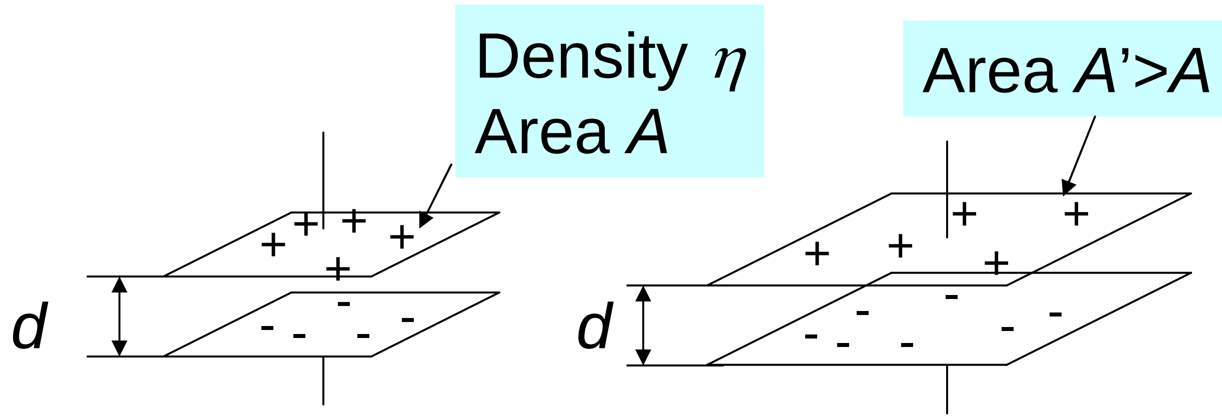
Lecture 10: Chapter 30, October 4 2005

- Calculating capacitance using steps discussed previously:

a) Parallel-Plate Capacitor
b) A Cylindrical Capacitor
c) A Spherical Capacitor

- Combination of capacitors
- The Energy Stored in a Capacitor
- Review problem for Quiz 5

$C = Q/\Delta V_c$



Which capacitance is larger?
Density η and separation d are the same, areas are different.

It will be shown that the structure of the expression for C:

$$C = \varepsilon_0 (Geometrical\_Factor)$$

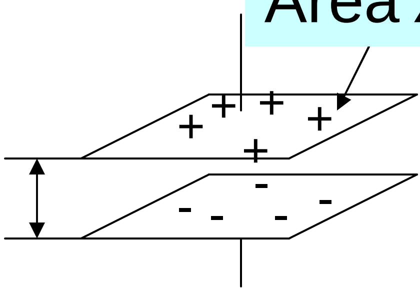
Different Geometries

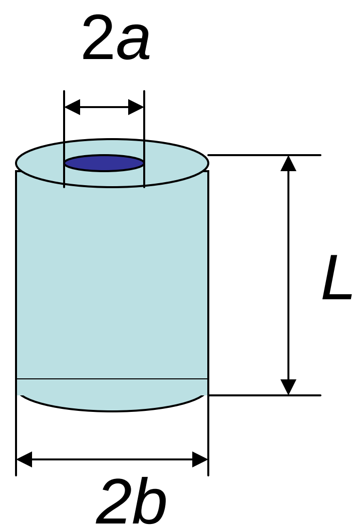
Parallel-Plate

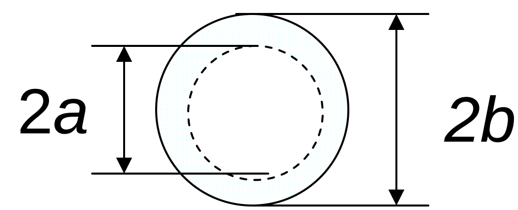
Cylindrical

Spherical

Density η
Area *A*







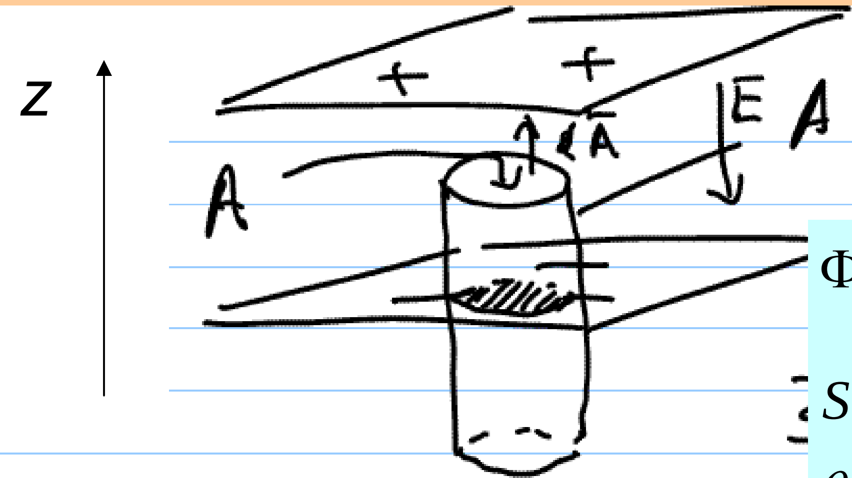
$$C = \varepsilon_0 \frac{A}{d}$$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Parallel-Plate Capacitor

