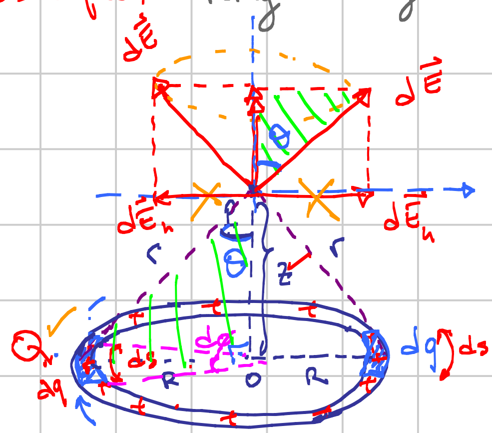
Lecture
$\vec{E}$ field (continued)

Note Title 1/24/2006

Example: Ring charge distribution (Continued)



$dE_{\text{horizontal}}$ cancel due to the symmetry

$$E = \int dE_{\text{vertical}}$$

$$dE_{\text{vertical}} = dE\cos\theta$$

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

$$r = \sqrt{z^2+R^2}\,, \quad \cos\theta = \frac{z}{r}$$

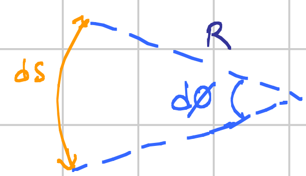
$$\cos\theta = \frac{z}{\sqrt{z^2+R^2}}$$

$$E = \int \underbrace{\frac{dq}{4\pi\epsilon_0\,(z^2+R^2)}}_{dE}\;\underbrace{\frac{z}{\sqrt{z^2+R^2}}}_{\cos\theta}$$

?

$$dq = \lambda\, ds\,, \qquad \lambda = \frac{Q}{2\pi R}$$

$$ds = R\,d\phi$$



$$ds = R\,d\phi$$

$$d\phi = \frac{ds}{R}$$

if $ds = R$

$$d\phi = \frac{R}{R} = 1 \text{ radian}$$

const.

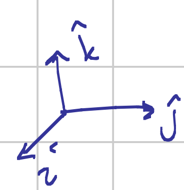
$$E = \int \frac{\overbrace{\frac{Q}{2\pi R}}^{\lambda}\;\overbrace{R\,d\phi}^{ds}\;z}{4\pi\epsilon_0\,(z^2+R^2)^{3/2}}$$

$$E = \frac{Q}{(2\pi)\,4\pi\epsilon_0}\,\frac{z}{(z^2+R^2)^{3/2}}\,\underbrace{\int_0^{2\pi} d\phi}_{\phi\big|_0^{2\pi}}$$

$$\phi\Big|_0^{2\pi} \rightarrow 2\pi - 0 = 2\pi$$

$$E = \frac{Q\,z}{\cancel{(2\pi)}(4\pi\epsilon_0)(z^2+R^2)^{3/2}}\,\cancel{2\pi}$$

$$\boxed{\vec{E} = \frac{Q\,z}{4\pi\epsilon_0\,(z^2+R^2)^{3/2}}\,\hat{k}}$$

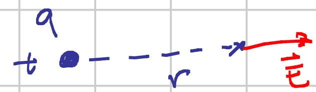


$E = ?$ for $z \gg R$, $\Rightarrow \frac{R}{z} \ll 1$

$$E = \frac{Q \cancel{z}}{4\pi\epsilon_0 z^{\cancel{3}\,2} \left(1 + \cancel{\frac{R^2}{z^2}}\right)^{3/2}} \qquad z^{\cancel{2}}\,\frac{z^3}{\cancel{z}} = z^3$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\epsilon_0 z^2}\,\hat{k}}$$

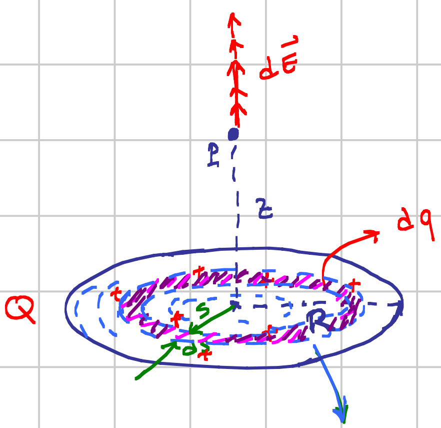
point charge $\vec{E} = \frac{q}{4\pi\epsilon_0 r^2}$



$\Downarrow$

Behaves like a point charge for $z \gg R$

Example: $\vec{E}$ field of a disc charge distribution with radius "R", and charge $+Q$, z- distance from the center along the axis.



ring charge		disc charge
Q	$\longrightarrow$	dq
z	$\longrightarrow$	z
R	$\longrightarrow$	s

$dq = \sigma\, dA$

dA: surface area of the ring

$$\sigma = \frac{Q}{\pi R^2}$$

$$dA = 2\pi s\, ds$$

$$dq = \sigma\, dA = \frac{Q}{\cancel{\pi} R^2}\, 2\cancel{\pi} s\, ds \qquad E_{ring} = \frac{Qz}{4\pi\epsilon_0 (z^2+R^2)^{3/2}}$$

$$E = \int dE_{ring} = \int \frac{dq\ z}{4\pi\epsilon_0 (z^2+s^2)^{3/2}}$$

const.

