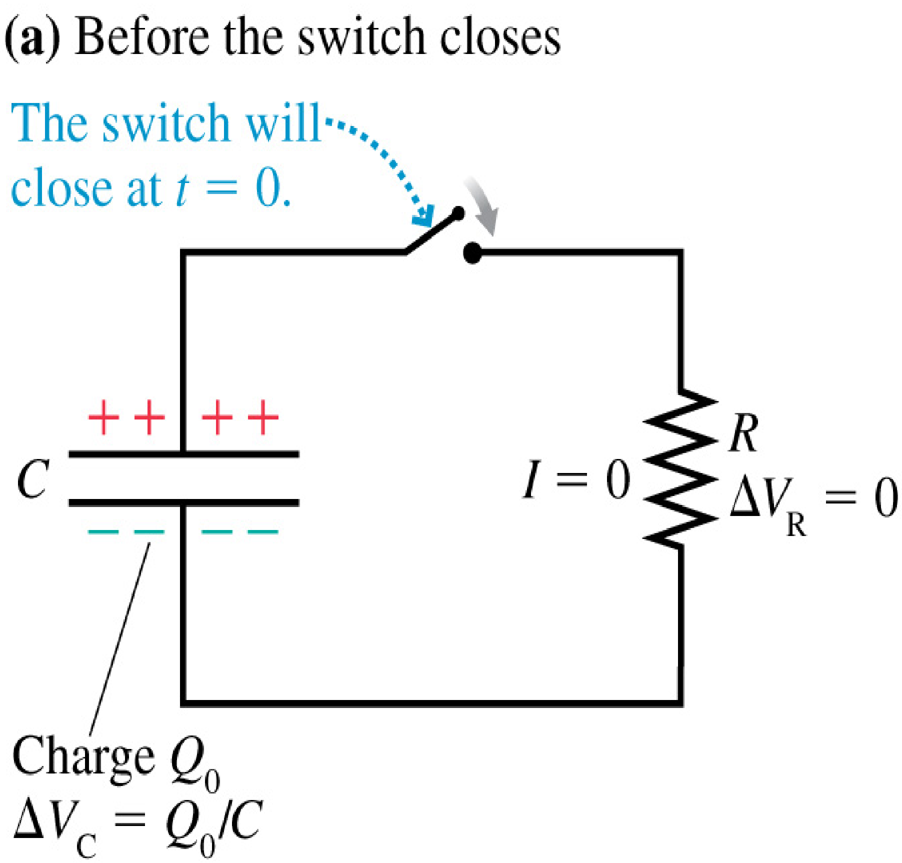
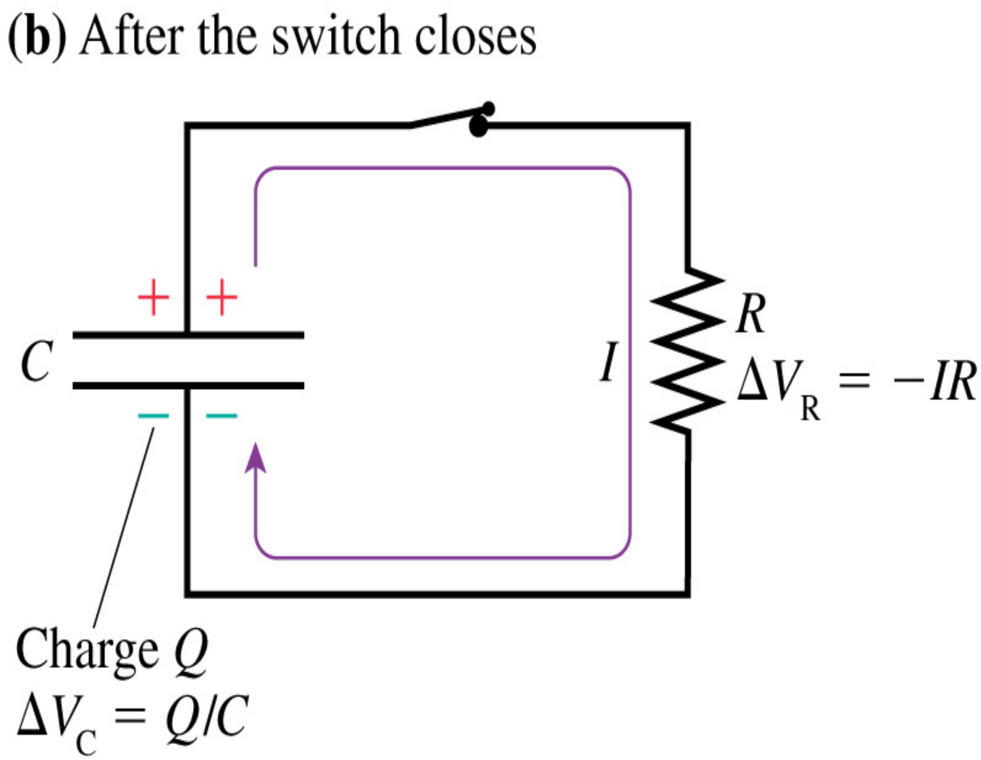
Lecture 13: Chapter 32, RC-circuit, October 18 2005