- **First Step**: Assume Charge *Q*
- **Second Step**: Calculating *E* from *Q* using Gauss's Law



A cylinder as a Gaussian surface:

$$\Phi_e = \oint \vec{E}\, d\vec{A} = 2 \int_{Faces} E dA$$

Since on the faces:

$$\theta = 180^0 \Rightarrow \cos\theta = -1$$

$$\Phi_e = -2E(z)A$$

$$\Phi_e = \frac{Q_{enc}}{\varepsilon_0}, \quad \underbrace{Q_{enc} = \eta \cdot A}$$

$$-2E(z)\cdot \cancel{A} = \frac{\eta \cdot \cancel{A}}{\varepsilon_0} \Rightarrow \boxed{E(z) = -\frac{\eta}{2\varepsilon_0}}.$$

η — surface density of charge, $\eta = \frac{Q}{A}$

Electric Field (E_{net}) inside a capacitor:

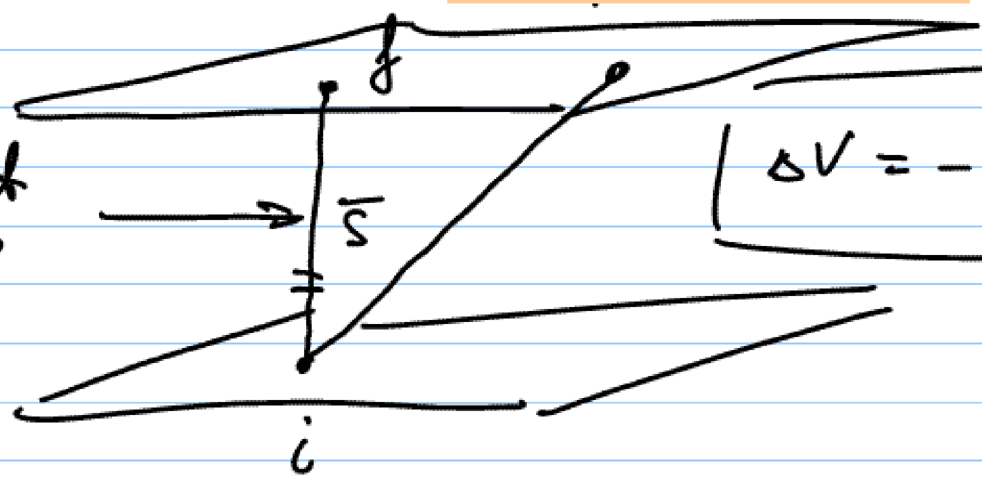
$$\boxed{E_{net} = E_+ + E_- = -\cancel{2}\frac{\eta}{\cancel{2}\varepsilon_0} = -\frac{\eta}{\varepsilon_0}.}$$

$$\boxed{E(\text{between 2 plates}) = -\frac{\eta}{\varepsilon_0}}$$

- **Third Step**: Finding ΔV using $\Delta V = -\int_i^f Eds$

Convenient path ⊥ to the plate

$\overline{E} \parallel d\overline{s}$



$$\boxed{\Delta V = -\int_i^f \overline{E}\cdot d\overline{s}}$$

$$\Delta V = -\int_i^f \left(-\frac{\eta}{\varepsilon_0}\right) \cdot dz =$$

$$= \frac{\eta}{\varepsilon_0} \int_i^f dz = \frac{\eta}{\varepsilon_0} (z_f - z_i) = \frac{\eta \cdot d}{\varepsilon_0}.$$

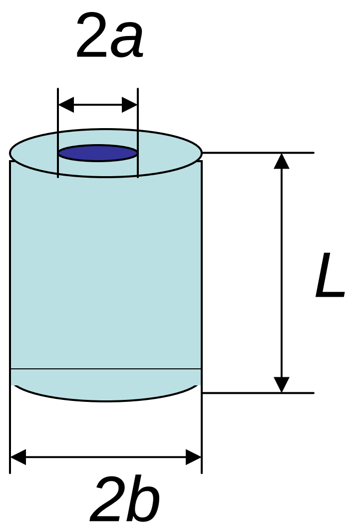
- **Last Step**: Calculate capacitance $C = Q/\Delta V$