$$E = \int \frac{\frac{Q}{R^2} 2s\,ds\ z}{4\pi\epsilon_0 (z^2+s^2)^{3/2}}$$

$$E = \frac{Qz}{4\pi\epsilon_0 R^2}\int_0^R \frac{2s\,ds}{(z^2+s^2)^{3/2}}$$

du

$u^{3/2}$

let $z^2+s^2 = u$

then $2s\,ds = du$

$$E = \frac{Qz}{4\pi\epsilon_0 R^2}\int_{u_1}^{u_2}\frac{du}{u^{3/2}} = \frac{Qz}{4\pi\epsilon_0 R^2}\int_{u_1}^{u_2} u^{-3/2}\,du$$

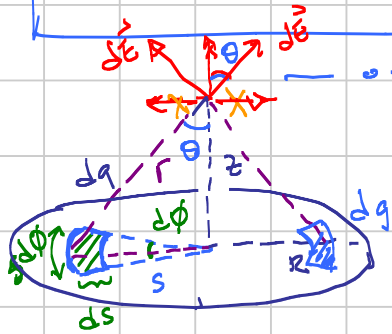
$$E = \frac{Qz}{4\pi\epsilon_0 R^2}\left.\frac{u^{-1/2}}{-1/2}\right|_{u_1}^{u_2} = \frac{Qz}{4\pi\epsilon_0 R^2}\left.\left(-2\frac{1}{\sqrt{u}}\right)\right|_{u_1}^{u_2}$$

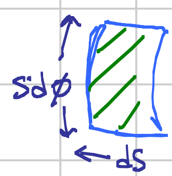
$$E = \frac{Qz}{\cancel{4}\pi\epsilon_0 R^2}\left.\left(-\cancel{2}\frac{1}{\sqrt{z^2+s^2}}\right)\right|_0^R$$

2

$$E = \frac{Qz}{2\pi\epsilon_0 R^2}\left\{\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right\}$$

$$\boxed{\vec{E} = \frac{Q}{2\pi\epsilon_0 R^2}\left\{1 - \frac{z}{\sqrt{z^2+R^2}}\right\}\hat{k}}$$





$dA = s\,ds\,d\phi$

$$dq = \frac{Q}{\pi R^2}dA = \frac{Q}{\pi R^2}s\,ds\,d\phi$$

$$dE = \frac{dq}{4\pi\epsilon_0 r^2}$$

$$r^2 = (z^2+s^2)$$

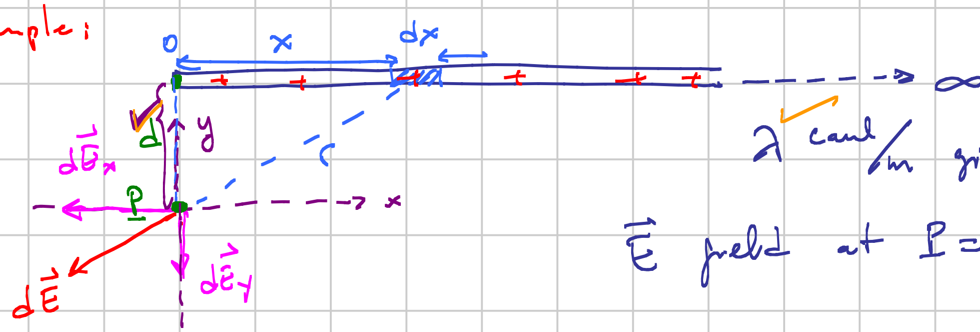
$$\cos\theta = \frac{z}{(z^2+s^2)^{1/2}}$$

$$E = \int \underbrace{dE\cos\theta}_{dE\ \text{verticals}}$$

$$E = \int_0^R\int_0^{2\pi}\frac{\frac{Q}{\pi R^2}s\,ds\,d\phi}{4\pi\epsilon_0(z^2+s^2)}\frac{z}{\sqrt{z^2+s^2}}$$

$$E = \frac{\cancel{2\pi}\, Q z}{\cancel{\pi} R^2\, 4\pi\epsilon_0} \int_0^R \frac{2 s\, ds}{(z^2+s^2)^{3/2}} = \text{same result as above}$$

Example:



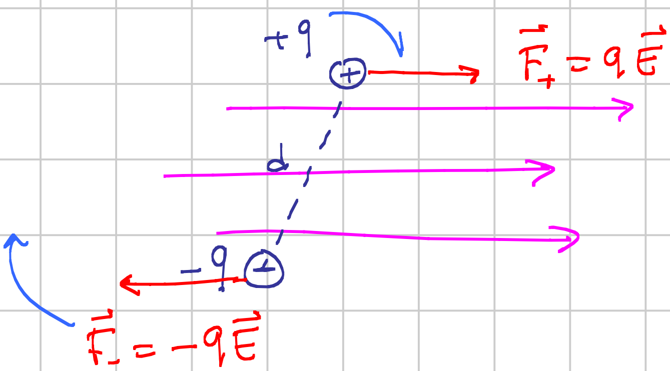
λ C/m given

$\vec{E}$ field at P = ?

Example: Electric dipole in an external $\vec{E}$ field.

$$\underbrace{q \Rightarrow \vec{E}}_{\text{coulomb's law}} \Rightarrow \text{other } q$$

$\vec{F} = q_{other}\, \vec{E}$



$\vec{E}$: External $\vec{E}$ field