Lecture

Note Title 4/4/2006

moving charge(s) $\Rightarrow$ $\vec{B}$ $\Rightarrow$ other moving charge(s)

Biot-Savart law

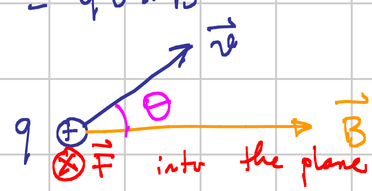
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{i\,d\vec{l}\times\vec{r}}{r^3}$$

Ampere's law

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0\, i_{enc}$$

for a single moving charge in an external $\vec{B}$ field

$$\vec{F} = q\vec{v}\times\vec{B}$$



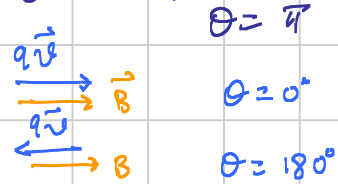
$$|\vec{F}| = F = qvB\sin\theta$$

—o—

$F = qvB\sin\theta$

① $F \propto q,\ v,\ B$

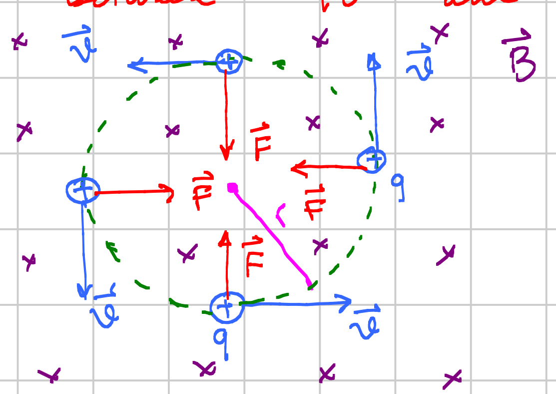
② $F_{min} = 0$ occurs when $\sin\theta = 0 \Rightarrow \theta = 0°$, $\theta = \pi$



③ $F_{max} = qvB$, occurs when $\theta = \frac{\pi}{2}$ rad. $= 90°$

—o—

Motion of a positive charged particle moving in an external unifor $\vec{B}$ field such that the angle between $q\vec{v}$ and $\vec{B}$ $= 90°$



$$F_{centripetal} = m\frac{v^2}{r}$$

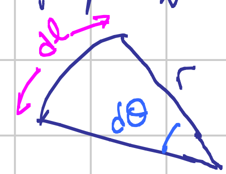
m: mass
v: speed
r: radius of the orbit.

$$F_{centripetal} = F_{magnetic}, \quad F_B = qvB\underbrace{\sin 90^\circ}_{1}$$

$$m\frac{v^{\cancel{2}}}{r} = q\cancel{v}B, \quad \boxed{r = \frac{mv}{qB}}$$

ω: angular velocity (angular frequency)

$$\omega = \frac{d\theta}{dt}$$



$$v = \omega r$$

$$dl = r\,d\theta$$

$$\omega = \frac{v}{r}$$

$$\underbrace{\frac{dl}{dt}}_{v} = r\underbrace{\frac{d\theta}{dt}}_{\omega}$$

$$\omega = \frac{\cancel{v}}{\frac{m\cancel{v}}{qB}} = \frac{qB}{m}$$

$$\boxed{\omega = \frac{qB}{m}}, \quad \omega = 2\pi f, \quad \omega = \frac{2\pi}{T}$$

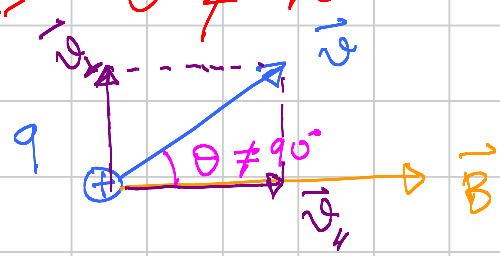
f: linear frequency
T: period

$$f = \frac{1}{T}$$

$$\boxed{f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\frac{qB}{m}}$$ → cyclotron-frequency

$$\boxed{T = 2\pi\frac{m}{qB}}$$

for $\theta \neq 90^\circ$



$$\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

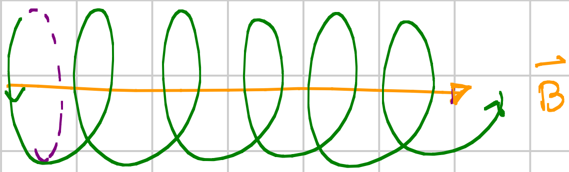
$$\vec F = q(\vec v_\perp + \vec v_\parallel)\times\vec B$$

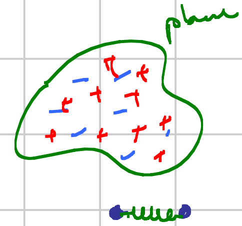
$$\vec F = q\,\vec v_\perp\times\vec B + q\,\vec v_\parallel\times\vec B$$

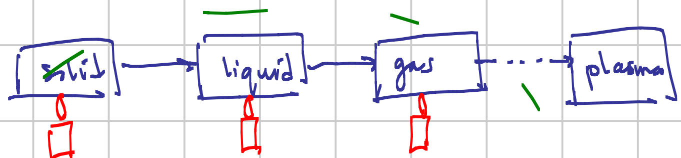
$$F = q\,v_\perp B\overbrace{\sin 90^\circ}^{1} + q\,v_\parallel B\,\underbrace{\cancel{\sin 0^\circ}}_{0}$$

