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PROBLEMS

1. Consider a system with $E_1 = 166658.484\ \text{cm}^{-1}$ and $E_2 = 148259.749\ \text{cm}^{-1}$.
   a. What is the wavelength (in μm) of the photons emitted or absorbed?
   b. What is the frequency (in Hz) of the emission?
   c. If the output has a spectral width of 0.2Å, what is the bandwidth in Hz?
2. Consider a system with $E_1 = 1.0$ eV, $E_2 = 1.1$ eV, and $A = 2.0 \times 10^6\ \text{s}^{-1}$. Let the density of occupied states in the levels be N_1 and N_2, respectively, and let $N_t = N_1 + N_2 = 2 \times 10^{16}\ \text{cm}^{-3}$.
   a. What is the wavelength (in μm) of the E_2–to–E_1 transition?
   b. What is N_2 at 300 °K?
   c. The system is now pumped to a steady state with $N_1/N_2 = 3$. How much pump power (in W/cm³) is absorbed?
   d. How much power (in W/cm³) is radiated by the spontaneous emission process in the steady state mentioned in (c)?
3. For a certain two-level system at thermal equilibrium at 600 °K, the densities of occupied states are $N_1 = 10^{+24}\ \text{cm}^{-3}$ and $N_2 = 10^{+14}\ \text{cm}^{-3}$. What is the wavelength (in μm) of the photons absorbed or emitted by the system?
4. Find the ratio N_2/N_1 of a two-level system at thermal equilibrium at room temperature (300 °K), if the transition wavelength falls in:
   a. Near IR region, say 1.0 μm.
   b. IR region, say 10.0 μm.
   c. Millimeter region, say 1.0 mm.
5. The ratio N_2/N_1 of a two-level system in thermal equilibrium at room temperature is $1/e$. Calculate the transition frequency of the system.
6. Given a two-level system at room temperature. If the number of photons radiated due to the stimulated emission process is the same as that due to the spontaneous emission process, what is the wavelength of the radiation? If the wavelength of radiation is 10.6 μm, what is the operating temperature of the system?
7. Consider a two-level system with $E_1 = 150856\ \text{cm}^{-1}$ and $E_2 = 159534\ \text{cm}^{-1}$. At $T = 300$ °K, $N_1 = 1.0 \times 10^{20}\ \text{cm}^{-3}$.

a. Calculate the wavelength (in μm) of the photons absorbed or radiated by the system.

b. Calculate N_2.

8. Consider a three-level system in which radiative decay takes place between the E_3–to–E_2 transition (*not* the E_2–to–E_1 transition), and pumping occurs from level E_1 to level E_3.

   a. Write the rate equations for this system. Identify each term.

   b. Can population inversion be achieved in the system? If not, why not? If it can, establish the condition for population inversion.

9. Derive expressions for the following parameters of a four-level system (Figure 3.4b).

   a. The critical population inversion, ΔN_c.

   b. The steady-state population inversion ΔN_0.

   c. The critical pumping transition probability, A_{Pc}.

   d. The steady-state optical power output, P_1 and P_2, through mirrors 1 and 2, respectively.

10. Consider a usual three-level system (Figure 3.4a). What is the pumping transition probability A_P required to achieve $N_2 = 0.1N_1$ in the steady state?

11. Consider the three-level system shown in Figure 3.4a, with the following energy levels and Einstein A coefficients:

    $E_1 = 0$ eV, $\quad E_2 = 1.2$ eV, $\quad$ and $\quad E_3 = 2.0$ eV

    $A_{32} = 5.0 \times 10^7\ \text{s}^{-1}$, $\quad A_{31} = 2.5 \times 10^7\ \text{s}^{-1}$, $\quad$ and $\quad A_{21} = 1.0 \times 10^6\ \text{s}^{-1}$

    a. What are the wavelengths (in μm) of the E_3–to–E_2, E_2–to–E_1, and E_3–to–E_1 transitions?

    b. At thermal equilibrium at 300 °K, what is the ratio $N_1{:}N_2{:}N_3$?

    c. At what temperature would $N_1{:}N_2 = 2{:}1$? What is the ratio $N_2{:}N_3$ at this temperature?

    d. Calculate B_{32}, B_{31}, and B_{21}. Take $\delta f = 1$ Hz.

    e. What is the lifetime of level 3?

    f. What percentage of states in level 3 would drop from level 3 to level 2?

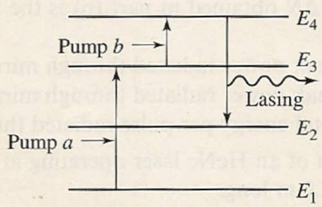
12. The three-level system discussed in problem 11 is pumped by some means such that a steady-state value of $N_3 = 1 \times 10^{14}$ states/cm³ is maintained. Assume that $N_t \approx 1 \times 10^{19}$ states/cm³ and pumping is very weak.

    a. What is the steady-state value of N_2?

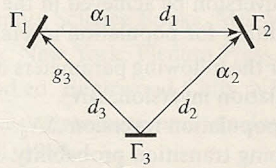
    b. How much pumping power (in W/cm³) is absorbed?

    c. How much power is radiated spontaneously in the 3–to–2 transition?

13. A four-level system is shown in the figure. Pumping is done with two sources, a and b, which maintain photon densities $\Phi_{Pa}\delta f$ and $\Phi_{Pb}\delta f$, with wavelengths corresponding to the E_1–to–E_3 and E_3–to–E_4 transitions, respectively. Lasing occurs between levels E_4 and E_2, with a photon density of $\Phi\ \delta f$. Write the rate equations for levels E_2 and E_4.



