Scintillation of nonuniformly correlated beams in atmospheric turbulence

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We investigated the scintillation properties of nonuniformly correlated (NUC) beams in atmospheric turbulence and have shown that NUC beams can not only have lower scintillation but also higher intensity than Gaussian–Schell model beams and even higher intensity than coherent Gaussian beams over certain propagation distances. © 2013 Optical Society of America

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Lasers provide reliable light sources for fast and secure free-space optical communications. It has been further demonstrated recently that the capacity of a free-space optical communications system can be significantly enhanced by employing orbital angular momentum multiplexing [1]. However, fluctuations in the atmosphere challenge the effectiveness of free-space optical communications over long propagation distances. In particular, turbulence-induced intensity fluctuations, also known as scintillations, severely degrade the performance of free-space optical communications systems.

It is well-known that partially coherent beams (PCBs) are a promising option for scintillation reduction [2]. However, both high average received intensity and low scintillation are preferred when an optical beam is used as the information carrier for free-space optical communications. It has been suggested that the beam with maximal mean transferred intensity is fully coherent [3]. As a frequently used model of PCBs, Gaussian–Schell model (GSM) beams deliver less power to the receiver than their fully coherent counterparts [4]. However, a new class of PCBs exhibits different features. Unlike GSM beams, these beams have a spatially variant correlation function [5], and also are referred to as nonuniformly correlated (NUC) beams [6]. Their propagation characteristics in the atmosphere have been investigated recently [7,8]. One intriguing feature of such beams is that focused modes are contained in the representation of a NUC beam and consequently the beam obtains a sharp on-axis maximum some distance beyond the source plane [5]. This suggests that a NUC beam can have higher intensity over a certain propagation distance. Therefore, it is natural to ask whether such NUC beams can have less scintillation and yet maintain high intensity on propagation through the atmosphere, fulfilling both of the aforementioned requirements for free-space optical communications.

In this letter, we analytically study the scintillation properties of NUC beams in weak turbulence. The cross-spectral density of a general PCB in the source plane ($z = 0$) has the form [9]:

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sqrt{S(\boldsymbol{\rho}_1)}\sqrt{S(\boldsymbol{\rho}_2)}\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2). \tag{1}$$

$S(\boldsymbol{\rho})$ is the spectral density and $\mu(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is the spectral degree of coherence. For an NUC beam, they are given by the following forms [5]:

$$S(\boldsymbol{\rho}) = \exp\left(-\frac{2\rho^2}{w_0^2}\right), \tag{2}$$

$$\mu_c(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left[-\frac{(\rho_2^2 - \rho_1^2)^2}{r_c^4}\right], \tag{3}$$

where w_0 is the width of Gaussian intensity profile and r_c is a measure of the correlation length of NUC beams. Equation (3) shows that μ_c depends on the radial locations of $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ (i.e., spatially variant). On the contrary, a GSM beam, whose spectral density is also given by Eq. (2), has a Gaussian correlation function that is homogeneous and isotropic [9]:

$$\mu_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left(-\frac{|\boldsymbol{\rho}_2 - \boldsymbol{\rho}_1|^2}{r_0^2}\right), \tag{4}$$

where r_0 is the coherence length of a Gaussian coherence function.

The cross-spectral density of a NUC beam may be expressed more illustratively by a mode representation [5]:

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \int p(v) H_0(\boldsymbol{\rho}_1, v) H_0^*(\boldsymbol{\rho}_2, v) \mathrm{d}v, \tag{5}$$

where

$$p(v) = \frac{1}{\sqrt{\pi} a} \exp\left(-\frac{v^2}{a^2}\right), \tag{6}$$

and

$$H_0(\boldsymbol{\rho}, v) = \exp\left(-\frac{\rho^2}{w_0^2}\right) \exp(-ikv\rho^2). \tag{7}$$

