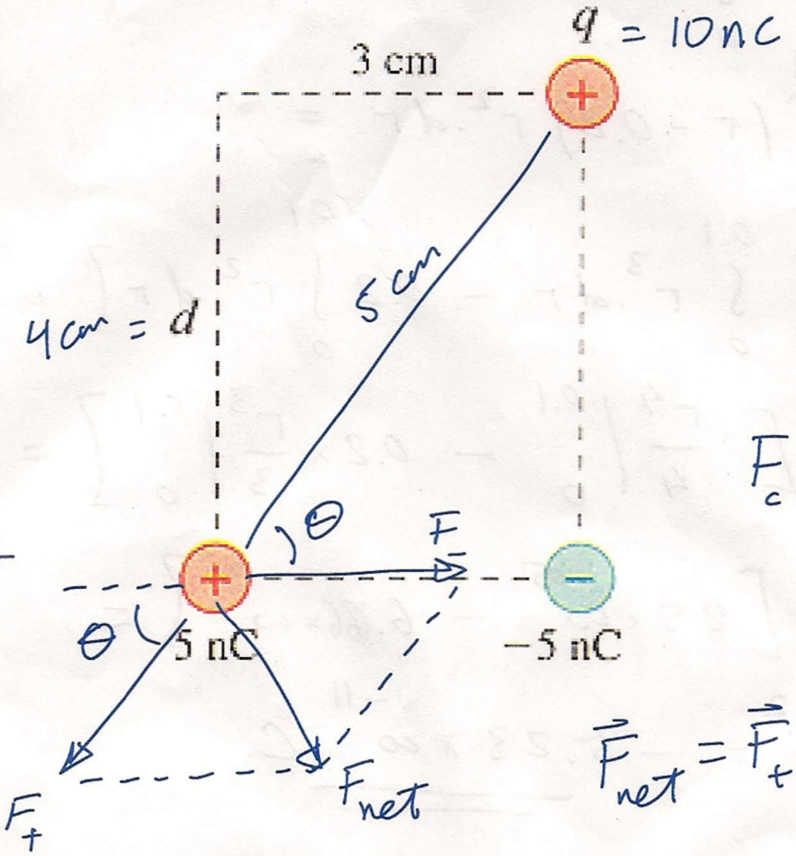
1. Consider the three charges in figure below, in which $d = 4$ cm, $q = 10$ nC, and the positive x-axis points to the right. What is the force $\vec{F}$ on the +5 nC charge? Give your answer as a magnitude and a direction.

1. Sketch (3)
2. F_+ (3)
3. F_- (1)
4. Magnitude F_{net} (1)
5. Direction F_{net} (2)



$$F_c = k\frac{q_1 q_2}{r^2}$$

$$\vec{F}_{net} = \vec{F}_+ + \vec{F}_-$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53^\circ$$

$$\vec{F}_+ = 9\times10^9 \; \frac{5\times10^{-9} \times 10\times10^{-9}}{25\times10^{-4}} \left(-\cos 53^\circ \,\hat{i} - \sin 53^\circ \,\hat{j}\right)$$

$$\therefore \; \vec{F}_+ = -1.08\times10^{-4}\,\hat{i} - 1.44\times10^{-4}\,\hat{j}$$

$$\vec{F}_- = 2.5\times10^{-4}\,\hat{i}$$

$$\rightarrow \; \vec{F}_{net} = (1.42\,\hat{i} - 1.44\,\hat{j})\ N$$

$$(2.02\times10^{-4}\ N,\ -45.4^\circ \text{ below } x\text{-axis})$$

2. An insulating sphere with radius R = 20 cm has a volume charge density of $\rho = 100\,(r - R)$ nC/m^3. What is the electric field strength and direction at a point r = 10 cm from the center?

$dq = \rho \cdot dV = 100(r-R)\cdot 4\pi r^2 \cdot dr$ Use SI system of units

(7) $q_{enc} = \int dq = \int_0^{0.1} 100(r-0,2)\, 4\pi r^2 \cdot dr \, (\times 10^{-9}) =$

$= 10^{-7} \times 4\pi \int_0^{0.1} (r-0.2)\, r^2 \cdot dr =$

$= 4\pi \times 10^{-7} \times \left[\int_0^{0.1} r^3 \cdot dr - 0.2 \int_0^{0.1} r^2 \cdot dr \right] =$

$= 12.57 \times 10^{-7} \left[\frac{r^4}{4}\Big|_0^{0.1} - 0.2 \times \frac{r^3}{3}\Big|_0^{0.1} \right] =$

$= 12.57 \times 10^{-7} \left[2.5 \times 10^{-5} - 6.66 \times 10^{-5} \right] =$

$\sim 52.3 \times 10^{-12} = -5.23 \times 10^{-11}$ C

Gauss' Law: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$E \cdot 4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$.