14. Consider a ring cavity with three mirrors as shown in the figure. The lengths of the three sections are d_i, where $i = 1, 2,$ and 3. The reflectivities of the mirrors are Γ_1, Γ_2, and Γ_3. The net losses of sections 1 and 2 are represented by attenuation constants α_1 and α_2. The net gain in section 3 is given by a gain constant g_3. The indices of these sections are approximately 1. Derive an expression for the rate of change of the photon density in the ring cavity for waves circulating in the clockwise direction.



15. The optical cavity parameters of a three-level laser are:
$\Gamma_1 = 0.95$, $\Gamma_2 \approx 0.99$, $\alpha = 0.01\text{cm}^{-1}$,
$d' = 10.0$ cm, $d = 50.0$ cm, $\mathcal{A} = 0.8$ cm^2, $V_a = \pi \times 10^{-2}$.
At the lasing wavelength of 0.693 μm, the parameters of the lasing medium are:
$B_{21} = 1.9 \times 10^{-9}$ m^{3}s^{-1}, $N_t = 1.6 \times 10^{19}$ cm^{-3}, $n = 1.76$,
$\tau_{21} = 3.0 \times 10^{-3}$ s, $\tau_{31} = 3.33 \times 10^{-6}$ s, $\tau_{32} = 5.0 \times 10^{-8}$s.
   a. Calculate the critical population inversion ΔN_c.
   b. Calculate the critical pumping transition probability A_{Pc}.
   c. What is the steady-state population inversion ΔN_0?
   d. Calculate the power output from the mirrors when the medium is pumped to $4A_{Pc}$.

16. Consider a Q-switched ruby laser ($\lambda = 0.6934$ μm). The active medium is a ruby rod doped with Cr^{+3} ions, resulting in a total population of $N_t = 1.6 \times 10^{19}$ cm^{-3}. The index of refraction of ruby is 1.76, $\alpha = 1.$ m^{-1}, d = 0.50 m, and $d' = 0.015$ m. Additional material parameters can be found in section 6 of this chapter. The cross-sectional area of the cavity is 1.2 cm^2. The radii of curvature of the mirrors are chosen such that a beam waist w_o of 5.0×10^{-4} m is produced. To account for the fact that πw_o^2 is much smaller than the cross-sectional area $\mathcal{A}$ of the cavity, a factor of $\pi w_o^2/\mathcal{A}$ should be added to the expression for the effective mode volume. Therefore, the effective mode volume V_a becomes $V_a = \pi w_o^2 d' n/(d_e \mathcal{A})$, where w_o, d', and d are given in meters.

   First, consider the *pumping stage.* During the pumping stage, the effective mirror reflectivities are $\Gamma_1 = 0.9$ and $\Gamma_2 = 0.1$.
   a. Calculate B_{12}.
   b. If the system is pumped at 10 times the critical pumping transition probability A_{Pc}, what is ΔN?

   Next consider the *lasing stage.* In the lasing stage, $\Gamma_1 = 0.9$ and $\Gamma_2 = 0.99$. The ΔN obtained in part (b) is the initial population inversion for the lasing stage.
   c. Calculate the peak power radiated through mirror 1.
   d. Calculate the peak power radiated through mirror 2.
   e. Calculate the total energy per pulse radiated through the two mirrors.

17. The gain bandwidth of an HeNe laser operating at 0.633 μm is about 1500 MHz. The laser cavity is 2.0 m long.

a. Estimate the number of longitudinal modes supported by the laser.
b. If all longitudinal modes are locked to generate narrow pulses, what is the pulse width Δt_{FWHP} (in s)?
c. What is the time interval (in s) between pulses?

18. A CO_2 laser ($\lambda \approx 10.6\mu m$) has an optical cavity length of 1.5 m and the gain medium has a gain bandwidth of 800 MHz. When an individual longitudinal mode is excited, the power output is 0.1 W.
a. How many longitudinal modes can be sustained by the laser medium?
b. If all longitudinal modes are excited and mode locked, estimate the peak power output.
c. Estimate the full width between half-power points of the pulses.

19. Pulses from a mode locked laser have a pulse separation of 10 ns and a pulse width (Δt_{FWHP}) of 0.5 ns. Assume that all longitudinal modes have the same amplitudes.
a. How many longitudinal modes are locked?
b. What is the frequency difference between two neighboring longitudinal modes?

20. A CO_2 laser ($\lambda \approx 10.6\mu m$) has a long optical cavity and a sufficient gain bandwidth to support several longitudinal modes. When *each* longitudinal mode is excited individually, the power output is 0.1 W. When *all* longitudinal modes are excited and locked, the output pulses have a repetition rate of 100 MHz, and each pulse has a pulse width (Δt_{FWHP}) of 0.5 ns.
a. Calculate the cavity length (in m).
b. Calculate the gain bandwidth (in MHz) of the active medium.
c. Estimate the peak power of the mode locked pulses.

21. The output of a mode locked laser has a pulse separation of 12 ns, a pulse width (Δt_{FWNL}) of 2.0 ns, and a peak power of 0.8 W. Assume that all longitudinal modes have the same amplitude.
a. How many longitudinal modes are locked?
b. Calculate the frequency difference between two neighboring modes.
c. Suppose the cavity length is increased by 25 percent. Factors or parameters which might affect the bandwidth of the gain medium, or the mirrors, are not changed. Estimate the peak power radiated by the laser with the lengthened cavity.