Here $k = 2\pi/\lambda$ is the wavenumber of beam and $a = 2/(r_c^2 k)$. Equations (5)–(7) show that an NUC beam can be generated by incoherent superposition of coaxial Gaussian modes represented by $H_0(\boldsymbol{\rho}, v)$. The initial radius of curvature R of these Gaussian modes is random and its reciprocal $v\,[R = 1/(2v)\,]$ follows a Gaussian distribution with a probability density $p(v)$. In Ref. [10], it has been shown that the scintillation properties of a PCB can be well approximated by a finite number of its constituent modes. Therefore, we studied the scintillation properties of an NUC beam through the discretized form of Eq. (5):

$$W(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \approx \sum_{n=1}^{N} A_n(\boldsymbol{\rho}_1) A_n^*(\boldsymbol{\rho}_2), \quad (8)$$

where

$$A_n(\boldsymbol{\rho}) = \sqrt{p(v_n)} H_0(\boldsymbol{\rho}, v_n). \quad (9)$$

The weights of these constituent modes are characterized by $p(v_n)$. We sampled the modes in the interval $|v| \leq 2a$. We have previously shown that a small number of modes can well approximate the exact scintillation [10]. Here, $\Delta v = 0.2a$, equivalent to 21 modes, was found by simulation to be sufficient to converge to the limiting scintillation value. The scintillation index of such a pseudo-NUC beam is written as Eq. (10):

$$\sigma^2(\boldsymbol{\rho}, L) = \frac{\sum_{m=1}^{N} \sum_{n=1}^{N} \langle I_m(\boldsymbol{\rho}, L) I_n(\boldsymbol{\rho}, L)\rangle}{(\sum_{n=1}^{N} \langle I_n(\boldsymbol{\rho}, L)\rangle)^2} - 1, \quad (10)$$

where $I_n(\boldsymbol{\rho}, L)$ is the instantaneous intensity of the nth mode on the receiver plane $z = L$ and the angle brackets stand for the average of the realizations of turbulence. On propagation in weak turbulence, $\langle I_n(\boldsymbol{\rho}, L)\rangle$ and $\langle I_m(\boldsymbol{\rho}, L) I_n(\boldsymbol{\rho}, L)\rangle$ in Eq. (10) can be derived within the framework of the Rytov approximation, given by [10]:

$$\langle I_n(\boldsymbol{\rho}, L)\rangle = |A_{0n}(\boldsymbol{\rho}, L)|^2 \exp\{2\,\mathrm{Re}[E_1^n(\boldsymbol{\rho}, L)]\} \times \exp[E_2^{nn}(\boldsymbol{\rho}, L)], \quad (11)$$

$$\langle I_m(\boldsymbol{\rho}, L) I_n(\boldsymbol{\rho}, L)\rangle = \langle I_m(\boldsymbol{\rho}, L)\rangle \langle I_n(\boldsymbol{\rho}, L)\rangle \times \exp\{2\,\mathrm{Re}[E_2^{mn}(\boldsymbol{\rho}, L)]\} \times \exp\{2\,\mathrm{Re}[E_3^{mn}(\boldsymbol{\rho}, L)]\}, \quad (12)$$

where $A_{0n}(\boldsymbol{\rho}, L)$ is the wavefield of the nth mode in the absence of the turbulence and can be calculated by the angular spectrum theory:

$$A_{0n}(\boldsymbol{\rho}, L) = \sqrt{p(v_n)} \frac{\exp(ikL)}{1 + i\alpha_n L} \exp\left[-\frac{\alpha_n k \rho^2}{2(1 + i\alpha_n L)}\right]. \quad (13)$$

Here, $\alpha_n = 2/(k w_0^2) + i2v_n$. For the quantities E_1^n, E_2^{mn}, and E_3^{mn} in Eqs. (11) and (12), their expressions can be obtained by a similar manner shown in Ref. [11]:

$$E_1^n(\boldsymbol{\rho}, L) = -\pi k^2 \int_0^L \mathrm{d}z \int \Phi_n(\boldsymbol{\kappa}) \mathrm{d}^2\kappa, \quad (14)$$

$$E_2^{mn}(\boldsymbol{\rho}, L) = 2\pi k^2 \int_0^L \mathrm{d}z \int \exp[i(\gamma_m - \gamma_n^*)\boldsymbol{\kappa}\cdot\boldsymbol{\rho}] \times \exp\left[-\frac{i(\gamma_m - \gamma_n^*)(L - z)\kappa^2}{2k}\right] \Phi_n(\boldsymbol{\kappa}) \mathrm{d}^2\kappa, \quad (15)$$

