Lecture

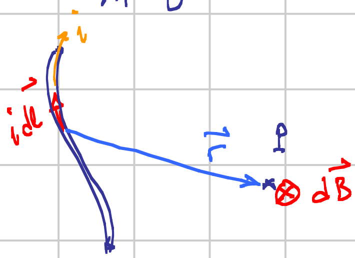
moving charge(s) $\Rightarrow$ $\vec{B}$ $\Rightarrow$ other moving charge(s)

Note Title 3/21/2006

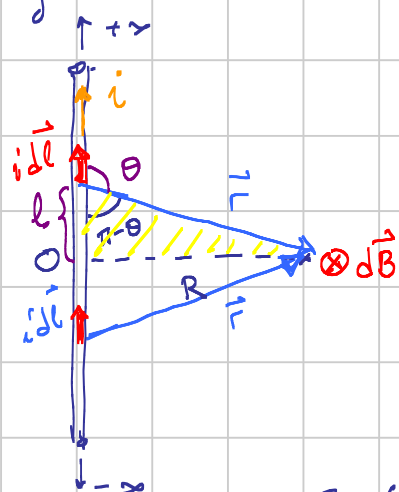
$\vec{F}_B$: magnetic force

Biot-Savart law

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\overbrace{i\,d\vec{l}}^{\text{source}} \times \vec{r}}{r^3}$$



Example: $\vec{B}$ field of a infinite, straight current carrying wire.



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{l}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\frac{i\,dl\,\cancel{r}\sin\theta}{r^{\cancel{3}\,2}}$$

$$B = \int dB = \frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{i\,dl\,\sin\theta}{r^2}$$

$\sin(\pi-\theta) = \sin\theta$, $r = \sqrt{l^2+R^2}$

$$\sin\theta = \sin(\pi-\theta) = \frac{R}{\sqrt{l^2+R^2}}$$

$$B = \frac{\mu_0}{\cancel{4}_{2}\pi}\,\cancel{2}\int_0^{\infty}\frac{i\,dl}{(l^2+R^2)}\,\frac{R}{\sqrt{l^2+R^2}}$$

$$B = \frac{\mu_0 i R}{2\pi}\int_0^{\infty}\frac{dl}{(l^2+R^2)^{3/2}},$$

let $l = R\tan\phi$

$$dl = \frac{R}{\cos^2\phi}\,d\phi$$

$$(l^2+R^2)^{3/2} = \left[R^2\tan^2\phi + R^2\right]^{3/2}$$

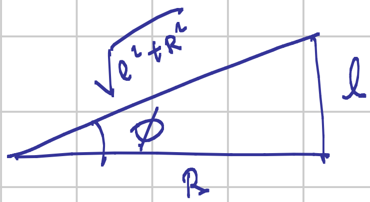
$$= \left[R^2\left(\frac{\sin^2\phi}{\cos^2\phi}+1\right)\right]^{3/2}$$

$$= R^3\left(\frac{\sin^2\phi + \cos^2\phi}{\cos^2\phi}\right)^{3/2}$$

$$= R^3\left(\frac{1}{\cos^2\phi}\right)^{3/2} = \frac{R^3}{\cos^3\phi}$$

$$B = \frac{\mu_0 i \cancel{R}}{2\pi}\int_{\phi_1}^{\phi_2}\frac{\cancel{R}/\cancel{\cos^2\phi}\, d\phi}{R^{\cancel{3}}/\cancel{\cos^3}\phi} = \frac{\mu_0 i}{2\pi R}\int_{\phi_1}^{\phi_2}\cos\phi\, d\phi$$

$$B = \frac{\mu_0 i}{2\pi R}\sin\phi\Big|_{\phi_1}^{\phi_2}, \qquad l = R\tan\phi$$



$\Longrightarrow$

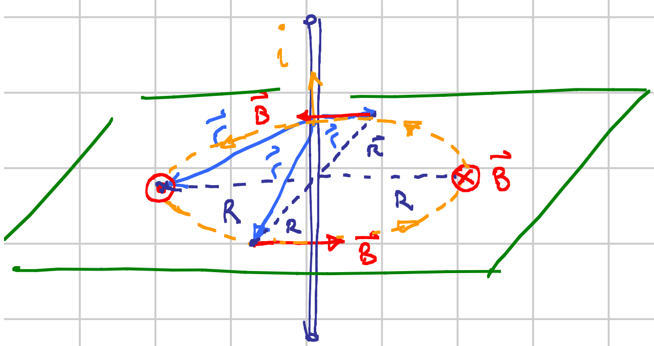
$$\tan\phi = \frac{l}{R}$$

$$\sin\phi = \frac{l}{\sqrt{l^2+R^2}}$$

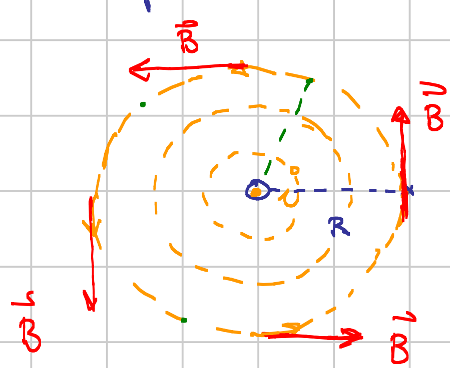
$$B = \frac{\mu_0 i}{2\pi R}\frac{l}{\sqrt{l^2+R^2}}\Bigg|_0^\infty$$