(3) $E = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2} = k \frac{q_{enc}}{r^2} = -9 \times 10^9 \frac{5.23 \times 10^{-11}}{10^{-2}} =$

$\boxed{-47.1 \frac{N}{C}}$

Direction toward the center of sphere

Grading:

q_{enc} (7)

E (3)

3. Two spherical drops of mercury each have a charge of 0.8 nC and a potential of 360 V at the surface. The two drops merge to form a single drop. What is the potential at the surface of the new drop?

$$V = k\frac{q}{r} \longrightarrow r = k\frac{q}{V}$$

ie radius of either sphere

$$r = \frac{9\times10^{9} \times 0.8\times10^{-9}}{360}$$

(3) $\longrightarrow$ $r = 2\times10^{-2}\,m$

$\therefore$ Volume of big drop $= 2\left(\frac{4}{3}\pi r^3\right)$

$$= \frac{4}{3}\pi r_0^{3}$$

where r_0 is the radius of big drop, $r_0 = \sqrt[3]{2}\, r = 2.52\,cm$

(3) $\longrightarrow$

Total charge of new drop $= 1.6\times10^{-9}\,C$

(2) $\longrightarrow$

$\therefore$ Potential at the surface of big drop $V_0 = \dfrac{9\times10^{9} \times 1.6\times10^{-9}}{2.52\times10^{-2}}$

(2) $\longrightarrow$ $V_0 = 570\,V$

4. The electric potential on a horizontal plane having 10-m long sides is given by

$$V = 3{,}000 - 5x^3 + 10\,x^2 - 2\,y^2 \text{ Volts}$$

Where x and y are measured from the origin of the coordinate system. A positive point charge of $+1\ nC$ with a mass of $1\ g$ is introduced into the chamber at position ($x = 2m,\ y = 2m$) with an initial speed of $0\ m/s$. What is the acceleration of the point charge at its initial position?

$$\vec{E} = -\vec{\nabla} V$$

$$= -\left[(-15x^2 + 20x)\,\hat{i} - 4y\,\hat{j}\right]$$

$$= (15x^2 - 20x)\,\hat{i} + 4y\,\hat{j}$$

(7) $\rightarrow$ $$\vec{E}\Big|_{(2,2)} = (20\,\hat{i} + 8\,\hat{j})\ N/C$$

$$\vec{a} = \frac{q\vec{E}}{m} = (20\,\hat{i} + 8\,\hat{j})\,10^{-6}\ m/s^2$$

(3) $\rightarrow$ $$\therefore\ \vec{a} = \left(21.5 \times 10^{-6}\ m/s^2,\ 21.8^\circ \text{ above } x\text{-axis}\right)$$

Some people can use:

$$|E| = \sqrt{(20^2) + (8)^2} = \sqrt{464} = 21.5\ \frac{N}{c}$$

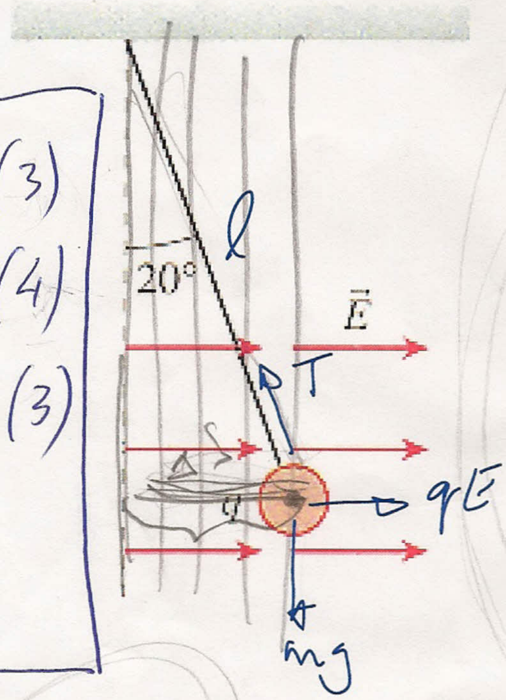
$$|a| = \frac{q\,|E|}{m} = \frac{10^{-9} \times 21.54}{10^{-3}} = 21.5 \times 10^{-6}\ \frac{m}{s^2}.$$