$$E_3^{mn}(\boldsymbol{\rho}, L) = -2\pi k^2 \int_0^L \mathrm{d}z \int \exp[i(\gamma_m - \gamma_n)\boldsymbol{\kappa}\cdot\boldsymbol{\rho}] \times \exp\left[-\frac{i(\gamma_m + \gamma_n)(L - z)\kappa^2}{2k}\right] \Phi_n(\boldsymbol{\kappa}) \mathrm{d}^2\kappa, \quad (16)$$

where $\gamma_n = (1 + i\alpha_n z)/(1 + i\alpha_n L)$ and $\Phi_n(\boldsymbol{\kappa})$ is the power spectrum of the turbulence.

The scintillation index of an NUC beam can be numerically evaluated by Eqs. (10)–(16) with a given turbulence model. In this Letter, the von Karman spectrum is used [11]:

$$\Phi_n(\boldsymbol{\kappa}) = 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \quad (17)$$

where turbulence strength is $C_n^2 = 10^{-15}\ \mathrm{m}^{-2/3}$, $\kappa_m = 5.92/l_m$ with the inner scale $l_m = 1$ mm and $\kappa_0 = 1/l_0$ with the outer scale $l_0 = 10$ m. Figure 1 shows that the on-axis scintillation index of an NUC beam is reduced from its coherent limit as the relative correlation length r_c/w_0 decreases. When the NUC beam becomes less coherent, its constituent modes have a broad range of initial radii of curvatures. These modes diverge/converge at different rates and propagate through different regions of turbulence. Therefore, their distortion in turbulence is less correlated, which is a key factor for PCB scintillation reduction [10]. However, when r_c is further reduced, the wavefronts of the additional modes are more curved (small $|R|$). These modes diverge fast on propagation and deliver negligible intensities to the on-axis detector. Therefore, their contributions to the scintillation reduction of the NUC beam are negligible and the on-axis scintillation index of such as beam saturates in the incoherent limit ($r_c \to 0$).

The on-axis scintillation index of a GSM beam is also shown in Fig. 1 for comparison. In Ref. [12], it has been shown that the cross-spectral density of a GSM beam represented by Eqs. (2) and (4) also can be expressed by a mode representation similar to Eq. (8), where its mode is given by

$$\psi_n(\boldsymbol{\rho}) = \sqrt{\tilde{\mu}_0(\mathbf{u}_{\perp n})} \sqrt{S(\boldsymbol{\rho})} \exp(ik\mathbf{u}_{\perp n}\cdot\boldsymbol{\rho}). \quad (18)$$

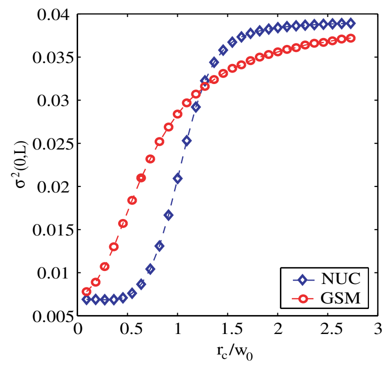

Fig. 1. On-axis scintillation index $\sigma^2(0, L)$ of an NUC beam and a GSM beam as a function of r_c/w_0. Here, $\lambda = 1.55\ \mu\mathrm{m}$, $L = 2$ km and $w_0 = 0.05$ m.

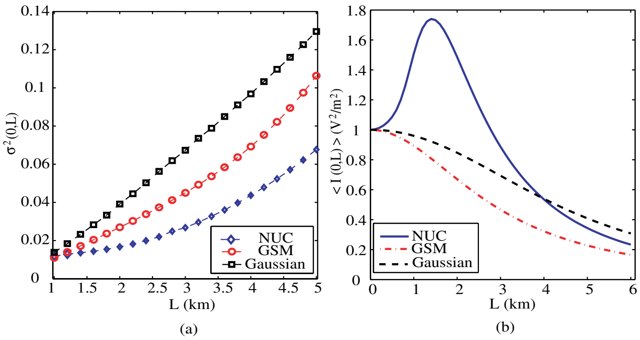

Fig. 2. (a) On-axis scintillation index $\sigma^2(0,L)$ and (b) On-axis average intensity $\langle I(0,L)\rangle$ of an NUC, a GSM, and a coherent Gaussian beam as a function of the propagation distance L. Here, $r_c = 0.91w_0$.

