Partially coherent beam propagation in atmospheric turbulence [Invited]

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Partially coherent beams hold much promise in free-space optical communications for their resistance to the deleterious effects of atmospheric turbulence. We describe the basic theoretical and computational tools used to investigate these effects, and review the research to date.   © 2014 Optical Society of America

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1. INTRODUCTION

Since the invention of the laser, the possibility of using visible light instead of radio for free-space electromagnetic communications has intrigued researchers. The extreme directionality of lasers allows a great degree of security, while optical frequencies permit a high rate of data transfer. The short wavelength of visible light also gives excellent resolution in applications such as imaging and laser radar. However, atmospheric turbulence, manifesting as random variations of the refractive index of the atmosphere, degrades the quality of an optical beam, resulting in excessive beam spreading of the beam, deviations of the beam direction (beam wander), and intensity fluctuations of the beam at the detector (scintillations). With sufficiently strong turbulence fluctuations, these effects can cause unacceptable data transmission errors over distances of less than a kilometer, severely limiting visible light for practical free-space applications.

However, it has long been known that partially coherent beams—beams that are partially randomized in time and/or space—often prove to be more resistant to the degrading effects of turbulence than their fully coherent counterparts. The earliest work can be dated to soon after birth of the laser and, not coincidentally, to the development of modern optical coherence theory. At the start of the 21st century, renewed interest in free-space optical communications, combined with the discovery of exotic beam classes and new types of partially coherent fields, has spurred much research into the behavior of randomized beams in turbulence.

In this article we review the progress on partially coherent (PC) beam propagation in atmospheric turbulence. In Section 2, we look at the progress in the field from the 1960s to the end of the 20th century, and give a simple description of how PC beams can "resist" turbulence. In Section 3, we discuss the most commonly used analytic model for studying the propagation of light through the atmosphere, and apply it to PC beams. Due to the complexity of turbulence, however, analytic models are often intractable, except in special cases; in Section 4, we describe the multiple phase screen model often used to computationally solve for beam evolution. In Section 5,

we review turbulence effects related to the second order (field–field) coherence properties of the field, and in Section 6, we consider effects related to the fourth order (scintillation) properties of the field.

2. HISTORICAL OVERVIEW

The earliest articles discussing the coherence of light in turbulence attempted to build on the classic texts of Tatarski [1] and Chernov [2] to construct robust and accurate propagation models for coherence functions in random media. Hufnagel and Stanley [3] estimated the modulation transfer function for an imaging system seeing through the atmosphere; several propagation models, including a geometric one, were used to estimate the spatial coherence of light at the input aperture. Beran [4] introduced an approximate solution for the mutual coherence function in random media by dividing the medium into a series of longitudinal slices. A slightly more general expression for the mutual coherence function was given by Taylor [5], giving results in accordance with geometrical optics.

These early formulations were all limited to very short propagation distances or, equivalently, a single-scattering approximation. Furthermore, the calculations were done only for exceedingly simple fields, namely, spherical waves, plane waves, or very wide Gaussian beams. In 1971, Lutomirski and Yura [6] introduced a new technique, now known as the *extended Huygens–Fresnel (eHF) principle* (to be discussed in Section 3), that can be used to derive propagation results over a broader range of circumstances and for more general fields. Yura [7] applied this method to derive an analytic expression for the mutual coherence function of a finite beam. It was later realized that essentially the same method had been independently derived in 1967 by Feizulin and Kravtsov [8].

The next round of published articles studied the general propagation characteristics of PC beams in turbulence. Kon and Tatarskii [9] evaluated the mutual coherence function of a Gaussian beam propagating in turbulence under the quadratic approximation; later, Belen'kii *et al.* [10] applied these results to turbulence-induced spreading of an optical image. In 1978, Leader [11] built upon the eHF principle to study

in detail the dependence of beam propagation on source coherence. Fante [12] derived the evolution in turbulence of a more general mutual coherence function with two frequencies.

About the same time, researchers' attention began to shift toward the more difficult problem of evaluating the intensity fluctuations of beams with partial spatial coherence in turbulence. Leader [13] investigated such fluctuations in the context of imaging random rough surfaces, using the "Huygens–Kirchhoff method" (essentially the eHF principle again). Fante [14], Banach *et al.* [15], and Banakh and Buldakov [16] studied the fluctuations of intensity over different receiver response times, noting dramatic differences between a "fast" and a "slow" detector. Fante [17] also considered the effect of temporal coherence on scintillation, at least for weak turbulence.

Through the 1990s, the study of atmospheric propagation of PC beams seems to have been largely ignored, with the exception of articles by Wu [18] and Wu and Boardman [19]; the former considers the propagation properties of model PC beams, while the latter considers the spatial coherence properties of the beams.