$$= \frac{\mu_0 i}{2\pi R}\frac{\cancel{l}}{\cancel{l}\sqrt{1+\frac{R^2}{l^2}}}\Bigg|_0^\infty = \frac{\mu_0 i}{2\pi R}(1-0)$$

$$\vec{B} = \frac{\mu_0 i}{2\pi R} \text{ into the plane}$$



top view



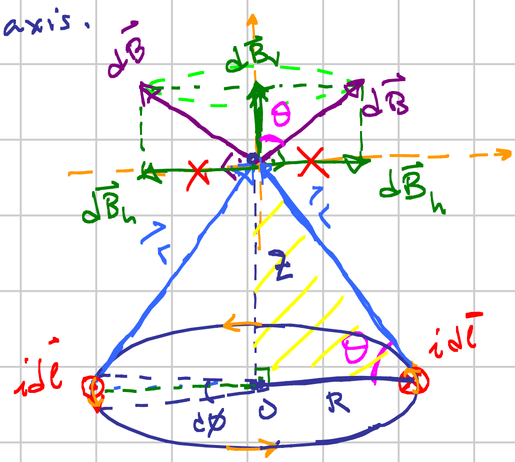
In order to determine the direction of $\vec{B}$ field generated by a straight current carrying wire:

① hold your Right Hand thumb in the direction of flow of current along the wire

② then curl your RH fingers about the thumb (wire), the sense rotation gives the direction of $\vec{B}$ field lines circling about the wire in the form of concentric circles.

③ $\vec{B}$ field is tangent to the field line passing through the point of interest.

Example: $\vec{B}$ field of a current loop along its axis.



$$d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,d\vec{l}\times\vec{r}}{r^3}$$

$dB_{horizontals}$ cancel due to the symmetry

B_{total} = Sum of all $dB_{verticals}$

$$B = \int dB_{verticals} = \int dB\cos\theta, \quad \cos\theta = \frac{R}{r}$$

$$r = \sqrt{z^2+R^2}, \quad d\vec{B} = \frac{\mu_0}{4\pi}\,\frac{i\,d\vec{l}\times\vec{r}}{r^3}$$

$$dB = \frac{\mu_0}{4\pi}\,\frac{i\,dl\,\cancel{r}\sin 90^\circ}{r^{\cancel{3}\,2}} = \frac{\mu_0\, i\, dl}{4\pi r^2}$$

$\vec{dl}$ R dØ

$$dl = R\,d\emptyset$$

const. dl

$$B = \int \frac{\mu_0\, i\, R\,d\emptyset}{4\pi(z^2+R^2)}\,\frac{R}{\sqrt{z^2+R^2}}$$

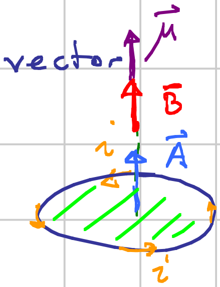
A: area of the loop

$$B = \frac{\mu_0\, i\, R^2}{4\pi(z^2+R^2)^{3/2}}\underbrace{\int_0^{2\pi} d\emptyset}_{2\pi} = \frac{\mu_0\, i\,R^2\,\cancel{2\pi}}{\cancel{4\pi}_{\,2}\,(z^2+R^2)^{3/2}}$$

$$\vec{B} = \frac{\mu_0 i A}{2\pi (z^2+R^2)^{3/2}} \hat{k}$$

for N turns: $$\vec{B} = \frac{\mu_0 N i \vec{A}}{2\pi (z^2+R^2)^{3/2}}$$

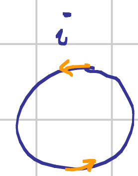
$\vec{\mu}_B$: magnetic dipole moment vector



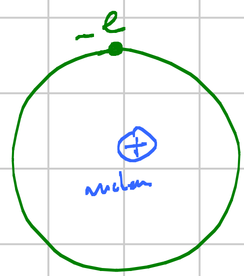
$$\boxed{\vec{\mu}_B = N i \vec{A}}$$

$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi (z^2+R^2)^{3/2}}$$

for $z \gg R$, then $\frac{R}{z} \ll 1$



$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi z^3 \left(1 + \cancel{\frac{R^2}{z^2}}\right)^{3/2}}$$



$$\boxed{\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi z^3}}$$