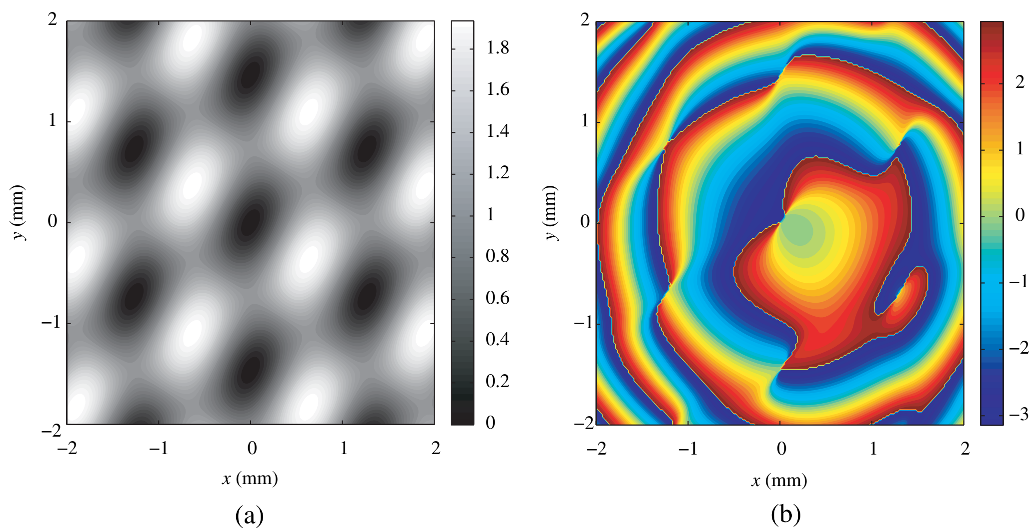


Fig. 7. (Color online) (a) Intensity pattern on the observation plane ($z = 2$ m). (b) Phase contour of the cross-spectral density on the observation plane. Here $\mu_1 = -1/\sqrt{2}$, $\mu_2 = 0$, and the reference point is at $(x_1, y_1) = (1\ \mathrm{mm}, 1\ \mathrm{mm})$.

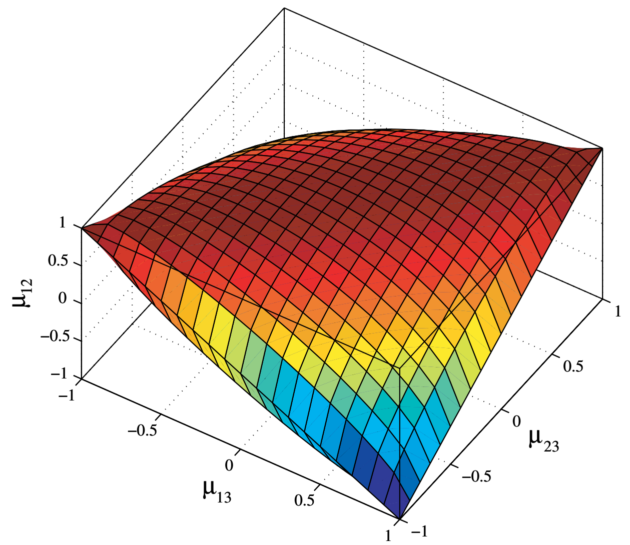


Fig. 8. (Color online) Illustration of the values of μ_{12} that give complete destructive interference, as a function of μ_{13} and μ_{23}.

density jumps by a factor of π as one crosses the diagonally oriented lines of zero intensity. These singularities, which are lines in a two-dimensional observation plane or planes in three-dimensional space, are nongeneric.

B. $\mu_1 = -1/\sqrt{2}$, $\mu_2 = 0$

This case is at first glance seemingly paradoxical: the light from pinholes 1 and 2 is completely uncorrelated, and these two pinholes are partially correlated with pinhole 3. Here we have nonzero field amplitudes at all the pinholes ($A_1 = 1$, $A_2 = 1$, and $A_3 = -\sqrt{2}$), yet we still have zeros of intensity on the observation screen. This is illustrated in Fig. 7. The field and correlation singularities are located very close to one another, as can be seen in the vicinity of the central axis.