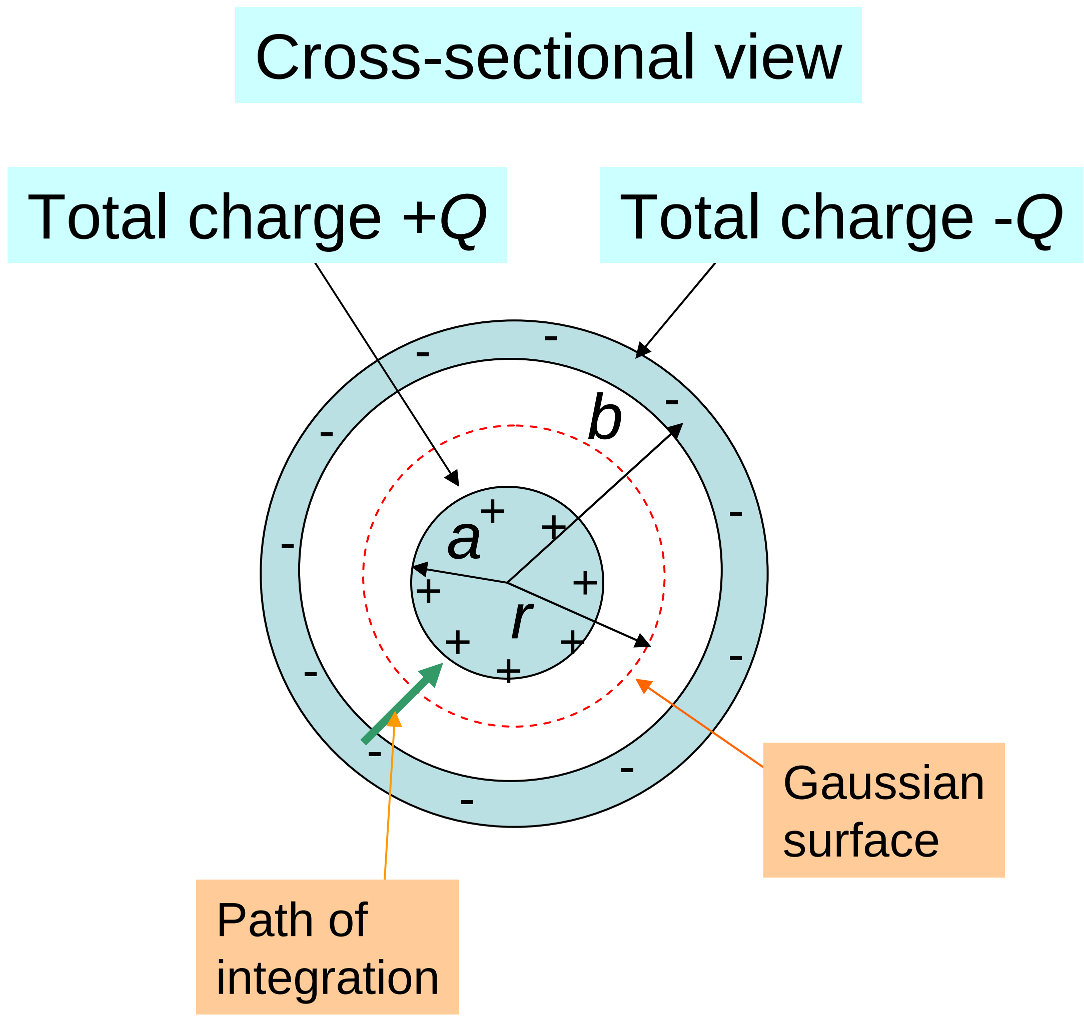
$$C := \frac{Q}{\Delta V} = \frac{Q \cdot \varepsilon_0}{\eta \cdot d} = \frac{\cancel{Q} \cdot \varepsilon_0 \cdot A}{\cancel{Q} \cdot d} = \boxed{\varepsilon_0 \frac{A}{d}},$$

$$\eta = \frac{Q}{A}$$

Cylindrical Capacitor

We did not solve the cylindrical geometry in the class, but I placed the solution below





- **First Step**: Assume Charge *Q*

- **Second Step**: Calculating *E* from *Q* using Gauss's Law.

As a Gaussian surface, we choose a cylinder of length *L* and radius *r*, closed by end caps.

$$\Phi_e = q_{enc} / \varepsilon_0$$

$$where\_\Phi_e = \oint \vec{E} d\vec{A} = E(2\pi r L)$$

$$q_{enc} = Q$$

$$E = \frac{Q}{2\pi\varepsilon_0 L r}$$

- **Third Step**: Finding ΔV using $\Delta V = -\int_i^f \vec{E} d\vec{s}$

We select the path from negative to the positive plate. For this path, the vectors ***E*** and ***ds*** will have opposite direction, so:

$$\vec{E} d\vec{s} = -Eds.\_Thus:$$

$$V = -\int_i^f \vec{E} d\vec{s} = \int_-^+ Eds =$$

$$-\frac{Q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{Q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

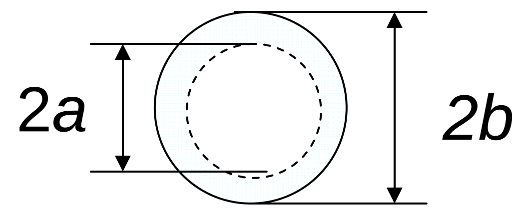
Where we used the fact that here *ds* = -*dr* (we integrated radially inward).