At the start of the 21st century, interest in the reduction of turbulence distortion by partial coherence grew dramatically. Gbur and Wolf [20] theoretically evaluated the spreading of PC beams in random media, noting that PC beams are in a sense less sensitive to turbulence; this result was demonstrated experimentally by Dogariu and Amarande [21]. Similar theoretical results were achieved by others using different methods: Ponomarenko *et al.* [22] used a Hilbert-space method, and Shirai *et al.* [23] applied a modal analysis. The average transmittance of PC beams in turbulence was analyzed by Baykal [24].

The promise of scintillation reduction by partial coherence has led an increasing number of researchers to study PC beams specifically for optical communications. The first in a new wave of studies were performed by Ricklin and Davidson [25,26], and Korotkova *et al.* [27].

Since this time there has been a vast amount of research on both the propagation and scintillation of PC beams in turbulence. It is to be noted that optimizing such beams for applications is a nontrivial problem: PC beams tend to have less scintillation on propagation, but also have a larger angular spread, resulting in less energy received at the detector. An appropriate balance of these seemingly exclusive traits will depend on the desired system specifications. Optimization is in principle possible; Schulz [28,29] has used variational and iterative methods to find the optimal mode structure of PC beams with either minimum scintillations or minimum spreading in turbulence. However, it is not clear how to produce these optimal beams in practice, and so research continues to find realizable sources with good propagation characteristics. Some of this research will be summarized in the final sections of this review.

3. ANALYTIC PROPAGATION MODELS

The atmosphere is, in a sense, an ideal medium to study using the traditional tools of scattering theory. The small variations of refractive index in atmospheric turbulence induce small phase fluctuations on a beam of light, and these fluctuations only become significant after appreciable propagation distances. Early researchers applied the Born and Rytov approximations to study atmospheric propagation; description of these methods can be found in [30], Chapter 13.

In recent years, however, the preferred strategy for studying propagation in turbulence is the eHF method noted previously. It was shown quite early that the eHF method is in good agreement with experimental data in weak and strong turbulence [31], even under simplifying phase approximations [32]. The method includes atmospheric turbulence as a phase distortion of the spherical wave in the standard Huygens–Fresnel integral, so that the field $U(\mathbf{r}, L, \omega)$ after propagation a distance $z = L$ is given by

$$U(\mathbf{r}, L, \omega) = -\frac{ik}{2\pi L} \exp(ikL)$$
$$\times \iint U_0(\boldsymbol{\rho}, \omega) \exp\left[\frac{ik|\boldsymbol{\rho} - \mathbf{r}|^2}{2L} + \psi(\mathbf{r}, \boldsymbol{\rho}, L)\right] d^2\rho, \quad (1)$$

where $U_0(\boldsymbol{\rho}, \omega)$ is the field in the source plane at $z = 0$, $\boldsymbol{\rho}$ and $\mathbf{r}$ are the transverse coordinates in the source and observation planes, respectively, and $\psi(\mathbf{r}, \boldsymbol{\rho}, L)$ is the phase distortion of the Huygens wavelet due to turbulence. The geometry is illustrated in Fig. 1. The field is taken for the moment to be monochromatic at frequency ω and with wavenumber k. In what follows, we will closely follow the notation of Andrews and Phillips [33]; we will also suppress the frequency dependence momentarily for brevity.

Of primary interest here are the second- and fourth-order average field moments, defined as

$$W_2(\mathbf{r}_1, \mathbf{r}_2, L) = \langle U(\mathbf{r}_1, L)U^*(\mathbf{r}_2, L)\rangle$$
$$= \iiint U_0(\boldsymbol{\rho}_1)U_0^*(\boldsymbol{\rho}_2)G_0(\boldsymbol{\rho}_1, \mathbf{r}_1)G_0^*(\boldsymbol{\rho}_2, \mathbf{r}_2)$$
$$\times M_2[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L]d^2\rho_1 d^2\rho_2 \quad (2)$$