How can we explain the presence of complete destructive interference even when $\mu_2 = 0$? Though the light from pinholes 1 and 2 is uncorrelated, it can be shown that the *sum* of fields from pinholes 1 and 2 is perfectly correlated with pinhole 3. To demonstrate this, we evaluate the cross-spectral density between the sum of fields 1 and 2 and the field 3,

$$W_{12,3} = \langle (U_1 + U_2)^* U_3 \rangle = \langle U_1^* U_3 \rangle + \langle U_2^* U_3 \rangle. \quad (25)$$

The spectral degree of coherence for this field may be written as

$$\mu_{12,3} = \frac{W_{12,3}}{\sqrt{I_{12}}\sqrt{I_3}},$$

where $I_{12} = \langle |U_1 + U_2|^2 \rangle = A_1^2 + A_2^2$ and $I_3 = A_3^2$. Furthermore, we may write

$$\langle U_1^* U_3 \rangle = A_1 A_3 \mu_1 = -1, \quad (26)$$

$$\langle U_2^* U_3 \rangle = A_2 A_3 \mu_1 = -1, \quad (27)$$

using the given values of μ_1 and A_i. Putting all of this together, we find that $\mu_{12,3} = -1$; i.e., the sum of U_1 and U_2 is fully (anti-)correlated with U_3.

4. ASYMMETRIC COHERENCE PROPERTIES OF THE THREE-PINHOLE INTERFEROMETER: ALL VALUES DIFFERENT

We may generalize the search for complete destructive interference further by returning to Eq. (12) and looking for real values of μ_{12}, μ_{23}, and μ_{13} that satisfy it. The solutions to the equation are plotted in Fig. 8. It can be seen that the two possible solutions to the quadratic equation form a pillowlike structure that includes, as limits, the fully coherent cases.

It is more difficult to find simple solutions for the required amplitudes for complete destructive interference but is readily done numerically. An example is shown in Fig. 9, with $\mu_{13} = 2/3$, $\mu_{23} = -1/4$, and $\mu_{12} = 0.5550$. Again one can see examples of both field singularities and correlation singularities, and the nature of these singularities can be established by varying the location of the reference point.

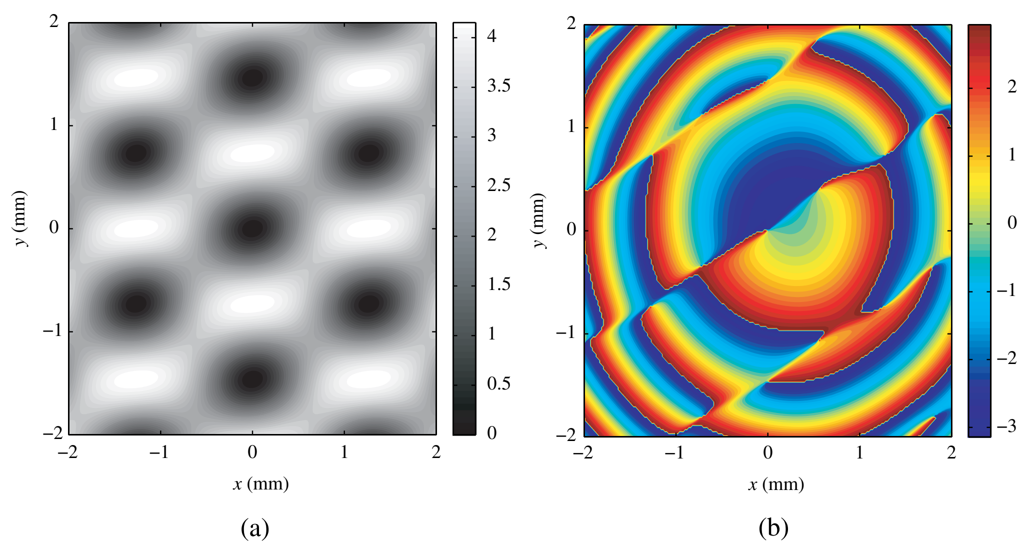
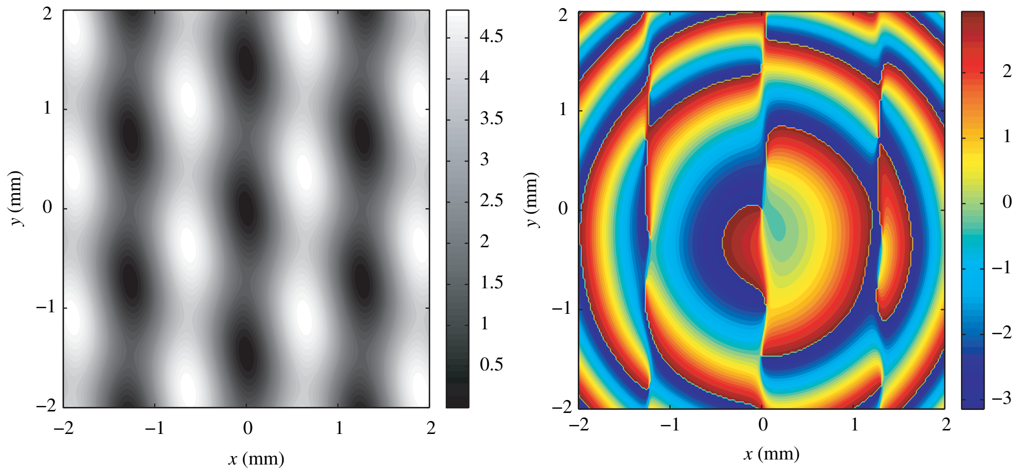