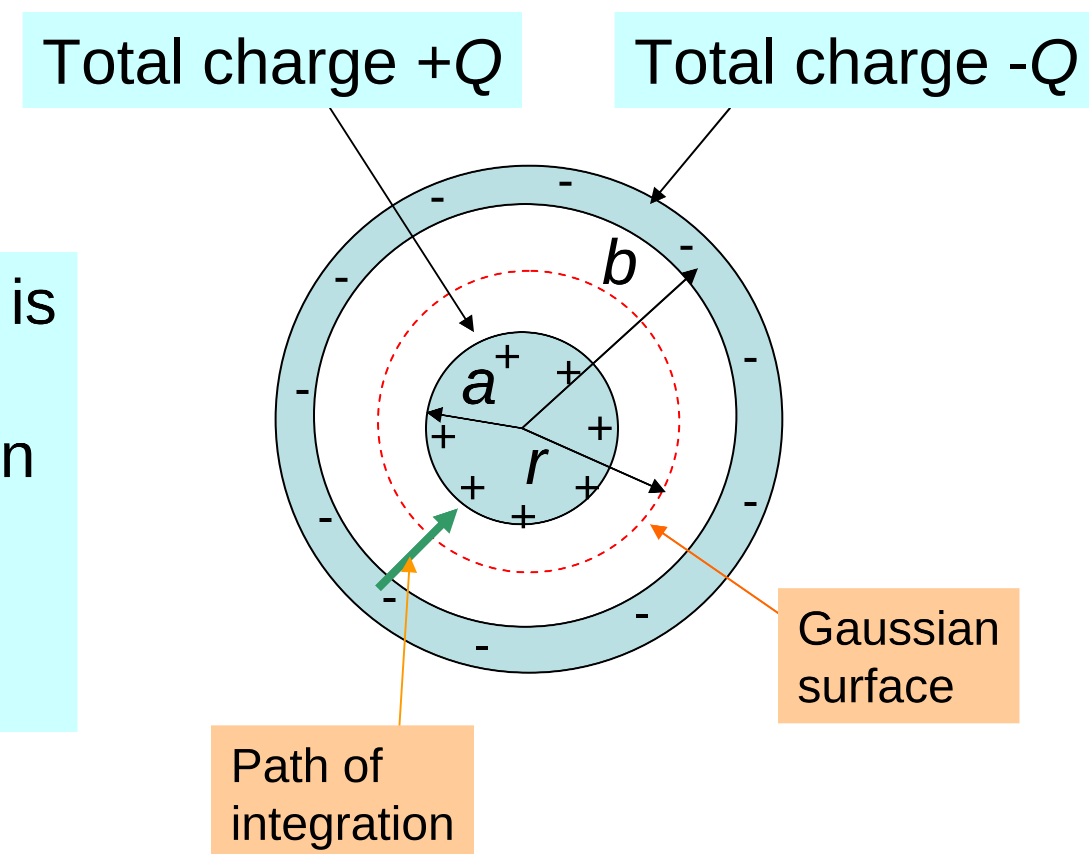
- **Last Step**: Calculate capacitance $C = Q/\Delta V$

$$C = 2\pi\varepsilon_0 \frac{L}{\ln(b/a)}$$

A Spherical Capacitor

- **First Step**: Assume Charge *Q*





The cross-sectional image is indistinguishable from the cylindrical case. However in 3-D case the spherical geometry is quite different from the cylindrical one.

Second Step: Calculating *E* from *Q* using Gauss's Law. As a Gaussian surface, we choose a sphere with radius *r*: $a < r < b$

$$\Phi_e = q_{enc} / \varepsilon_0$$

$$\Phi_e = \oint \vec{E} d\vec{A} = E(r) 4\pi r^2$$

$$q_{enc} = Q$$

$$E(r) 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$$

• **Third Step**: Finding ΔV using $\Delta V = -\int_i^f \vec{E} d\vec{s}$

$$V = \int_{-}^{+} Eds = -\frac{Q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} =$$

$$\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{Q}{4\pi\varepsilon_0}\frac{b-a}{ab}$$

Where again we substituted –*dr* for *ds*

• **Last Step**: Calculate capacitance $C = Q/\Delta V$

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

An Isolated Sphere

Let us first divide numerator and denominator by *b*:

$$C = 4\pi\varepsilon_0 \frac{a}{1 - a/b}$$

If we now let $b \to \infty$, external sphere infinitely expands leaving just one central sphere. Substituting *R* for *a*:

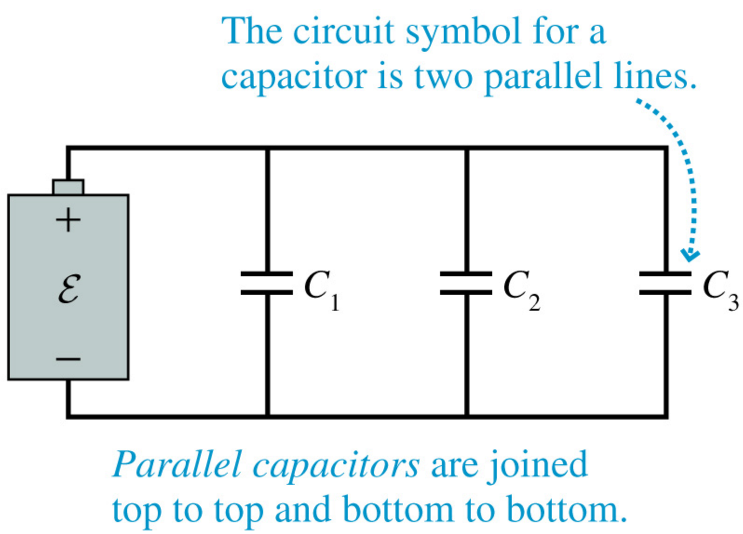
$$C = 4\pi\varepsilon_0 R$$

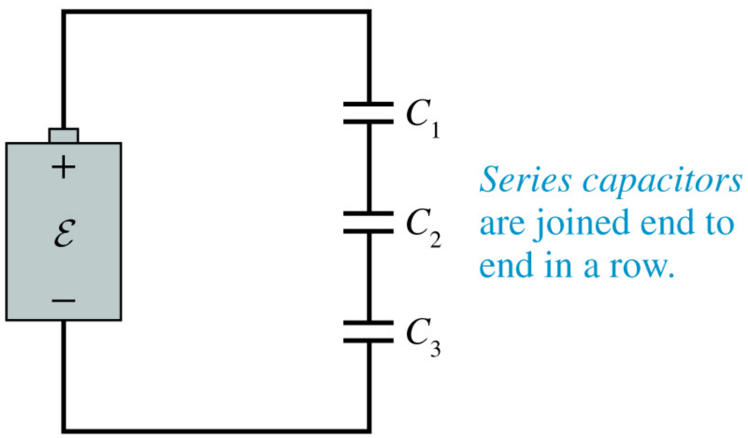
This is the capacitance of isolated single sphere