The capacitor is charged

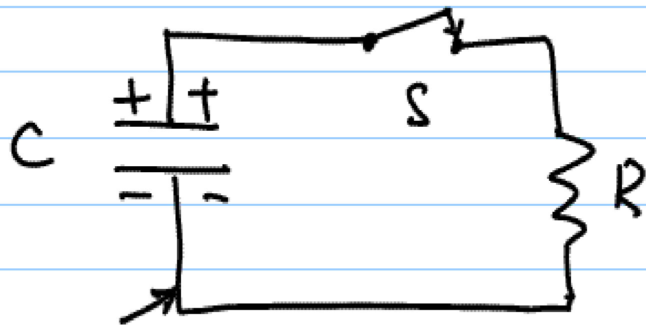
The capacitor is discharging

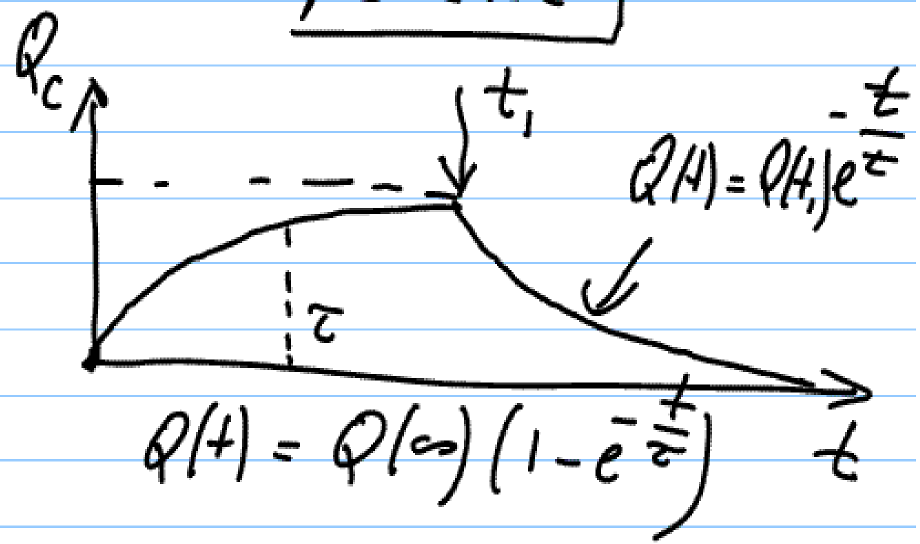
RC-circuit

Note Title 3/22/2005

Discharging capacitor

$\tau = RC$





Kirhoff's rule:

Loop rule

$$\Delta V_c - IR = 0$$

$$I = \frac{dq}{dt}$$

q — charge through R

$dq = -dQ$ $\quad Q$ — charge on the capacitor

$$I = -\frac{dQ}{dt}$$

$$C = \frac{Q}{\Delta V_c} \Rightarrow \Delta V_c = \frac{Q}{C}$$

$$\frac{Q}{C} + \frac{dQ}{dt} R = 0 \qquad \frac{dt}{C} + \frac{dQ}{Q} R = 0.$$

$$\frac{dQ}{Q} R = -\frac{dt}{C}$$

$$\frac{dQ}{Q} = -\frac{1}{RC} dt$$

$$\int_{Q_0(0)}^{Q(t)} \frac{dQ}{Q} = -\frac{1}{RC} t \Big|_0^t$$

$$\ln|Q| \Big|_{Q_0}^{Q(t)} = -\frac{t}{RC} \Rightarrow \ln|Q(t)| - \ln|Q_0| = -\frac{t}{RC}$$