and

$$W_4(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, L) = \langle U(\mathbf{r}_1, L)U^*(\mathbf{r}_2, L)U(\mathbf{r}_3, L)U^*(\mathbf{r}_4, L)\rangle$$
$$= \iiint \iiint U_0(\boldsymbol{\rho}_1)U_0^*(\boldsymbol{\rho}_2)U_0(\boldsymbol{\rho}_3)U_0^*(\boldsymbol{\rho}_4)$$
$$\times G_0(\boldsymbol{\rho}_1, \mathbf{r}_1)G_0^*(\boldsymbol{\rho}_2, \mathbf{r}_2)G_0(\boldsymbol{\rho}_3, \mathbf{r}_3)G_0^*(\boldsymbol{\rho}_4, \mathbf{r}_4)$$
$$\times M_4[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L]d^2\rho_1 d^2\rho_2 d^2\rho_3 d^2\rho_4, \quad (3)$$
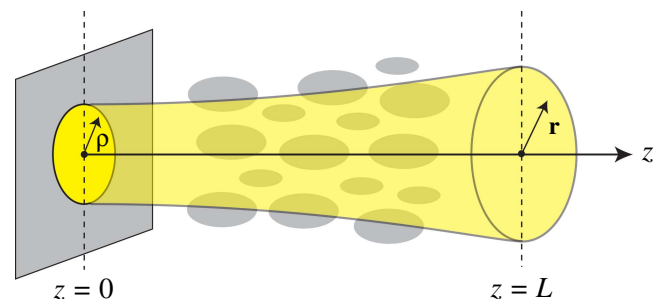

Fig. 1.   Illustration of the geometry of beam propagation through the atmosphere.

where we have defined

$$M_2[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L] = \langle \exp[\psi(\mathbf{r}_1, \boldsymbol{\rho}_1, L) + \psi^*(\mathbf{r}_2, \boldsymbol{\rho}_2, L)] \rangle, \quad (4)$$

$$M_4[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L] = \langle \exp[\psi(\mathbf{r}_1, \boldsymbol{\rho}_1, L) + \psi^*(\mathbf{r}_2, \boldsymbol{\rho}_2, L) \\ + \psi(\mathbf{r}_3, \boldsymbol{\rho}_3, L) + \psi^*(\mathbf{r}_4, \boldsymbol{\rho}_4, L)] \rangle, \quad (5)$$

where $\{\mathbf{r}_i, \boldsymbol{\rho}_i\}$ represents the set of relevant arguments, and

$$G_0(\boldsymbol{\rho}, \mathbf{r}) = -\frac{ik}{2\pi L} \exp(ikL) \exp\left[\frac{ik|\boldsymbol{\rho} - \mathbf{r}|^2}{2L}\right] \quad (6)$$

is the free-space Green's function in the Fresnel approximation. The angle brackets $\langle \rangle$ represent an ensemble average over a large number of realizations of the turbulence; from these moments we can derive properties such as the average beam width and the average variance of the intensity. It should be noted, however, that specific realizations of these field moments can look very different from the averages.

The average phase terms can be calculated using the method of cumulants (see, for instance, [34], Section 15.10). To second order, we may write

$$\langle \exp[\Psi(\mathbf{r}, L)] \rangle = \exp\left[K_1 + \frac{1}{2} K_2\right], \quad (7)$$

where

$$K_1 = \langle \Psi(\mathbf{r}, L) \rangle, \quad (8)$$

$$K_2 = \langle \Psi^2(\mathbf{r}, L) \rangle - \langle \Psi(\mathbf{r}, L) \rangle^2. \quad (9)$$

From these results, it follows that the moments of the phase can be written as

$$M_2[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L] = \exp[2E_1(0, 0; 0, 0) + E_2(\mathbf{r}_1, \mathbf{r}_2; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2)], \quad (10)$$

$$M_4[\{\mathbf{r}_i, \boldsymbol{\rho}_i\}, L] = \exp[4E_1(0, 0; 0, 0) + E_2(\mathbf{r}_1, \mathbf{r}_2; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \\ + E_2(\mathbf{r}_1, \mathbf{r}_4; \boldsymbol{\rho}_1, \boldsymbol{\rho}_4) + E_2(\mathbf{r}_3, \mathbf{r}_2; \boldsymbol{\rho}_3, \boldsymbol{\rho}_2) \\ + E_2(\mathbf{r}_2, \mathbf{r}_4; \boldsymbol{\rho}_2, \boldsymbol{\rho}_4) + E_3(\mathbf{r}_1, \mathbf{r}_3; \boldsymbol{\rho}_1, \boldsymbol{\rho}_3) \\ + E_3^*(\mathbf{r}_2, \mathbf{r}_4; \boldsymbol{\rho}_2, \boldsymbol{\rho}_4)], \quad (11)$$

where we have introduced

$$E_1(0, 0; 0, 0) = -2\pi^2 k^2 L \int_0^\infty \kappa \Phi_n(\kappa) d\kappa, \quad (12)$$

$$E_2(\mathbf{r}_1, \mathbf{r}_2; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = 4\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) J_0[\kappa|(1 - \xi)\Delta\mathbf{r} \\ + \xi\Delta\boldsymbol{\rho}|] d\kappa d\xi, \quad (13)$$

$$E_3(\mathbf{r}_1, \mathbf{r}_2; \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = -4\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) J_0[\kappa|(1 - \xi)\Delta\mathbf{r} \\ + \xi\Delta\boldsymbol{\rho}|] \exp[-iL\kappa^2\xi(1 - \xi)/k] d\kappa d\xi. \quad (14)$$

We have now introduced $\Delta\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$ and $\Delta\boldsymbol{\rho} \equiv \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$, as well as the power spectral density of refractive index fluctuations $\Phi_n(\kappa)$.