Combinations of Capacitors

Parallel Capacitors

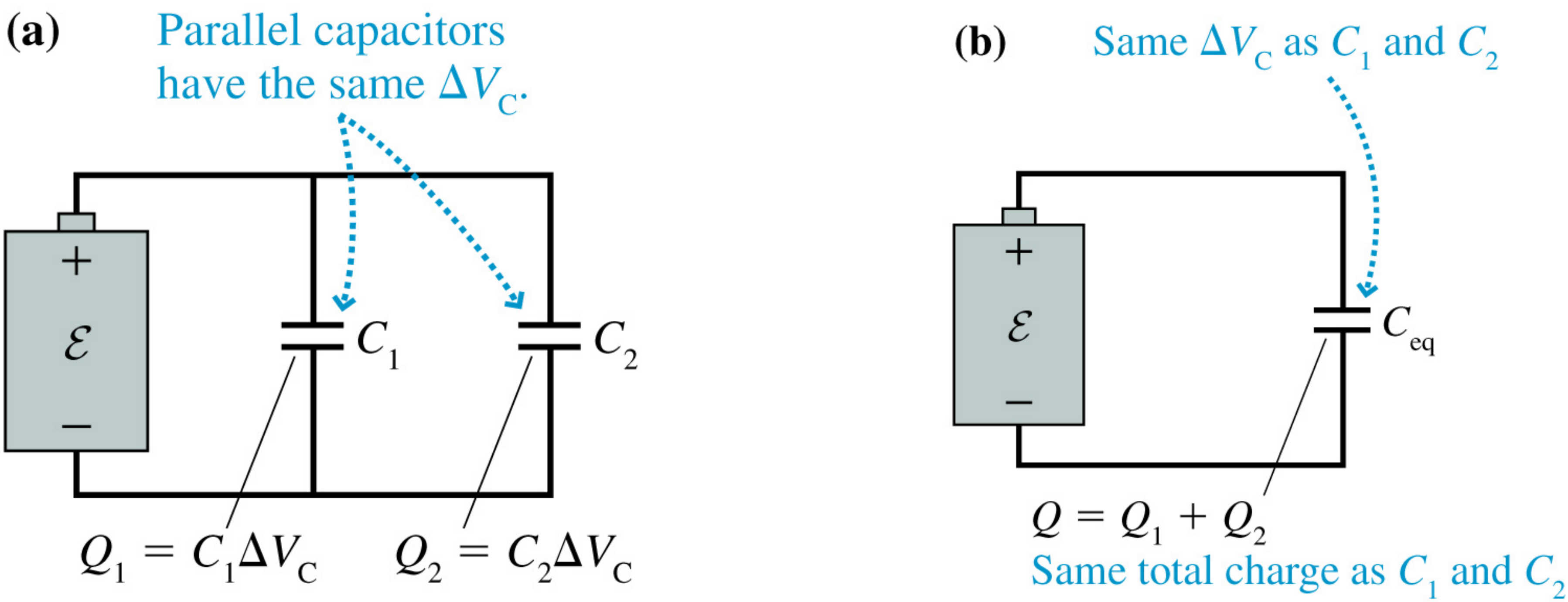
Series Capacitors





Replacing two parallel capacitors with an equivalent one

Equivalent capacitance is a *single* element which has same property, charge and potential as a pair of capacitors

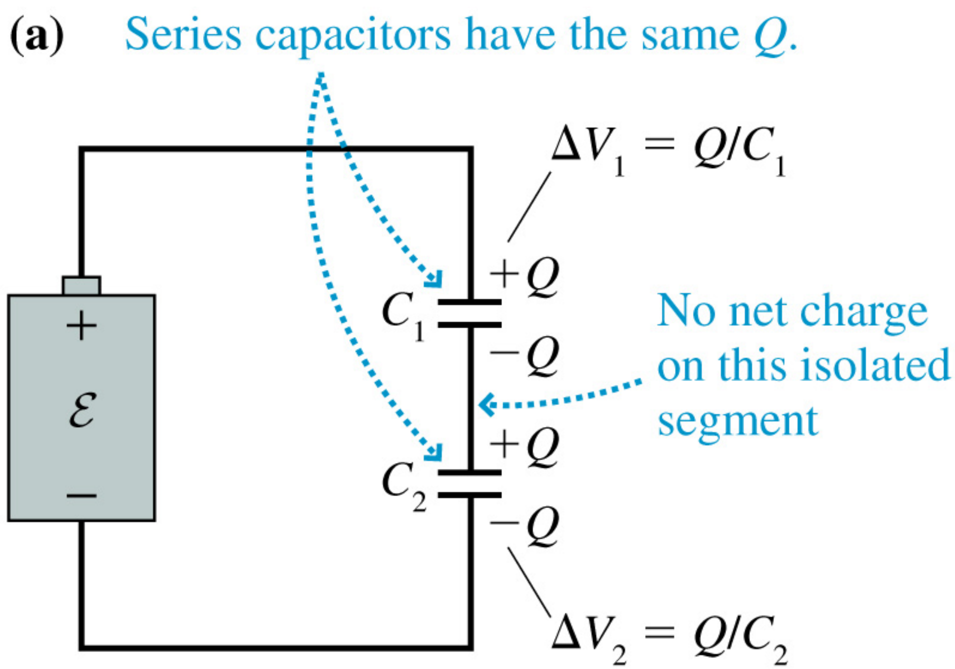


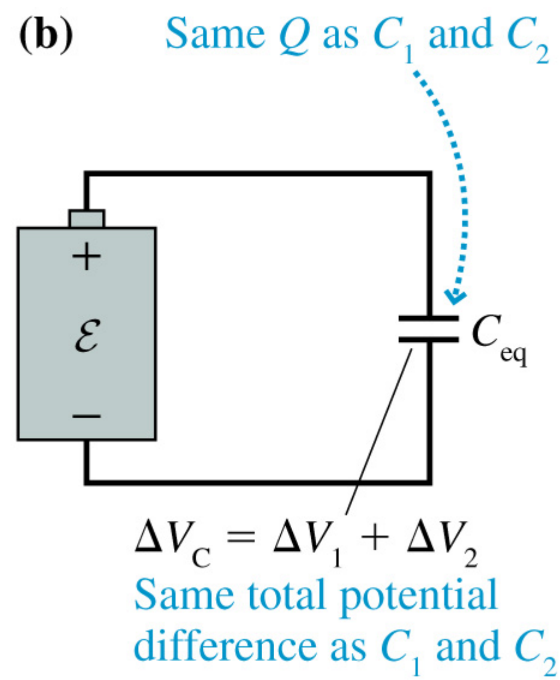
$C_{eq} = Q/(\Delta V_c) = (Q_1+Q_2)/\ \Delta V_c = C_1 + C_2$

$C_{eq} = C_1 + C_2 + \ldots$ (parallel capacitors)

Replacing two series capacitors with an equivalent one

In this case the magnitude of charges (Q) on all capacitors including equivalent one are equal – "chain" charging process.

