MATH RESEARCH AT UNC CHARLOTTE 2024

Project 6: Knot theory with a graph theory approach.

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Project Description A central issue in knot theory is how to distinguish two different knots. This is a difficult problem in general because a knot can be deformed into various different geometric shapes. These seemingly very different knots are in fact topologically equivalent. Then there is the problem of how to tabulate knots. The common approach is to tabulate knots using their minimum crossing numbers. This task has been completed for knots with minimum crossing numbers up to 13. For some special classes of knots such as the two bridge knots and the torus knots (these classes contain infinitely many knots), a complete tabulation has been done. In this project, we will study another special class of knots called the *pretzel knots*.

In this project, the students will be using graph theory as a tool to tackle the enumeration of all alternating pretzel knots, with methods and theories developed in the mentor's recent work and ongoing work [1–4]. For any oriented knot diagram D, we can construct an edge weighted simple graph $G_S(D)$, called the *Seifert graph* of D, from the Seifert decomposition $S(D)$ of D by identifying each Seifert circle to a vertex of $G_S(D)$. If there exist $k \geq 1$ crossings between two Seifert circles C_1 and C_2 in $S(D)$, then the two corresponding vertices v_1 and v_2 in $G_S(D)$ are connected by an edge with weight k. It is known that two alternating knot diagrams are topologically equivalent if and only if there exists a sequence of so called flyping moves that transform diagrams the other, and the flyping moves on the diagram can be interpreted using moves on the Seifert graphs of these diagrams. The ultimate goal is to determine, for a giving minimum crossing number, how many distinct (oriented) alternating knots there are with that minimum crossing number.

References

[1] Yuanan Diao, Michael Lee Finney, Dawn Ray. The number of oriented rational links with a given deficiency number, *Journal of Knot Theory and its Ramifications*, **30**(9) 2150065 (20 pp), 2021.

[2] Y. Diao, C. Ernst, G. Hetyei and P. Liu. A diagrammatic approach to the braid index of alternating links, *Journal of Knot Theory and its Ramifications*, **30**(5), 2150035 36 pp), 2021.

[3] Y. Diao and V. Pham. Writhe-like invariants of alternating links, *Journal of Knot Theory and its Ramifications*, **30**(1) 2150004 (17 pp), 2021.

[4] Dawn Ray and Yuanan Diao. The average genus of oriented rational links with a given crossing number, *Journal of Knot Theory and its Ramifications*, **32**(02) 2350010 (12 pp), 2023.