To apply these formulas, we must have a model of the atmosphere from which a formula for $\Phi_n(\kappa)$ can be deduced. The simplest model with some predictive power is illustrated in Fig. 2, and shows the energy cascade theory of turbulence. When wind speeds exceed the critical threshold for turbulence to form, large turbulent eddies of roughly constant refractive index are created with a large characteristic scale L_0, called the *outer scale*. These large eddies undergo a process in which they break up continually into smaller and smaller sizes, eventually reaching a critical minimum size l_0, called the *inner scale*, at which their energy is dissipated completely. The outer scale can range widely from meters to tens of meters, while the inner scale is typically of the order of millimeters.

Starting from a dimensional argument, Kolmogorov noted that this process could be represented by the simple power spectral density

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3}, \qquad 1/L_0 \ll \kappa \ll 1/l_0. \quad (15)$$

Here C_n^2 is called the *structure parameter* and is a measure of the overall turbulence strength. Its values range from 10^{-17} m$^{-2/3}$ for weak turbulence to as high as 10^{-13} m$^{-2/3}$ for strong turbulence.

The Kolmogorov spectrum is an exceedingly simple expression, but does not treat the inner and outer limits in a satisfactory manner. A more sophisticated model is the Tatarskii spectrum:

$$\Phi_n(\kappa) = 0.033 C_n^2 \kappa^{-11/3} \exp(-\kappa^2/\kappa_m^2), \qquad 1/L_0 \ll \kappa, \quad (16)$$

with $\kappa_m = 5.92/l_0$, which accounts for the inner scale (high spatial frequencies). A yet more detailed spectrum is the von Karman spectrum:

$$\Phi_n(\kappa) = 0.033 C_n^2 \frac{\exp(-\kappa^2/\kappa_m^2)}{(\kappa^2 + \kappa_0^2)^{11/6}}, \quad (17)$$

with $\kappa_0 = 1/L_0$, which takes into account both inner and outer scales. Nevertheless, even this analytic model for the spectrum misses important physical details; more advanced ones can be found in [33].

We can now include partial coherence into the equations for the field moments. Considering first the second-order
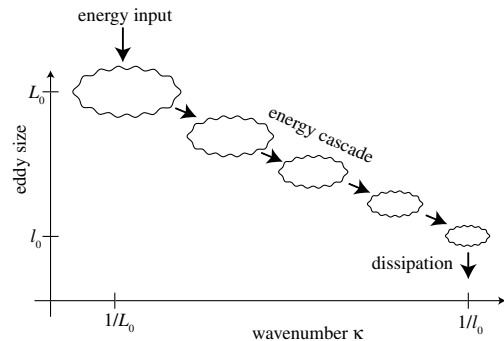

Fig. 2. Simple illustration of a model for atmospheric turbulence.

moment, we may average the moment over a set of monochromatic field realizations using the space–frequency representation of the source cross-spectral density $W_0(\mathbf{r}_1, \mathbf{r}_2, \omega)$, i.e.,

$$W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \omega) = \langle U_0(\boldsymbol{\rho}_1, \omega) U_0^*(\boldsymbol{\rho}_2, \omega) \rangle_\omega, \qquad (18)$$

where the subscript "ω" represents an average over a specially constructed set of monochromatic field realizations; see [35], Chapter 4 for more details. The resulting second-order moment $W_2(\mathbf{r}_1, \mathbf{r}_2, z, \omega)$ is the cross-spectral density of the field after propagating a distance z through the atmosphere.

It is to be noted that the time scales of fluctuations are extremely important in performing averages in the simple manner described. Because the fluctuations of turbulence are typically slow (of the order of milliseconds) compared to the fluctuations of the optical source, the medium may be treated as effectively "frozen" for the time of field averaging, and the averages may be taken independently.