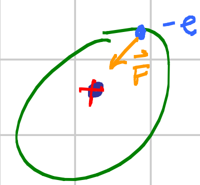
$$F = q\,v_\perp B$$

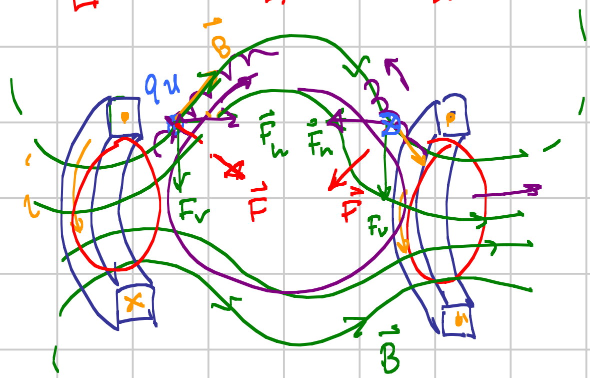
$\vec v_\parallel$ = constant

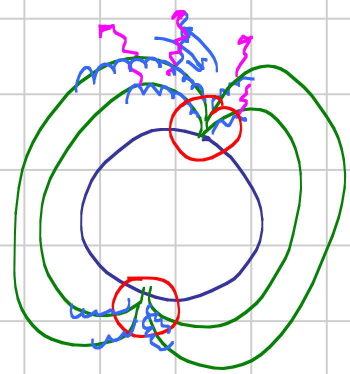








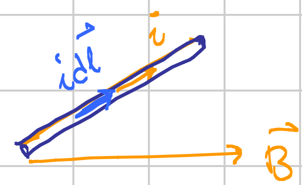




for a single moving charge

$$\vec F_B = q\,\vec v\times\vec B$$

$$i = \frac{dq}{dt} \Rightarrow dq = i\,dt$$



$$dq = i\,dt$$

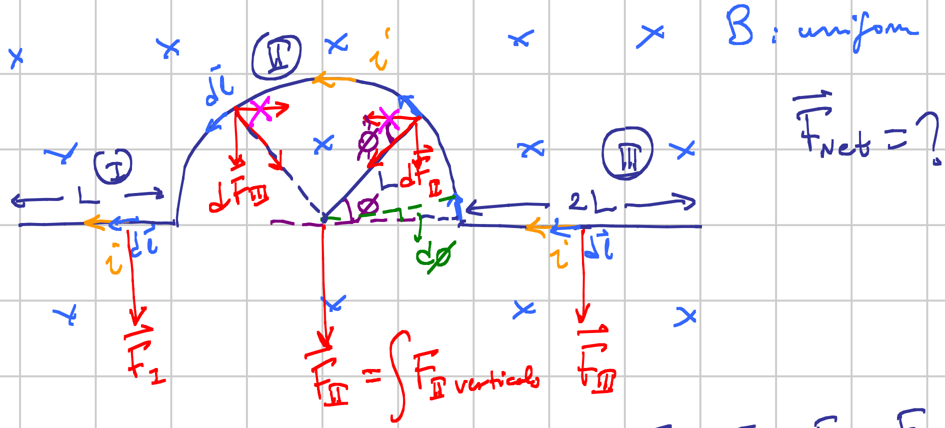
$$\vec v = \frac{d\vec\ell}{dt}$$

$$d\vec F_B = dq\,\vec v\times\vec B$$

$$d\vec F_B = i\,\cancel{dt}\,\frac{d\vec\ell}{\cancel{dt}}\times\vec B$$

$$\boxed{\begin{aligned} d\vec F_B &= i\,d\vec\ell\times\vec B \\ \vec F_B &= \int d\vec F_B = \int_0^\ell i\,d\vec\ell\times\vec B \end{aligned}}$$

Example:



$$F_T = F_I + F_{II} + F_{III}$$

$$\vec{F}_I = \int i\,d\vec{l} \times \vec{B}$$

const

$$F_I = \int i\,dl\,B \underbrace{\sin 90^\circ}_{1} = iB\int_0^L dl = iBL$$

$$F_{III} = iB\int_0^{2L} dl = 2iBL$$

$$F_{II} = \int dF_{II}\ \text{verticals}$$

dF_{II} horizontals cancel due to the symmetry

const

$$F_{II} = \int dF_{II}\sin\phi = \int i\,\underbrace{dl}_{L\,d\phi}\,B\underbrace{\sin 90^\circ}_{1}\sin\phi$$

$$F_{II} = iBL\int_0^{\pi}\sin\phi\,d\phi$$

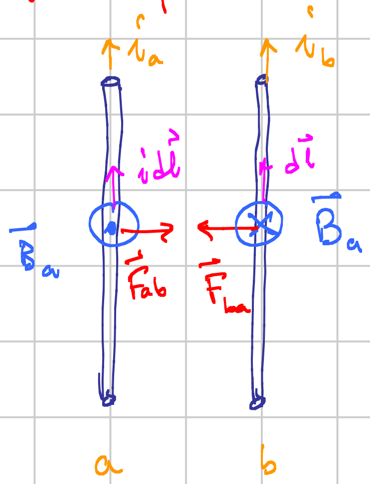
$$F_{II} = iBL\,(-\cos\phi)\Big|_0^{\pi} = iBL\,(+1+1) = 2iBL$$

$$F = F_I + F_{II} + F_{III} = iBL + 2iBL + 2iBL$$

$$\boxed{\vec{F} = 5iBL\,(-\hat{j})}$$

y x

Two parallel wires



F_{ba}: force on wire "b" due to wire "a"