$$\ln \frac{Q(t)}{Q_0} = -\frac{t}{RC}$$

$$\ln y$$

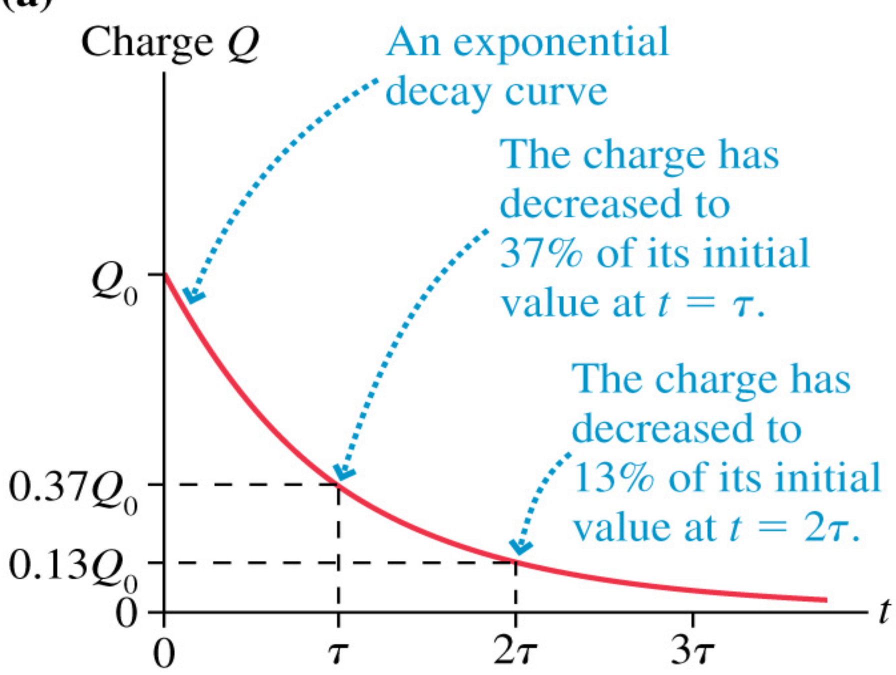
$$e^{\ln y} = y$$

$$e^{\ln \frac{Q(t)}{Q_0}} = e^{-\frac{t}{RC}}$$

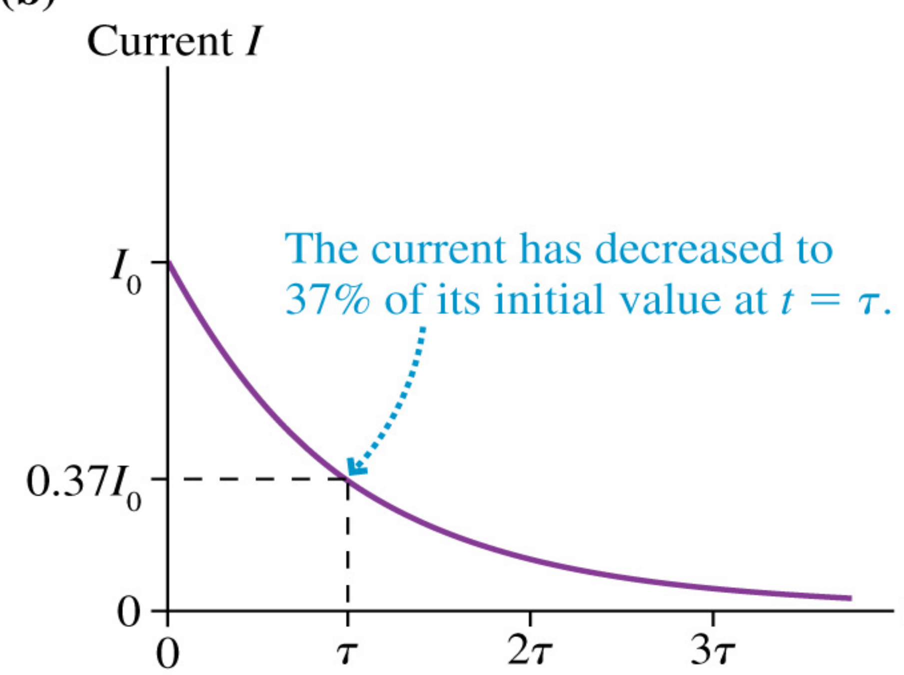
$$\tau = RC$$

$$Q(t) = Q_0 e^{-\frac{t}{RC}} = Q_0 e^{-\frac{t}{\tau}}$$

(a)