The cross-spectral density may be expanded in the form of a *coherent mode representation*, as first demonstrated by Wolf [36]. This representation may be written as

$$W(\mathbf{r}_1, \mathbf{r}_2, \omega) = \sum_n \lambda_n(\omega) \phi_n(\mathbf{r}_1, \omega) \phi_n^*(\mathbf{r}_2, \omega), \qquad (19)$$

where n is a (possibly multiple) sum over a set of so-called coherent modes that are mutually orthogonal when integrated over the domain of interest (in this case, the source plane $z = 0$) that satisfy

$$\iint W(\mathbf{r}_1, \mathbf{r}_2, \omega) \phi_n(\mathbf{r}_2, \omega) \mathrm{d}^2 r_2 = \lambda_n(\omega) \phi_n(\mathbf{r}_1, \omega), \qquad (20)$$

and $\lambda_n(\omega) > 0$.

The coherent mode representation provides a nice physical picture for the mechanism by which PC beams are more resistant to turbulence; this is illustrated in Fig. 3. A coherent single mode laser will send all of its energy through a single path in the turbulence; this mode will interfere with itself, producing laser speckle and, therefore, intensity fluctuations, if it even hits the detector at all. A PC beam will send energy via multiple modes that travel through different channels toward the detector. Multiple modes will likely hit the detector and, because of their mutual incoherence, their individual speckle patterns will wash out, producing a more regular intensity at the detector and less scintillation.
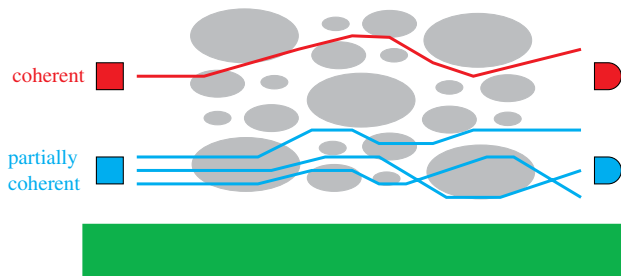

Fig. 3. Simple illustration of the turbulence resistance of PC beams. A PC beam will simultaneously "sample" multiple paths through the turbulence, and the mutually incoherent beams will not produce interference speckle at the detector. Figure adapted from [37].

The scintillation of an optical beam is typically characterized by the *scintillation index*, defined as

$$\sigma_I^2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} - 1, \qquad (21)$$

where I is the intensity of the field at a single point, $I(\mathbf{r}) = |U(\mathbf{r})|^2$. The denominator of Eq. (21) can be derived from the second-order field moment and the numerator from the fourth-order field moment. For a plane wave in weak turbulence, the scintillation index takes on the specific form

$$\sigma_1^2 = 1.23 C_n^2 k^{7/6} z^{11/6}, \qquad (22)$$

and is known as the *Rytov variance*. The Rytov variance is often used as a loose measure of the strength of atmospheric turbulence, with $\sigma_1^2 \ll 1$ implying weak turbulence and strong turbulence otherwise.

As was noted in [14–16], the response time τ_d of the detector is an important factor in the scintillation measured. If the detector response is much longer than the coherence time τ_c of the light, i.e., $\tau_d \gg \tau_c$, the inherent fluctuations in the source intensity will be averaged out, and we may write

$$\langle U_0(\boldsymbol{\rho}_1) U_0^*(\boldsymbol{\rho}_2) U_0(\boldsymbol{\rho}_3) U_0^*(\boldsymbol{\rho}_4) \rangle = W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) W_0(\boldsymbol{\rho}_3, \boldsymbol{\rho}_4), \qquad (23)$$

i.e., all the intensity fluctuations come from the turbulence, and none from the source.

If $\tau_d \ll \tau_c$, then the detector will be sensitive to source fluctuations, as well. Most methods of creating PC sources result in fields with Gaussian statistics (see [35], Chapter 7), for which we may write

$$\langle U_0(\boldsymbol{\rho}_1) U_0^*(\boldsymbol{\rho}_2) U_0(\boldsymbol{\rho}_3) U_0^*(\boldsymbol{\rho}_4) \rangle = W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) W_0(\boldsymbol{\rho}_3, \boldsymbol{\rho}_4) \\ + W_0(\boldsymbol{\rho}_1, \boldsymbol{\rho}_4) W_0(\boldsymbol{\rho}_3, \boldsymbol{\rho}_2). \qquad (24)$$

These intensity fluctuations will result in a scintillation that is *higher* than that of a fully coherent beam.

These observations suggest that any application of PC fields to scintillation reduction must take careful consideration of the time scales involved. To the times already considered we can add the duration of an optical bit in a communications signal, τ_b, which must be longer than both the coherence time and the detector response. If we also consider the characteristic time of turbulence fluctuations τ_t, we require the following set of inequalities to be satisfied to have effective scintillation reduction in optical communications:

$$\tau_c \ll \tau_d \ll \tau_b \ll \tau_t. \qquad (25)$$

These inequalities also imply that a scintillation-reducing source must have a spectral bandwidth significantly larger than the communications bandwidth.