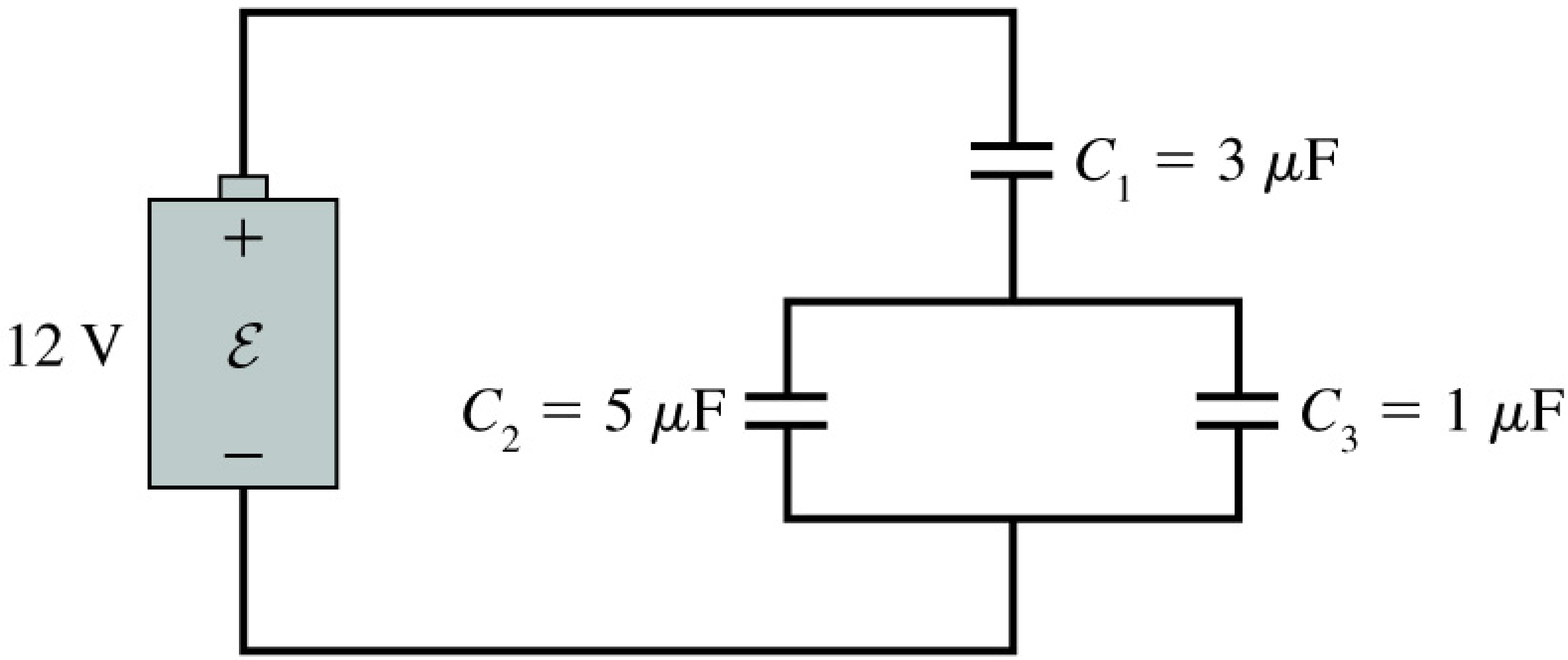




$$1/C_{eq} = \Delta V_c/Q = (\Delta V_1 + \Delta V_2)/Q = 1/C_1 + 1/C_2$$

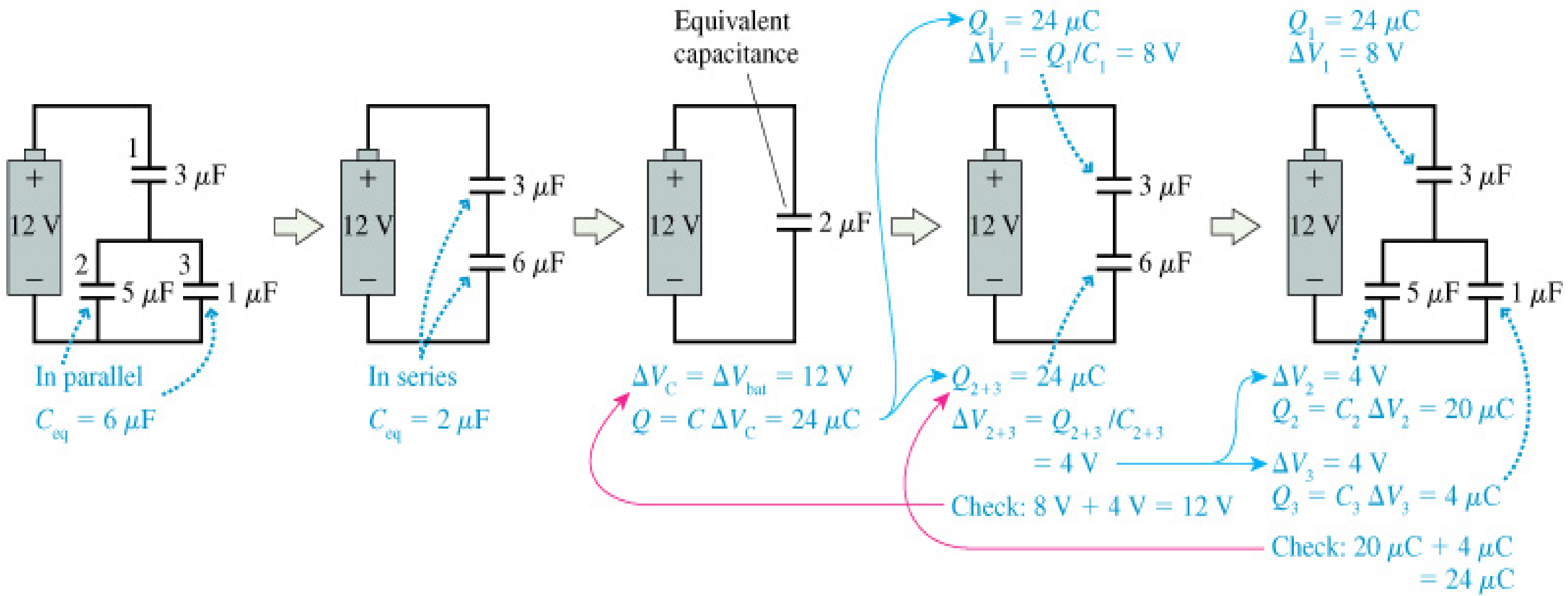
$$C_{eq} = (1/C_1 + 1/C_2 + \ldots)^{-1} \text{ (series capacitors)}$$

A Capacitor Circuit



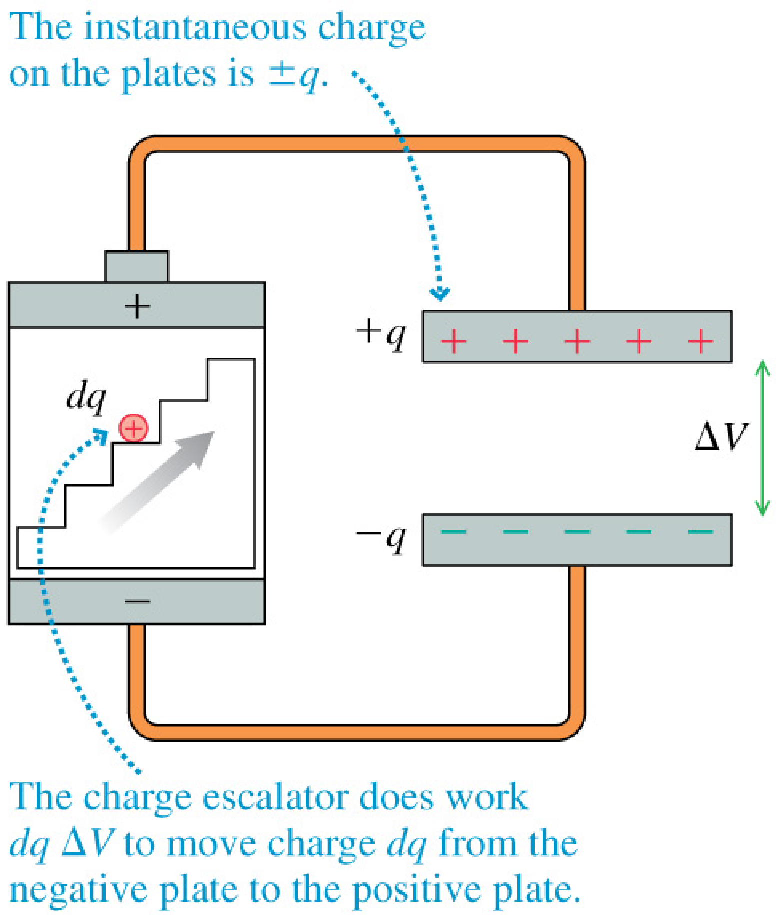
Typical question: to find charges and potentials across each capacitor

Analyzing the Capacitor Circuit



The Energy Stored in a Capacitor

Each little charge *dq* added to the charge *q*(*t*) on the capacitor increases its potential energy: $dU = dq\,\Delta V = q\,dq/C$



The energy stored in a charged capacitor:

$$U_c = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C} = \frac{1}{2}C\left(\Delta V_c\right)^2$$

Since $\Delta V_c = Q/C$

End of Lecture 10
Reading: Chapter 30
Review for Quiz 5
Home Work 5