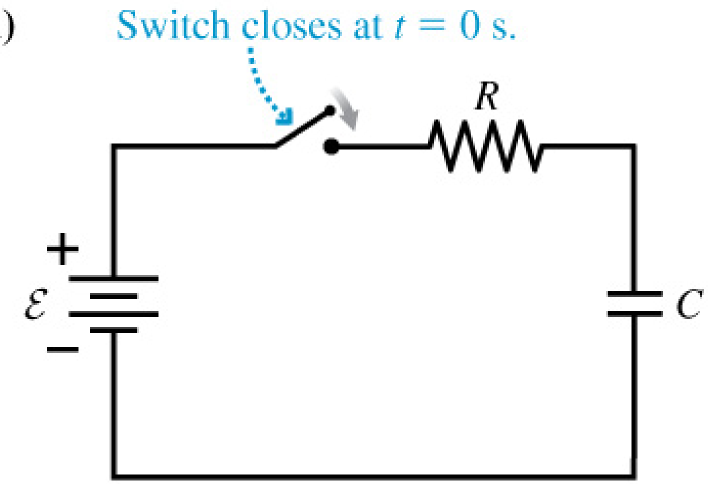


(b)

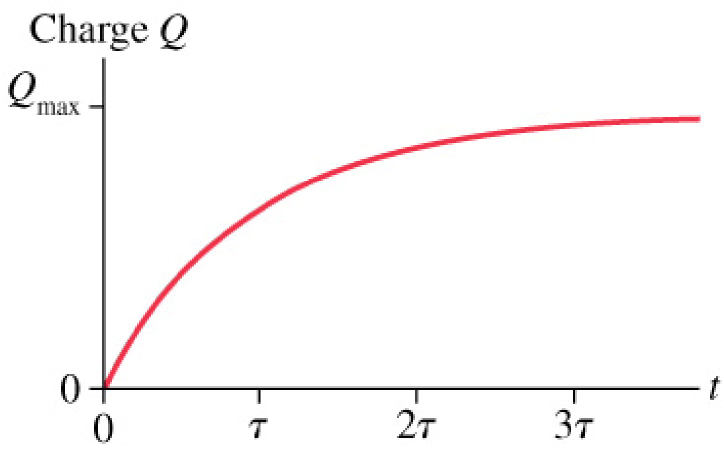


Charging Capacitor

(a)



(b)



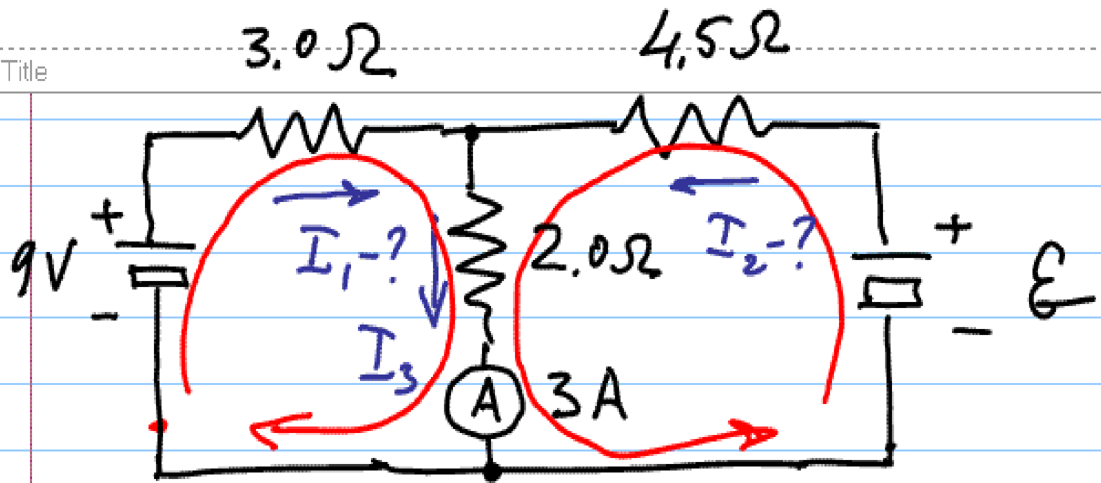
As a homework problem, you can show that the capacitor charge at time *t* is:

$$Q = Q_{max}(1 - e^{-t/\tau})$$

A Couple of Problems from Chapter 31

Problem 57 Chapter 31

Note Title 3/17/2005



$|I_3| = 3A$

Find $I_1, I_2, \mathcal{E}$

- Decide on direction of the currents
- Decide about direction of loops

Red color used for loops

$$+9V - I_1 \cdot 3 - 3 \cdot 2 = 0 \Rightarrow 3I_1 = 9 - 6 = 3 \Rightarrow I_1 = 1A$$

$$+\mathcal{E} - I_2 \cdot 4.5 - 3 \cdot 2 = 0$$

$\mathcal{E}$ – not known

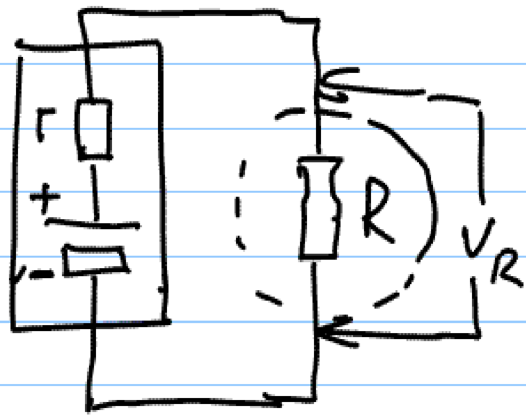
I_2 – not known

$$I_3 = I_1 + I_2 \Rightarrow I_2 = I_3 - I_1 = 3 - 1 = 2A$$

$$\mathcal{E} = 6 + I_2 \cdot 4.5 = 6 + 9 = 15V$$

Problem 56 Chapter 31

Note Title 3/21/2005

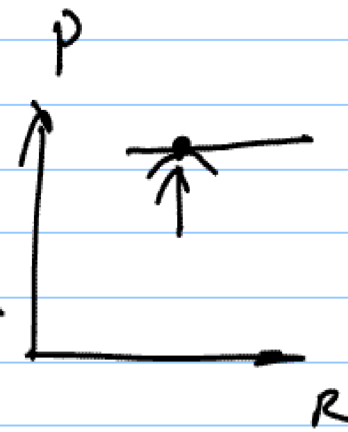


$$P_R = I \cdot V_R = I^2 \cdot R = \frac{V_R^2}{R}$$

$$V_R = I \cdot R$$

$$I = \frac{V_R}{R}$$

$$I = \frac{\mathcal{E}}{r+R}$$



$$P_R = \frac{\mathcal{E}^2 \cdot R}{(r+R)^2} = \underbrace{\left(\mathcal{E}^2 \cdot R\right)}_{u} \underbrace{\frac{1}{(r+R)^2}}_{v} = u \cdot v$$

$$\frac{\partial P_R}{\partial R} = 0$$

$$P' = u' \cdot v + v' \cdot u$$

$$\frac{\partial P}{\partial R} = \frac{\partial u}{\partial R} \cdot v + \frac{\partial v}{\partial R} \cdot u$$

$$\frac{\partial P_R}{\partial R} = \underbrace{\varepsilon^2}_{\frac{\partial u}{\partial R}} \frac{1}{(r+R^2)^2} + \frac{(-2)\cdot\varepsilon^2\cdot R}{\underbrace{(r+R)^3}_{\frac{\partial v}{\partial R}}} =$$

$$= \frac{\varepsilon^2(r+R) - 2\varepsilon^2 R}{(r+R)^3} = \frac{\varepsilon^2[r+R-2R]}{(r+R)^3} =$$

$$= \frac{\varepsilon^2[r-R]}{(r+R)^3} = 0 \qquad \boxed{r = R}$$

End of Lecture 13
Reading: Chapter 31
Review for Quiz 6
HW7