Though the eHF principle has been shown to agree well with experimental data, there are a number of limitations worth noting. First, the simplest formulas given above, and they are by no means simple, involve light propagating in uniform, homogeneous turbulence throughout the entire source-to-detector path. This excludes situations such as ground-to-air or ground-to-satellite links, which can be accounted for, albeit with additional complexity.

Even for the simplest Kolmogorov model of turbulence, however, the field integrals are generally intractable. In many cases, the integrals are evaluated by making use of a quadratic approximation for the turbulence phase moments, though it has long been known that this approximation leaves out important features of the turbulence [38]. Furthermore, the complexity of the models often leads to their misuse, as was discussed recently [39].

The expressions can be evaluated by numerical integration, but this can be extremely time consuming: the fourth-order field moment requires an eightfold integral that must be evaluated for each set of observation points $\mathbf{r}_i$. If analytic insight into the problem is not required, it is much more efficient to use computational methods to propagate the field, as described in the next section.

4. COMPUTATIONAL PROPAGATION MODELS

The standard method for numerically simulating the propagation of waves in atmospheric turbulence is a multiple phase screen method, in which the extended random medium is described by a collection of thin random phase screens with the appropriate turbulence statistics. We briefly review this method, following the clear discussion of Martin and Flatté [40].

A monochromatic scalar wave propagating in an inhomogeneous medium will satisfy the Helmholtz equation:

$$[\nabla^2 + n^2(\mathbf{r})k^2]U(\mathbf{r}) = 0, \qquad (26)$$

where $n(\mathbf{r})$ is the refractive index of the medium and $k = \omega/c$ is the free-space wavenumber. If we restrict ourselves to paraxial fields of the form

$$U(\mathbf{r}) = V(\mathbf{r})e^{ikz}, \qquad (27)$$

where $V(\mathbf{r})$ is a slowly varying function in z, we get the simplified equation

$$2ik\partial_z V + \nabla_\perp^2 V + k^2(n^2 - 1)V = 0, \qquad (28)$$

where $\nabla_\perp^2 = \partial_x^2 + \partial_y^2$. The refractive index of the atmosphere is quite close to unity; we write $n = 1 + \Delta n$, where $\Delta n \ll 1$, and then to a good approximation we may write

$$2ik\partial_z V + \nabla_\perp^2 V + 2\,k^2 \Delta n V = 0. \qquad (29)$$

This is the *parabolic approximation* to the wave equation.

The small variations of the refractive index, and the absence of absorption, suggest that the effect of the medium can be described by phase approximations. Because these changes build slowly, it can be argued that the entire extended medium can be well modeled by a finite number of phase screens, equally spaced by δ_z, with appropriate statistics. It is shown in [40] that the relationship between the power spectrum $\Phi_n(\kappa)$ of the turbulence and the spectrum $\Phi_\theta(\kappa)$ of the phase screens may be related by

$$\Phi_\theta(\kappa) = 2\pi k^2 \delta_z \Phi_n(\kappa). \qquad (30)$$

The screens themselves may be calculated by creating an $N \times N$ field of random complex numbers (with real and imaginary components between 0 and 1), multiplying this field at spatial frequencies $(j\Delta_k, n\Delta_k)$ by $\Delta_\kappa^{-1}\sqrt{\Phi_\theta(j\Delta_k, n\Delta_k)}$, and then taking the two-dimensional discrete inverse Fourier transform to get a complex random phase array $\theta_1 + i\theta_2$, with $\Delta_\kappa = 2\pi/N\Delta$, and Δ is the spatial sampling interval of the array. The arrays θ_1 and θ_2 are both random phase screens with the appropriate statistics. More details of this technique can be found in [41].

The propagation of a wavefield through turbulence is therefore accomplished by free-space propagation of the field from screen to screen; this can be achieved by Fourier transform techniques as follows:

1. Field passing through screen acquires phase, $V(\boldsymbol{\rho}, z) \rightarrow V(\boldsymbol{\rho}, z)\exp[i\theta(\boldsymbol{\rho})]$.
2. Discrete Fourier transform of field, $V(\boldsymbol{\rho}, z) \rightarrow \tilde{V}(\boldsymbol{\kappa}, z)$.
3. Fresnel propagation to next screen, $\tilde{V}(\boldsymbol{\kappa}, z) \rightarrow \tilde{V}(\boldsymbol{\kappa}, z)\exp[-i\kappa^2\delta_z/2k] = \tilde{V}(\boldsymbol{\kappa}, z + \delta_z)$.
4. Inverse discrete Fourier transform, $\tilde{V}(\boldsymbol{\kappa}, z + \delta_z) \rightarrow V(\boldsymbol{\rho}, z + \delta_z)$.