Fig. 9. (Color online) (a) Intensity pattern and (b) phase contour of the cross-spectral density, with $\mu_{13} = 2/3$, $\mu_{23} = -1/4$, and $\mu_{12} = 0.5550$. The corresponding field amplitudes are $A_1 = -0.6550$, $A_2 = 0.5042$, and $A_3 = 0.5627$. The reference point is at $(x_1, y_1) = (1\ \text{mm}, 1\ \text{mm})$.



(a) (b)

Fig. 10. (Color online) (a) Intensity pattern and (b) phase contour of the cross-spectral density, with $\mu_{12} = -1/2$, $\mu_{23} = \sqrt{3}/2$, and $\mu_{13} = 0$. The corresponding field amplitudes are $A_1 = -0.3536$, $A_2 = -0.7071$, and $A_3 = 0.6124$. The reference point is at $(x_1, y_1) = (1\ \text{mm}, 1\ \text{mm})$.

The special case described in Subsection 3.B may also be generalized. If we look for solutions under the condition that $\mu_{13} = 0$, we readily find that Eq. (12) simplifies to the form

$$\mu_{12}^2 + \mu_{23}^2 = 1, \tag{28}$$

i.e., that the values lie on the unit circle. In Fig. 10, we show the intensity and phase on the observation plane for the values $\mu_{12} = -1/2$, $\mu_{23} = \sqrt{3}/2$. Again, the presence of both correlation singularities and field singularities is confirmed.

5. CONCLUSIONS

It has been known for some time that intensity zeros are rare (i.e., nongeneric) in partially coherent wave fields. In this paper we have demonstrated that there are nevertheless a wide variety of conditions in which a three-pinhole interferometer can produce perfect zeros of intensity, with their associated field singularities. Contrary to the more restrictive cases studied in previous work, it is found that this interference involves the intensity of light emanating from the pinholes as well as its state of coherence.

These conditions can in principle be generalized further. Only real values of the spectral degree of coherence were considered here, whereas in general this quantity can be complex. Also, the general conditions were made analytically tractable by requiring a zero on the central axis of the interferometer; clearly it is possible to create conditions for complete destructive interference in which there is no on-axis zero.

Finally, one can consider the presence of destructive interference in an interferometer with $N > 3$ pinholes. Similar behavior is expected, except that the determination of the solutions becomes increasingly complicated with increasing N, involving the determinant of an $N \times N$ matrix.

These results illustrate the depth of the relationship between field singularities and correlation singularities and furthermore provide a more general setting to study their connections.

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