Here, $\tilde{\mu}_0(\mathbf{u}_{\perp n}) = \pi r_0^2 \exp(-k^2 r_0^2 u_{\perp n}^2/4)$ is the Fourier transform of μ_0 given in Eq. (4). Equation (18) shows that the constituent modes of a GSM beam are collimated Gaussian modes with random propagation direction vectors, represented by Eq. (18). Their projection in the source plane $\mathbf{u}_{\perp n}$ follows a Gaussian distribution specified by $\tilde{\mu}_0$. This mode representation suggests that the scintillation properties of a GSM can also be studied through a finite number of the constituent modes. Its scintillation index in weak turbulence can be calculated by Eqs. (10)–(12), where the corresponding expressions of E_1^n, E_2^{mn}, E_3^{mn} and $\psi_{0n}(\boldsymbol{\rho}, L)$ are available in Ref. [10], except with a different coefficient $\sqrt{\tilde{\mu}_0(\mathbf{u}_{\perp n})}$ for $\psi_{0n}(\boldsymbol{\rho}, L)$. Here, the weights of a GSM beam's constituent modes are characterized by $\tilde{\mu}_0(\mathbf{u}_\perp)$ whose width is $B = 2/(kr_0)$. We sampled the modes in the interval $|\mathbf{u}_\perp| \leq 2.4B$ and $\Delta u_{\perp x} = \Delta u_{\perp y} = 0.6B$. These 49 modes were found by simulation to be sufficient to calculate the scintillation indices of the GSM beam used in this paper. We assumed that an NUC beam and a GSM beam have comparable coherence properties by letting their spectral degree of coherence μ_c and μ_0 have the same width at half-maximum (FWHM) when $\boldsymbol{\rho}_1 = 0$, namely $r_c = 0.91r_0$. Therefore, the on-axis scintillation index of a GSM beam is also plotted as a function of r_c/w_0 in Fig. 1. The comparison shows that an NUC beam scintillates less than a GSM beam in the low coherence regime.

Figure 2 illustrates the on-axis scintillation indices of these two kinds of PCBs as well as a coherent Gaussian beam as a function of propagation distance. Their corresponding average on-axis intensities along propagation are also plotted, using the extended Huygens–Fresnel principle [11] with a quadratic phase approximation for the turbulence fluctuation [4]. It can be clearly seen that an NUC beam with appropriate coherence length ($r_c = 0.91w_0$ in Fig. 2) can have both lower scintillation and higher average transferred intensity than a GSM beam and even a coherent Gaussian beam over a certain distance on propagation through atmospheric turbulence. A rough understanding of this behavior of the NUC beam arises from its spatially variant correlation. Let us write Eq. (3) in terms of $\bar{\rho} = (\rho_1 + \rho_2)/2$ and $\Delta\rho = \rho_2 - \rho_1$, so that

$$\mu_c = \exp\left[-\frac{(\Delta\rho)^2}{r_c^4/(4\bar{\rho}^2)}\right]. \tag{19}$$

The width of the correlation function, given by $r_c^2/(2\bar{\rho})$, decreases monotonically with the average radial distance $\bar{\rho}$ from the beam center, implying the beam represents a central coherent core surrounded by an incoherent shell. The core maintains high intensity while the shell aids in reducing scintillation.

Note that for a specific propagation distance, there is a corresponding perfectly focused mode in the mode representation of an NUC beam. It is problematic to study the scintillation properties of such a mode by the Rytov approximation [11]. However, the scintillation properties of an NUC beam can be characterized by its pseudo version Eq. (8), which is not affected by removing the perfectly focused mode. Therefore, the scintillation properties of an NUC beam can be studied by the formulae developed in this Letter.

In conclusion, we believe we have demonstrated NUC beams are the first kind of PCBs discovered so far that possess both low scintillation and high average transferred intensity on propagation in atmospheric turbulence. This conclusion suggests NUC beams are potentially useful in free-space optical communications.

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