The sampling of the screens must be chosen such that their spatial frequencies include the relevant turbulence ranges, notably the inner and outer scale frequencies. The spacing of the screens must also satisfy a pair of additional conditions. They should be spaced such that the real extended medium scintillation is weak over the interscreen distance, i.e., $\sigma_I^2(\delta_z) < 0.1$. Also, one should require that less than 10% of the total scintillation take place over the interscreen distance, i.e., $\sigma_I^2(\delta_z) < 0.1\sigma_I^2(L)$.

At this point the argument may seem rather circular: in order to calculate the scintillation of a beam, we need to know the scintillation of the beam! However, the Rytov variance, given by Eq. (22), can usually be used as an estimate of the scintillation for use in the inequalities above.

The simple method discussed here suffers from a significant limitation: the absence of low spatial frequencies. Due to the nature of discrete Fourier transforms, the lowest spatial frequency included in the simulation is roughly equal to the inverse of the width of the simulation domain; any spatial frequencies lower than this are not represented. However, the lowest spatial frequencies are exactly those which result in the wander of the centroid of the beam. These effects become especially important over long propagation distances and strong turbulence. To incorporate them, a variety of so-called subharmonic methods have been introduced, which put the low frequencies back into the simulation "by hand." Worthwhile discussions can be found in [42–44].

To incorporate partial coherence into the simulations, a PC beam can be treated as an incoherent superposition of coherent beams, each of which are propagated through the same realization of turbulence. The coherent mode representation of the beam, formally given in Eq. (19), is one such way to decompose the cross-spectral density. Alternatively, if the beam is Schell-model (see [35], Section 5.3), i.e., the degree of coherence $\mu(\mathbf{r}_1, \mathbf{r}_2)$ is independent of the origin of position,

$$W(\mathbf{r}_1, \mathbf{r}_2) = \sqrt{I(\mathbf{r}_1)}\sqrt{I(\mathbf{r}_2)}\mu(\mathbf{r}_2 - \mathbf{r}_1), \qquad (31)$$

then the degree of coherence can be written in terms of its Fourier representation:

$$\mu(\mathbf{r}_2 - \mathbf{r}_1) = \iint \tilde{\mu}(\mathbf{K}) \exp[-i\mathbf{K} \cdot \mathbf{r}_2] \exp[i\mathbf{K} \cdot \mathbf{r}_1] \mathrm{d}^2 K. \quad (32)$$

On substitution into Eq. (31), it is clear that the cross-spectral density may be expressed as an incoherent sum of beams with different inclination phases.

This method produces a source that has no inherent intensity fluctuations, meaning that it is in agreement with the inequalities of Eq. (25). This is the regime of effective scintillation reduction, which makes this formulation appropriate for most practical applications.

5. BEAM PROPAGATION IN TURBULENCE

Despite the fact that the general spreading and propagation characteristics of PC beams have been known for some time, research nevertheless continues. It is spurred by the relatively recent introduction of "special" classes of coherent and PC beams with unusual propagation characteristics, such as Bessel and Airy beams. Furthermore, in seeming contradiction to conventional wisdom, appropriately prepared electromagnetic beams can have unusual polarization changes in turbulence. Finally, the propagation and polarization of light can be used to measure the turbulence itself. In this section, we highlight some of these possibilities.

Extending beyond ordinary Gaussian beams, Cai and He studied the propagation of elliptical Gaussian beams [45] and dark hollow beams [46] in the atmosphere, finding that in both cases the beams will eventually become circular. More exotic possibilities involve the use of so-called Bessel beams [47], which can be nondiffracting over large propagation distances, and Airy beams [48], which can exhibit transverse acceleration. Chen and Pu [49] have investigated Bessel beams in turbulence, while Chu [50] has studied Airy beams. Perhaps unsurprisingly, the turbulence always "wins" over sufficiently long distances, erasing the special features of the beam and reducing it to a Gaussian profile. However, Çil *et al.* [51] have noted that the beam wander of Bessel–Gaussian beams can be less than a standard Gaussian.

Similar results arise for PC beam propagation. Chen *et al.* [52] studied the degradation of a PC Bessel–Gaussian beam in turbulence, finding its reduction to a Gaussian profile. The spreading and directionality of PC Hermite–Gauss beams were investigated by Ji *et al.* [53]. In recent years, a new analytic class of non-Schell-model PC beams was introduced by Gori and Santarsiero [54]; the propagation characteristics of such beams were then studied by Tong and Korotkova [55].