5. An electric field $\vec{E} = 90000\,\hat{i}$ N/C causes the ball in figure below to hang at a *20°* angle, where the length of the massless rigid rod connecting the ball to the pivot point is *10 cm,* and the ball's mass is *5.0 g*. If the ball is swung back to the perpendicular position indicated by the dotted line, what is the change in its electric potential energy?

Sketch Forces (3)

Calc. q (4)

Calc. ΔU (3)



$$T\cos\theta - mg = 0$$

$$T\cos\theta = mg \quad (1)$$

$$-T\sin\theta + qE = 0$$

$$T\sin\theta = qE \quad (2)$$

$$\frac{(2)}{(1)} \rightarrow \tan\theta = \frac{qE}{mg}$$

$$\therefore\ q = 1.98 \times 10^{-7}\ C$$

$\Delta V \cdot q$

$\Delta V = E \cdot \Delta s$

$\Delta U = E \cdot q \cdot \Delta s$

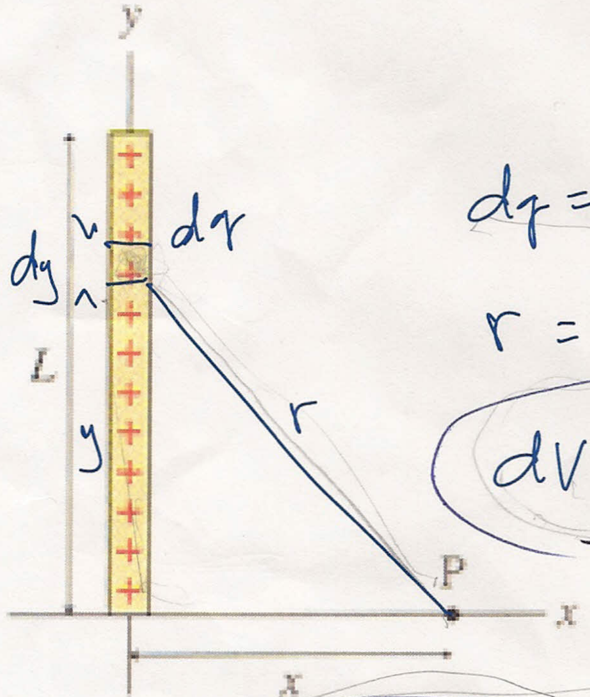
q = ?

$$\Delta U_e = -\int_i^f \vec{F}_e \cdot d\vec{s}$$

$$= +(qE)\, l \sin\theta$$

i.e. $\Delta U_e = 6.09 \times 10^{-4}\ J$

6. The figure below shows a thin rod of length L with a linear charge distribution of $\lambda = \lambda_0 y$, where λ_0 is a constant. Find an expression for the electric potential at distance x from the end of the rod.



$$dq = \lambda\, dy = \lambda_0 y\, dy$$

(3)

$$r = (y^2 + x^2)^{1/2}$$

$$dV = k\frac{dq}{r} = k\lambda_0 \frac{y\,dy}{(y^2+x^2)^{1/2}}$$

$$V = \int dV = \int_0^L k\lambda_0 \frac{y\,dy}{(y^2+x^2)^{1/2}} = k\lambda_0 (y^2+x^2)^{1/2}\Big|_0^L$$

(4)

$$\therefore\ V = k\lambda_0\left[(L^2+x^2)^{1/2} - x\right]$$

(2) for taking $\int$.

Taking an integral:

$$\int \frac{y\,dy}{(y^2+x^2)^{1/2}} = \frac{1}{2}\int \frac{dz}{z^{1/2}} = \frac{1}{2}\int z^{-\frac{1}{2}}\,dz = 2\left(\frac{1}{2}\right) z^{\frac{1}{2}} =$$

$$= \sqrt{y^2+x^2}$$

$$y^2 + x^2 = z$$

$$dz = 2y\cdot dy$$