Though it has long been appreciated that the polarization of a uniformly polarized beam is unchanged on propagation, due to the weak scattering of turbulence, the same is not true for beams with a nonuniform state or degree of polarization. Such nonuniformly polarized beams can be considered a coherent or incoherent superposition of different orthogonally polarized modes, each of which propagates through the atmosphere via a different channel. Early research by Salem *et al.* [56] illustrated that the degree of polarization changes dramatically on propagation; however, unlike in free space, it surprisingly returns to its original value after an appreciable distance. Not long after this work, Korotkova *et al.* [57] showed that the state of polarization also in general changes on propagation. Similar results apply to the propagation of light through tissue, as was shown by Gao [58] and Gao

and Korotkova [59]. Several studies have also looked at the propagation characteristics of coherent radial and azimuthally polarized light; see, for instance, [60,61].

A number of new tools have been introduced to model the propagation of PC beams in the atmosphere. Among them is a reformulation of the eHF method in the angular spectrum regime, both in the scalar [62] and electromagnetic [63] cases.

The relationship between partial coherence and a random medium can be exploited to probe the structure of the medium itself. Ponomarenko and Wolf [64] introduced a technique to fully measure the correlation function of turbulence from the correlations of scattered light. McKinney *et al.* [65] tested a strategy for measuring the scattering parameter of an optically diffusive medium using PC light. More recently, a technique called *variable coherence tomography* was introduced in [66,67] to use variable coherence states to deduce the structure of a random media; this method was adapted to include polarization effects by Tyo and Turner [68]. It has also been shown by Gu and Gbur [69] that the evolution of correlation singularities in turbulence can be used as a crude measure of turbulence strength.

6. SCINTILLATION EFFECTS IN TURBULENCE

A broad research effort has been dedicated to studying the scintillation of a variety of special beam classes over the past decade; from this work has come a number of surprises. In this section, we consider some of the highlights.

A number of coherent beams of special form have been found to have less scintillation than a comparable Gaussian. Strömqvist Vetelino and Andrews [70] studied an annular Gaussian beam and found it to have superior scintillation characteristics; Baykal [71] considered higher-order annular beams. The scintillation of elliptical Gaussian beams were investigated by Cai *et al.* [72], and also found to have reduced scintillation under certain circumstances. In weak turbulence, Eyyuboğlu *et al.* [73] demonstrated that Bessel–Gauss beams can also have somewhat reduced scintillation, as can modified Bessel–Gauss beams [74]. Flat-topped Gaussian beams have also shown some advantages [75].

Partial coherence can provide further scintillation reduction. Berman *et al.* [76] introduced a design for a communication system based on partial coherence, and suggested suppression by "orders of magnitude." Baykal *et al.* [77] observed scintillation reduction as a function of decoherence for a beam consisting of multiple incoherent Gaussians. Beams appropriately tailored with nonuniform correlations, of the Gori type mentioned in Section 5, have also been shown to provide scintillation reduction over simpler types of PC beams [78]. In the temporal domain, Kiasaleh [79] noted scintillation reduction in a multiwavelength Gaussian beam.

In a study of Bessel-correlated beams, Gu and Gbur [80] made the surprising observation that a small array of incoherent beamlets can provide scintillation nearly indistinguishable from a more general PC beam. This suggests that such arrays may be good enough for any scintillation reduction based on partial coherence. An unusual example of such an array was provided by Gu and Gbur [81], who used an incoherent collection of Airy beams to achieve reduction.

Experimental tests have confirmed the theoretical improvements. Voelz and Fitzhenry [82] introduced a pseudo PC beam

for laser communication and laboratory tested its effectiveness. Researchers at the University of Arizona calculated the scintillation reduction characteristics of two- [83] and multi-Gaussian [84] beam arrays, and verified their results experimentally [85].

Beams with nontrivial polarization can also achieve surprising reductions. Korotkova [86] noted that even simple unpolarized beams will typically have less scintillation than comparable polarized beams. The polarization properties of light were considered in a pair of studies [87,88] on the use of PC beams for laser radar. An electromagnetic "cosine-Gaussian Schell-model beam" has also been investigated [89].

Curiously, even fully coherent but nonuniformly polarized beams can achieve scintillation reduction. The orthogonal modes of a nonuniformly polarized beam propagate differently through turbulence and do not produce an interference pattern at the detector, acting as an effective two-mode PC source. Scintillation reduction was demonstrated by Gu *et al.* [90], using a beam that would later be recognized as a Poincaré beam [91]. It was later shown by Gu and Gbur [92] that the benefits of nonuniform polarization even extend to an incoherent array of such beamlets.

As can be seen, many options exist for scintillation reduction. Optimizing the effect, which involves a trade-off in beam spreading and scintillation, will no doubt depend on the desired characteristics of the system it is applied to.

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